



Math Objectives

- You have seen before that for any triangle, you can find a circle that contains all three vertices (called the circumscribed circle or the circumcircle). When the four vertices of a quadrilateral lie on a circle, it is a special situation and deserves a special name: cyclic quadrilateral. In this activity, students will investigate some of the properties of cyclic quadrilaterals.

Vocabulary

- circumscribed circle
- perpendicular bisector
- supplementary angles

About the Lesson

- The time varies for this activity depending on whether the .tns file is given to or created by the students.
- Students can either create the .tns file by following the instructions given in *Cyclic_Quadrilaterals_Create.doc*, or they can use the pre-constructed file entitled *Cyclic_Quadrilaterals.tns*.
- Students might work independently or with a partner to complete this activity.
- This activity is intended to be introduced by the teacher and completed by the students, as they discover and justify their conjectures.
- The student worksheet helps guide students through the activity and provides a place for students to record their answers.

TI-Nspire™ Navigator™ System

- Use Class Capture to observe students' work as they proceed through the activity.
- Use Live Presenter to have a student illustrate how he or she used a certain tool.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- Once a function has been graphed, the entry line can be shown by pressing **ctrl** **G**.





Lesson Materials:

Create Instructions
 Cyclic_Quadrilaterals_Create.pdf
Student Activity
 Cyclic_Quadrilaterals_Student.pdf
 Cyclic_Quadrilaterals_Student.doc
TI-Nspire document
 Cyclic_Quadrilaterals.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.



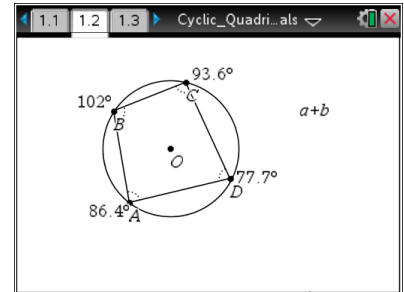
Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the arrow until it becomes a hand () . Press   to grab the point and close the hand () .

Move to page 1.2.







Part 1 – Investigating a Cyclic Quadrilateral







On page 1.2, there is a constructed cyclic quadrilateral with interior angles measured.




Drag any of the vertices of the quadrilateral, making sure the sides do not cross.

The expression symbol $a + b$ is provided so that pairs of angles can be added. Use the expression to find the sums of pairs of angles. Some of the angles are adjacent angles and some are opposite angles.

1. Press **Menu > Actions > Calculate**. Move the cursor over the expression $a + b$. When it begins to blink, press . Then a box will appear with the message “Select a? (or press VAR).” Move the cursor to the measure for angle A and press  or . When the message “Select b? (or press VAR)” appears, move to the measure for angle B and press  or . A “ghost” sum will appear. Use the Touchpad arrows to move the sum to a location on the screen, and place it there by pressing .

Obtain the other angle sums in the same way. Move the cursor over the expression $a + b$. When it begins to blink, press . Then a box will appear with the message “Select a? (or press VAR).” Move the cursor to the measure for angle C and press  or . When the message “Select b? (or press VAR)” appears, move to the measure for angle D and press  or . A “ghost” sum will appear. Use the Touchpad arrows to move the sum to a location on the screen, and place it there by pressing .

Repeat for other angle sums: A and C , B and D . Press  when finished finding the sums.

Note that the letters a and b could be used for any angle measures, not just those for angles A and B . Make sure to keep track of where the different sums are placed on the screen.



a. $m\angle A + m\angle B =$

Sample answer: Answers may vary.

b. $m\angle C + m\angle D =$

Sample answer: Answers may vary.

c. $m\angle A + m\angle C =$

Answer: 180°

d. $m\angle B + m\angle D =$

Answer: 180°

- e. Describe the pattern that you see in any of these sums. Be sure to drag vertices to check your pattern.

Answer: Opposite angles of a cyclic quadrilateral are supplementary or their sum is equal to 180 degrees. The opposite angles are supplementary no matter where the vertices are dragged along the circle.

Teacher Tip: Students may come up with a pattern for the sum of consecutive angles if the cyclic quadrilateral happens to also form another special quadrilateral.

Continue to verify patterns. Use the **Redefine** tool to redefine one of the vertices to be off the circle to investigate. Press **Menu > Actions > Redefine**. First click on any of the vertices of the quadrilateral and then click outside or inside the circle. The quadrilateral is no longer a cyclic quadrilateral. Press **esc**.

Drag different vertices again to check the patterns. Then redefine the vertex you are exploring to be back on the circle to re-establish the cyclic quadrilateral.

Teacher Tip: The **Redefine** tool is a powerful tool in TI-Nspire. It can be used to answer “what if” questions as is evidenced here. If this is the first time students have seen it, you may want to demonstrate the use of it first.



2. After redefining a vertex off of the circle, what happened to the sum of the opposite angles?

Answer: The sum of the opposite angles is no longer 180° .

3. How would you describe opposite interior angles in a cyclic quadrilateral?

Answer: In a cyclic quadrilateral, opposite interior angles are supplementary. An angle inscribed in a semicircle intercepts a 180° arc, so its measure will be 90° . Opposite angles of a cyclic quadrilateral intercept arcs whose measures sum to 360° , so the sum will be $\frac{1}{2}$ of 360° or 180° .

4. Determine if you can create each of the following quadrilaterals. Explain why or why not.
- a. Trapezoid

Answer: Yes (sometimes). It is cyclic only if it is isosceles.

- b. Isosceles trapezoid

Answer: Yes. Two parallel chords of a circle intercept congruent arcs between their endpoints. These congruent arcs have congruent chords that are the non-parallel sides of the trapezoid.

- c. Kite

Answer: Yes (sometimes), convex kites. The line of symmetry of the kite will be a diameter of the circle, and the non-vertex angles will be right.

- d. Rectangle

Answer: Yes. For a figure to be cyclic, its perpendicular bisectors must be concurrent at the center of its circumscribed circle. A parallelogram has concurrent perpendicular bisectors only when it is a rectangle.

- e. Square

Answer: Yes. Same as a rectangle.

- f. Rhombus


Answer: No. A rhombus is cyclic only if it is a square.



Teacher Tip: If two sides of a cyclic quadrilateral are parallel, what kind of quadrilateral can it be? (Rectangle, square or isosceles trapezoid)
You might have students explore this on page 1.5 of the .tns file.

Move to page 1.3.

Part 2 – Perpendicular Bisectors

This page contains the cyclic quadrilateral $ABCD$. Construct the perpendicular bisectors of each side by pressing **Menu > Construction > Perpendicular bisector**. Move the cursor to a side (make sure it says *side*) and . Repeat for all sides. Press **esc** when you are finished.

Drag different vertices of the quadrilateral to look for the relationship between the quadrilateral and the perpendicular bisectors.

5. In a cyclic quadrilateral, the perpendicular bisectors _____.

Answer: are concurrent. The point of concurrency is the center of the circle.

Teacher Tip: Encourage students to redefine one of the vertices off of the circle to see what happens to the perpendicular bisectors. Challenge students to try other types of construction, such as altitudes, angle bisectors, and so on.

Move to page 1.4.

Part 3 – Quadrilaterals

6. Josephine tells Bobby that quadrilateral $ABCD$ is a cyclic quadrilateral. He says it isn't. Who is correct? How do you know?

Answer: Josephine is correct. The perpendicular bisectors are concurrent. The opposite angles are supplementary.

Teacher Tip: Encourage students to try out the constructions on the given quadrilateral as well as finding angle measurements. Challenge students to find the concurrent point and then construct a circle using that point and a vertex. Tech savvy students might just “unhide” the original circle.



Move to page 1.5.

7. Explore more properties of cyclic quadrilateral $ABCD$ as instructed by your teacher.

Extension – Explorations with Cyclic Quadrilaterals

- Parallel lines can be drawn as secant lines through the circle and a cyclic quadrilateral can be formed from those lines. This will construct an isosceles trapezoid.
- All vertices are equidistant from the center of the circle.
- When two opposite vertices are collinear with the center of the circle, the other two vertices will be right angles.
- The altitude of a triangle is a segment from a vertex perpendicular to the opposite side or to an extension of the opposite side of the triangle. The analogous construction in a quadrilateral is a maltitude. A maltitude is a segment from the midpoint of a side perpendicular to the opposite side of a quadrilateral. The maltitudes of a cyclic quadrilateral are concurrent.
- There are many diagonal relationships in a cyclic quadrilateral. If P is the intersection of the diagonals, then $\overline{AP} \times \overline{CP} = \overline{BP} \times \overline{DP}$ (sometimes known as the Chord-Chord Power Theorem).
- Ptolemy's Theorem: The sum of the products of the two pairs of opposite sides is equal to the product of the diagonals in a cyclic quadrilateral.