A Tale of Two Temperatures

Concepts

- Measurement
- Data Collection
- Data Analysis
- Algebra
- Physical Science

Materials

- TI-73 Explorer™
- CBL 2™
- Two temperature sensors
- Unit-to-unit cable
- A cool and a warm liquid
- *DataMate* APP for TI-73
- Tape
- TI Connectivity USB Cable (Optional)

Overview

Students will collect data for changing temperature, display it in a plot, and create a model for the relationships between the units of Fahrenheit and Celsius. The slope and *y*-intercept of the model will be examined to underscore the conversion from one measure to the other. Additionally, students will collect data in units of Fahrenheit and Celsius over a range of values and will create a conversion model for this data, exploring the relationship between the units of one measure to the other. They will analyze the measures made by the sensors and the CBL 2™ device as related to the True conversion formulas.

Introduction

The need to convert between the temperature measures of Fahrenheit and Celsius becomes more important as American citizens have become more involved in the study of science and with the rest of the world.

The equations for converting

$$C^\square = \frac{5}{9}\left(F^\square - 32\right) \qquad F^\square = \frac{9}{5}C^\square + 32$$

are often hard to use and harder to remember. Most people have points of conversions to use as a basis for approximating the measures in other units. For example, 20 degrees Celsius is 68 degrees Fahrenheit, so if one observes a temperature of 70 degrees Fahrenheit, then they can figure that it is more than 20 degrees Celsius, maybe 21 degrees. This method is not exact and could lead to some problems if the magnitudes of the temperatures are important to the situation.

To develop a feel for the values of temperature in both Fahrenheit and Celsius, you will collect a range of temperatures in both measurement units and then make comparisons of the two.

The Problem

The Celsius scale is set up based on the boiling point of water being 100 and the freezing point of water being 0. So there are 100 degrees of Celsius between these two critical points.

In the Fahrenheit scale, the boiling point of water is set at 212 and the freezing point at 32, so you have 180 degrees of Fahrenheit between these points. This is a ratio of 180 to 100 or 100 to 180 depending on your perspective. In addition, the starting point is shifted by 32 degrees. These factors make for a complicated and therefore sometimes inaccessible conversion formula.

In this investigation, you will develop the conversion formula for changing from one unit of measure for temperature to the other. To do this, use two temperature sensors with the CBL 2™, and collect the pairs of temperatures (Fahrenheit and Celsius) over a range of values. To do this, two things are important:

- The need to keep the sensors close to each other so that they will be measuring the same "part" of the liquid, and
- The need to cover a range of temperatures from cold to hot, so that you will have a large variance of conversion points.

Collecting Data

1. Create two containers of water, one hot (close to boiling) and one cold (close to freezing).
   - Be careful that the hot container does not spill or splash on others and burn them.

2. Place the two temperature sensors in the CBL 2™, one in Channel 1 and the second in Channel 2.

3. Tape the two sensors together along the white insulated connectors, about 4 centimeters from the ends of the sensor so that either the black tips, if you have older sensors, or the ends of the stainless steel sensors are side by side.

4. Place the two sensors in the cold solution, and hook up the TI-73 and the CBL 2™ as shown in Figure 1.

5. Load the *DataMate* application onto your handheld.
   - This may be transferred from another handheld, the CBL 2™ or downloaded from a computer using the TI Connectivity Cable USB.
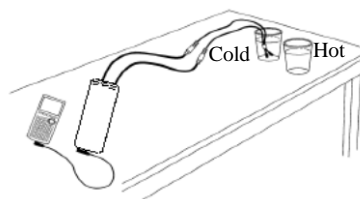

Figure 1

6. Once the two sensors have been in the cold liquid for a few minutes and they are close to the temperature of the liquid, run the *DataMate* application.

7. Press the 9 key, scroll to the *DataMate* application, and press β (Figure 2).
   - If the sensors are not recognized, or if there are residual setting from past experiments, press : on the TI-73 to reset the CBL 2™ (Figure 3).

8. You will need to set one of the temperature sensors for Fahrenheit and the other for Celsius.
   - To make changes to one of the sensors, press Ψ to access the **1: SETUP** menu (See Figure 3).
   - You will want to change the first sensor to Fahrenheit (Figure 4).

   - While the arrow is on CH1, press β, and then press Ψ (Figure 5).

   - In our case, we now want to press P to select the stainless temperature sensor for Fahrenheit (Figure 6). You will now see the sensor in Channel 1 change from Celsius to Fahrenheit.

   - To accept the changes, press Ψ (Figure 7).
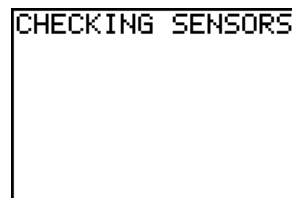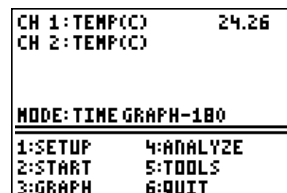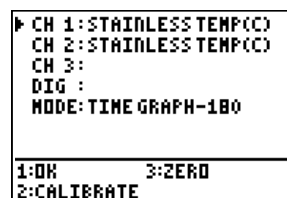
Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7

9. At this point, we need to adjust the time for data collection.

    - The time that the CBL 2™ will use is given next to the **MODE:** setting on the screen.

    - In our Figures, the setting in current use is 180 seconds (See Figure 8). Since all we want in this experiment is enough variety of temperatures in going from a cold liquid to a hot liquid, we can use a 30 second interval instead.

    - To change this setting, press Ψ again to get to the **SETUP** menu, scroll down to the **MODE** line, and press β (Figure 8).

    - Now press Z to select the **TIME GRAPH** option (Figure 9). The screen will now display the current settings.

    - To change the time settings, press Z again, and enter the desired settings (Figure 10). In this case, we will choose 0.5 as the time between samples and 60 as the number of samples (Figure 11). This will give an overall time of 30 seconds for the experiment.
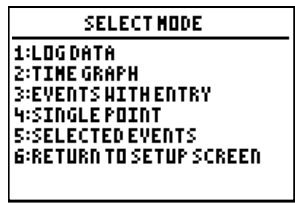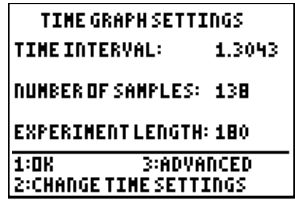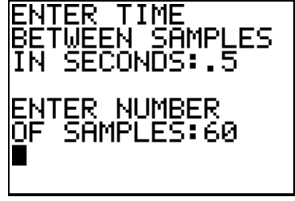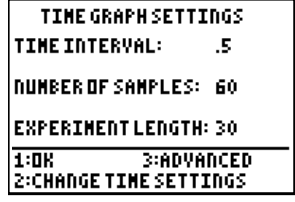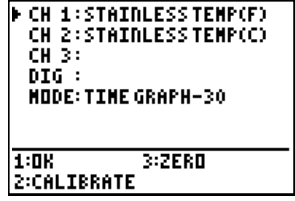
    - When the settings have been entered, press Ψ for **OK**. You will now be given a display of the overall experiment settings (Figure 12). When you agree with the settings, press Ψ for **OK** again (Figure 13).

10. To begin data collection, press Z.

11. Once you start collecting data, quickly move the two sensors to the hot water and let them rest in that container for the rest of the experiment.

Figure 8

Figure 9

Figure 10

Figure 11

Figure 12

Figure 13

12. When the CBL 2™ beeps, the SELECT THE PLOT menu will appear.

    - Since we are interested in a relationship between Fahrenheit and Celsius, select **CH2 VS. CH1** (Figure 14).

    - Explore the plots. If the graph does not give a satisfactory plot (no straight-line graph for Fahrenheit vs. Celsius, see Figure 15), press β from the graphing screen, and then Ψ to get back to the MAIN SCREEN.

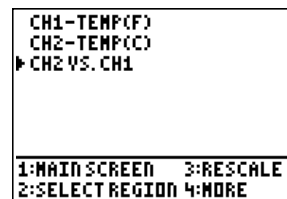   - Set up the experiment again, and repeat the process by first pressing Z to start data collection.

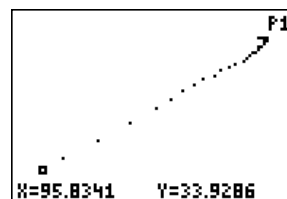## Displaying Data in a Plot

1. Exit the *DataMate* application, if you have not already done so.

    - Select 6: QUIT from the MAIN SCREEN.

2. Look at the lists of data by pressing 3 (Figure 16).

    - Since it may be difficult to remember that time is given in list L1, temperature (˚F) in list L2, and temperature (˚C) in list L3, we will store the data in lists called FDEG, and CDEG.

    - To store list L2 in **FDEG** and L3 in list **CDEG**, we will first move our cursor to the next clear space to create new lists of Fahrenheit and Celsius degrees measured by the sensors (Figure 17).

    - Press − τ to get to the text editor (Figure 18).

    - Select the letters to spell the list name **FDEG**, and then select the word 'Done' and press β to return to the list editor.

    - Repeat this process for **CDEG** (Figure 19).

Figure 14

Figure 15

Figure 16

Figure 17

Figure 18

Figure 19

3. To copy the lists L2 and L3 into **FDEG** and **CDEG** respectively, go to the top of each list until the list name appears at the bottom of the screen.

   - Press – ϖ to get a list of all list names in the handheld.

   - Select L2 to place in **FDEG** and L3 to place in **CDEG** (See Figures 20 to 23).

4. Examine the plot of Celsius vs. Fahrenheit again by setting up the plot as shown below.

   - Press – ε, and make sure that all the plots (except maybe Plot1) are Off.

     - Select 1:Plot1 from the PLOT menu, and set up a scatterplot.
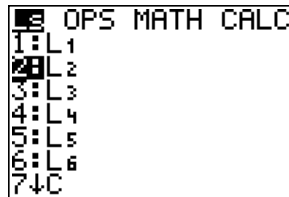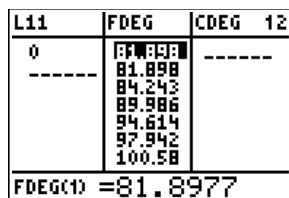
Figure 20
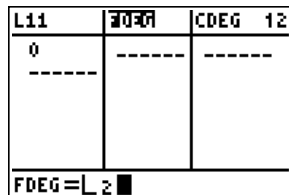
Figure 21

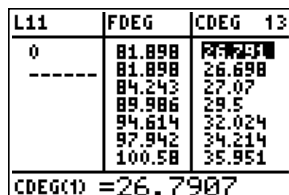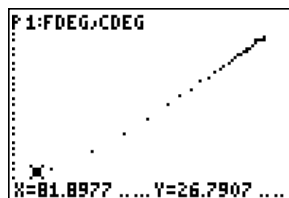**Figure 22**

Figure 23

   - Press (, and select option 7:ZoomStat from the ZOOM menu to explore the data on the plot (Figure 24).

5. Change your Mode setting to display 4 digits by pressing . and selecting 4.

Figure 24

Creating a Model

Go to the Home screen to do a Manual-Fit on this data so that you have a formula (equation) that will model the relationship between Fahrenheit and Celsius degrees based on your data.

1. Press : : and then – ϖ !.

2. Select option 3:Manual-Fit from the CALC menu (Figure 25).

Figure 25

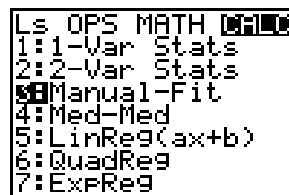3. From the Home screen, press − }, and select option 2:Y-Vars from the VARS menu (Figure 26).

Figure 26

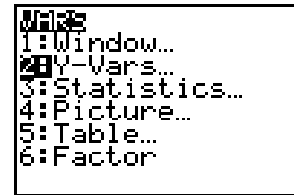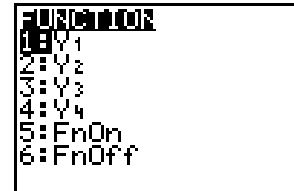4. Select option 1:Y₁ from the FUNCTION menu (Figure 27).

Figure 27

5. Press β from the Home screen to start the process of setting the line of best fit (Figure 28).

6. Move your cursor to the left side of the plot, and select a starting point for the Manual-Fit line.
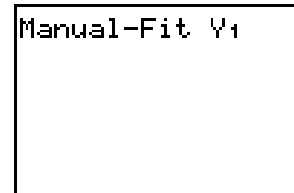
Figure 28

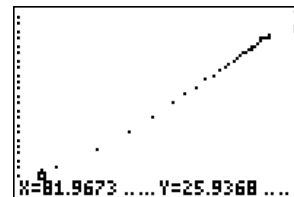7. Press β when you are satisfied with the start (Figure 29).

X=81.9673 ..... Y=25.9368 ....

Figure 29

8. Move to the right side of the plot, and select an ending point for this new line (Figure 30).

X=106.6022 ... Y=34.5013 ....
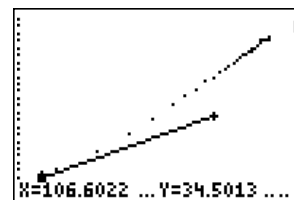
Figure 30

9. Press β again to tag this point (Figure 31).

10. If this is the line that you want, press β again, or move the line with the cursor keys until you have what you want.
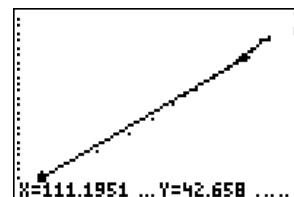
X=111.1951 ... Y=42.658 .....

Figure 31

11. Press β to save the formula and draw the line (Figure 32).

12. Press & to see the function (Figure 33).

This line looks like it reflects what is shown in the data, but it is just our "sight" that says the fit is a good one. You need a more analytical (numerical) statement of the closeness of the model to the data.

One way to get this information would be to look at how far off the numbers you collected with the sensor measuring Celsius are from those predicted by the model equation in Y1.

1. To set up this list, go to the list editor by pressing 3.
   - Move to the next clear space to create a new list of Celsius degrees (from the model) for the Fahrenheit degrees measured by the other sensor (Figure 34).
   - Press − τ to get to the text editor.
   - Select the letters to spell the list name for the model, and then select the word 'Done' to return to the list editor.

2. Now you need to have the function Y1 calculate the values of Celsius for the given values of Fahrenheit in the list **FDEG**.
   - Press β while still in the heading of the list named **MODEL**, and then key in the expression to fill the list (Figure 35).
   - Press − }, and select option 2:Y-Vars from the VARS menu.
   - Select option 1:Y1 from the FUNCTION menu, and press Δ (Figure 36).

   - You want to evaluate this function at the values in list **FDEG**, so press − ϖ, and select the list **FDEG** from the Ls menu (Figure 37).

   - Press E β to close the expression and to calculate and

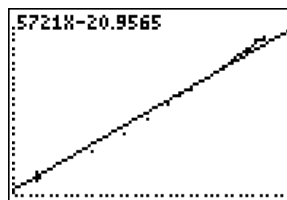**Figure 32**

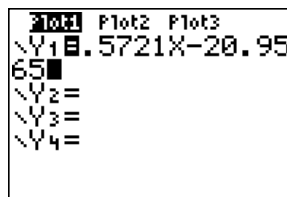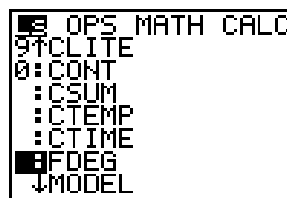**Figure 33**

Figure 34

Figure 35

Figure 36

Figure 37

fill the list (Figures 38 and 39). Examine this list and the list **CDEG** by moving up and down the list.

3. To get a list of values showing how far the model is off from the measured data, create a list called ERROR.

   - Move to the top of the next open list, and key in the name as before.

   - In this list, place the difference from the measured and the model. At the heading of the list named ERROR, press β.

   - Select the list names from the list of lists by pressing – ϖ and finding the names in the LS menu.

   - Press the subtract symbol T between the list names (Figure 40).

   - When finished, press β to fill the list (Figure 41). Use the # and ∃ keys to move up and down this list to see how much difference there are in the numbers.

4. To get a better feel for this ERROR, set up a plot of the data showing how far off each value was for every value of Fahrenheit.

   - Press – ε, and turn Plot1 off.

   - Set up Plot2 as shown in Figure 42.

   - Press &, move the cursor to the = sign for Y1, and press β to deselect it.

   - Press (, and select option 7:ZoomStat from the ZOOM menu.

     -) on the plot, examining the data. Notice that in the sample data in Figure 43, the maximum error (farthest the error gets from the horizontal axis) is about 1.15 degrees Celsius at 94.6 degrees Fahrenheit.

   - Find the approximate value of your maximum error in the list, and calculate the percent error for that value.

Figure 38

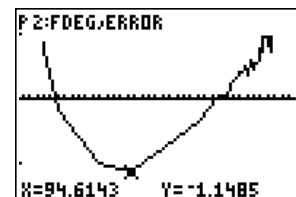**Figure 39**

Figure 40

Figure 41

Figure 42

Figure 43

In this case, at approximately 94.6 degrees Fahrenheit, the measured value of Celsius is 32.0 degrees and the calculated value is 33.2 degrees (Figure 44).

- This difference of 1.2 degrees is about 3.75% (Figure 45).

Figure 44

Figure 45

Analyzing the Measured Data

Now you can view the Truth!

1. Move to the Y= editor by pressing &.

2. Highlight the = sign by moving your cursor to the = sign in Y1, and pressing β.

3. Turn Plot1...On and Plot2...Off by moving your cursor to the words at the top of the screen and pressing β. The plot is on when it is highlighted.

4. Add a new expression to the Y2 line. Press I – □, and select option 5:Temp from the CONVERSIONS menu (Figures 46 and 47).

Figure 46

Figure 47

5. Select option 2:degF from the TEMP menu and then option 1:degC (Figures 48 and 49).

6. Set up the window so that you can see the graph of the Manual-Fit formula (Y1), with the set of collected values (Plot1) and the exact conversion (Y2).
   - Press (, and select option 7:ZoomStat from the ZOOM menu.

Figure 48

   - Press), and explore the graph, using the # and ∃ keys to switch between the three items (Figures 50 to 52).

Figure 49

- Trace the plot, and identify a value to explore in the assigned region.
- Switch the trace to one of the functions, and key in the number for the desired X (Fahrenheit) value.
- Repeat the process for the other function.

7. To continue to explore the model and the Truth, set up the table starting at the minimum temperature and exploring the values by one-degree-Fahrenheit steps.
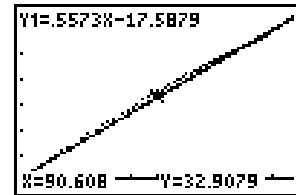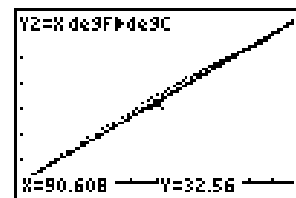
- Press – φ, and set up the screen as shown in Figure 53.

- Press – ι, and explore the values (Figure 54).

- To have greater control on the TABLE option, you can set the window as shown in Figure 55 so that you only get the values that are asked for (in terms of X).

- Press – φ, and set up as shown in Figures 56 and 57.
- Return to the table by pressing – ι as before.

Figure 50

Figure 51

Figure 52

Figure 53

Figure 54

Figure 55

The real way to look at how close your model is to the correct relationship between Fahrenheit and Celsius temperatures is to look at the percent error. You can find this using the following formula where the Model values come from Y1 and the Truth comes from Y2:

$$\frac{|Model - Truth|}{Truth} * 100$$

Figure 56

Figure 57

8. To have the TI-73 do this calculation, enter the formula above into Y3.
   - Press &, turn off the first two functions as before, and move down to Y3.
   - Press 1 ∀ and move down to select the 1:abs(option (Figure 58).
   - Press − }, and select Y1 as before.
   - Press T, and then repeat the previous step to select Y2.
   - Close the parentheses, set up a division (EΦ), and select Y2 again.
   - Finish the expression by pressing M Ψ _ _ (Figure 59).

Figure 58

Figure 59

9. To look at the variation of the values of Celsius temperature generated over the range of Fahrenheit degree measures, you will need to set the Window up to cover that territory.
   - Press ɜ, and move to the Xmin option.
   - Press − ϖ ∀ ∀ to move to the MATH menu. Select option 1:min(.
   - Select the name of the list that contains the measures in Fahrenheit degrees by pressing − ϖ, and selecting from the LS menu.
   - Press β to have the smallest value of the list **FDEG** placed in Xmin (Figure 60).
   - Repeat the process, using the maximum value for Xmax (Figure 61).

Figure 60

Figure 61

   - Figure 62 shows the calculated values.

10. You can let the TI-73 set your values for Ymin and Ymax.

- Press (, and select option 9:ZoomFit from the ZOOM menu.
- Press)□ and explore the percent error values for each Fahrenheit temperature reported in this range (Figure 63).

Figure 62

11. To compare the model that you have with the True relationship stated at the start of this investigation, you will need to bring the information from Y1 to the Home screen.
    - From the Home screen, press : :.
    - Recall the information stored in Y1 by pressing – Π, and – }.
    - Select option 2:Y-Vars from the VARS menu.
    - Select option 1:Y1 from the FUNCTION menu. Press ⊆ (Figure 64).

Figure 63

12. Now examine the parts of the conversion equation below.

$$C^{\square}_{\square\square} = \frac{5}{9}\left(F^{\square} - 32\right)$$

- The 5/9 is multiplied by the measure of temperature in Fahrenheit and by 32.
- Look at the decimal value of 5/9 by pressing P = Λ β > β (Figure 66).
- Compare this value to the value before the X in the Y1 equation.

Note: You may need to adjust the FLOAT line in your MODE settings if the numbers of digits shown are not the same.

Figure 64

Figure 65

13. Now examine the other value in the expression (-20.9565). This comes from multiplying 5/9 * 32.
    - Key in 5/9 as before. Before you press β, press ∀ M α [Z β (Figure 67).
    - Convert to a decimal with the > β sequence.
    - In the formula in Y1, each of the numbers has units.

Figure 66

Figure 67

Going Further

1. What is the ratio of 180 to 100, and 100 to 180, in fractional form? (Use improper fractions if needed.)

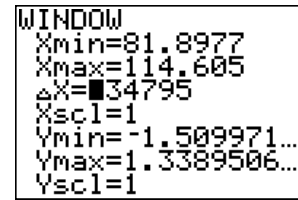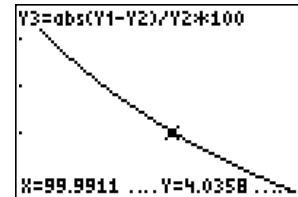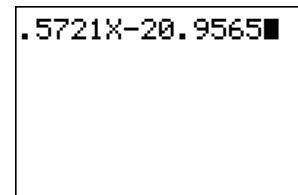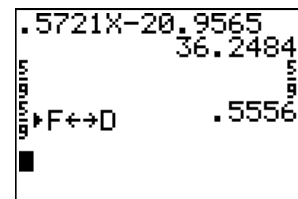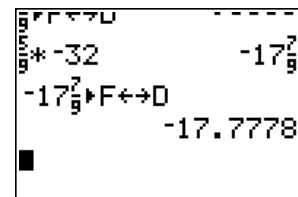How do these appear in the two conversion formulas for Fahrenheit and Celsius?

2. Give the model for converting from Celsius to Fahrenheit, using the Manual Fit and the data from your experiment.

3. Compare the values from questions 22 (5/9) and 23 (5/9*-32) on the student data sheet, calculating the percent error.

4. What is the meaning of the 9/5 in the formula to convert from degrees Celsius to degrees Fahrenheit?

5. How big is a degree Celsius in terms of Fahrenheit degrees?

6. Why do you calculate percent error?

For a classroom-ready format of this activity, including student handouts and worksheets, visit the Activities Exchange online using the following address: http://education.ti.com/educationportal/activityexchange/activity_list.do?cid=us. Search for 'tale two temperatures.'

Teacher Notes

Classroom Management and Safety

The sensors should be taped together so they are close to

the same point in the liquid, measuring the same temperature. The cool liquid should be close to freezing, while the warm liquid should be close to boiling.

Care should be taken to let the sensors come to the temperature of the cool liquid before starting the experiment. The warm liquid should be monitored so that students don't accidentally knock it over and burn themselves.

The method of heating the liquid should be monitored closely. Using very hot tap water might work fine.

The Set Up

1. The greater the difference in the temperatures collected from start to finish, the better the fit will be. Ice water and boiling water will give the best results.

2. Make sure the sensors stay together so that they collect the temperature of the same point/body of water. If one sensor touches the side of the container and the other does not, there will be a variation in the temperatures that are being measured.

3. The *DataMate* Application can be transferred from the CBL 2™ or from another TI-73 handheld. To do this, follow the normal Link procedures for transferring programs and applications.

4. The Mode setting for the number of decimals should be set at 2 decimals in the program and changed later to 4. This may cause some difficulty with older or younger children. Setting it at 2 (except for the Manual-Fit) might be best for younger students, while more digits might be better for the older child.

5. Encourage the students to think of other pairs of data that they could collect using this concept (pressure in atmospheres versus pounds per square foot, for example). The key is that you must have two sensors to plug into Channels 1 and 2, and they must be set up to collect data in the different units.
   However, this is easily done with the *DataMate* application since it recognizes the sensors and allows the user to select the units to be used for each sensor.

6. To test the temperatures in the GAUGE mode, see *Appendix C: CBL/CBR APPS*. The *DataMate*

application will display current temperatures from the Main Screen.

7. To examine the mean percent error in a more numerical way, return to the Home screen, and clear it by pressing : :. Now create the expression in Figure 68 using the CATALOG (– |) menu, the appropriate Window, and the Y-Vars variables from the VARS menu (– }). Press β when completed.

```
mean(seq(Y3,X,Xm
in,Xmax,ΔX))
            1.3842
■
```

Figure 68

Going Further - Key

1. Looking for 5/9 and 9/5 as the "slope" in the two conversion formulas (Figure 69).

$$C^\square_{\boxed{|||}} = \frac{5}{9}\left(F^\square - 32\right) \qquad F^\square_{\boxed{|||}} = \frac{9}{5}C^\square + 32$$

You would expect something like the formula shown below with the C = X and the F = Y.

$$\frac{\left|\,Model - Truth\,\right|}{Truth} * 100$$

Figure 69

2. Use the formula below with the Truth being 5/9 and 5/9∗-32:

$$F^\square_{\boxed{|||}} = \frac{9}{5}C^\square + 32$$

3. The 9/5 is the rate of change of Fahrenheit per degree Celsius.

4. A degree Celsius is 1.8 degrees Fahrenheit.

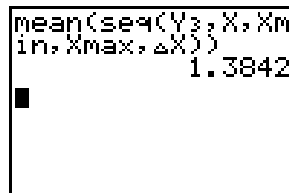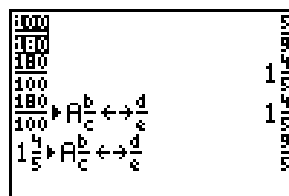5. To show how strongly we can embrace our conclusions and to point to a built-in level of uncertainty.