EXPLORATIONS

Activity 4

Oliver's Method

Objective

- To discover a relationship between the Greatest Common Factor (GCF) and the Least Common Multiple (LCM) of two numbers

Materials

- TI-73 calculator
- Student Worksheet

In this activity you will:

- find a relationship between Greatest Common Factor (GCF) and Least Common Multiple (LCM)
- justify why Oliver's method works

You will need to know this math vocabulary:

- greatest common factor (GCF) also known as greatest common divisor (GCD)
- least common multiple (LCM)
- prime factorization
- relatively prime
- multiplicative inverse or reciprocal

Introduction

Is there a relationship between Greatest Common Factors (GCF) and Least Common Multiples (LCM)? Oliver Richard came up with his own unique method of finding the LCM after the GCF is known.

This is what he does. To find the LCM of 20 and 36:

Step 1: Write the two numbers as a fraction $\frac{20}{36}$

Step 2: Simplify the fraction $\frac{20 \div 4}{36 \div 4} = \frac{5}{9}$

Step 3: Take the original fraction and multiply it by the reciprocal of the simplified fraction $\frac{20 \times 9}{36 \times 5} = \frac{180}{180}$

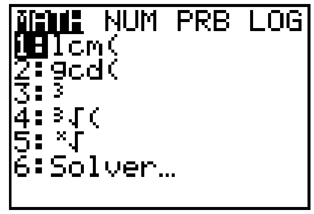
180 is the LCM of 20 and 36.

Problem

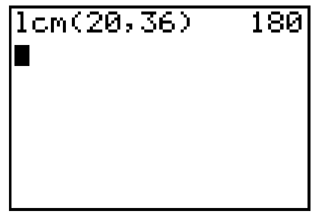
Through repeated examples on Table 1 of the Student Worksheet, you will look for a relationship between the GCF, LCM, and the two numbers. You will then discuss your findings with your group. Your second task will be to justify why Oliver's method works or doesn't work.

Activity

1. Go to the Home screen ([2nd] [QUIT]) and press [CLEAR]. You will find the LCM of 20 and 36 on the calculator. Press [MATH] [ENTER] and type **2 0** [,] **3 6** [)] [ENTER].

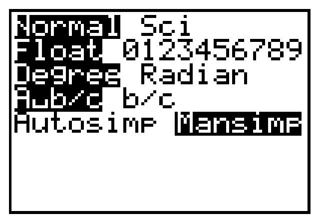


2. To find the GCD of two numbers, follow the same steps as screen shots at the right except select **2:gcd(**. Press [MATH], select **2:gcd(** and type **2 0** [,] **3 6** [)] [ENTER].

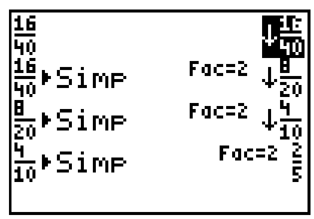


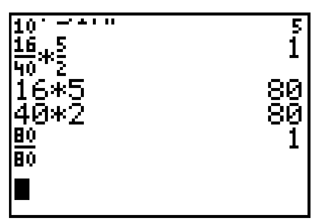
✎ Go to the Student Worksheet and complete Table 1 using the calculator's LCM and GCD functions. Answer questions 1 through 4 on the Student Worksheet.

3. Test Oliver's method using some of the problems in Table 1. Set [MODE] as shown at the right. Enter **16** [b/c] **40** [ENTER]. Press [SIMP] [ENTER] [SIMP] [ENTER] [SIMP] [ENTER] until the numerator and denominator are **relatively prime**. (GCF of the numerator and denominator is 1.)



4. Type in the original fraction and multiply it by its simplified reciprocal. This shows that the product of a number and it's **reciprocal** (or **multiplicative inverse**) is 1. Next find the unsimplifed form of 1 by breaking the multiplication into numerator times numerator and denominator times denominator.





✎ Answer questions 5 through 8 on the Student Worksheet.

Student Worksheet

Name ______________________

Date ______________________

Activity 4

Oliver's Method

Record your results on the table below. Then answer the questions about the activity.

Table 1

(a, b)	GCD	LCM	GCD x LCM	a x b
(16,40)				
(18,72)				
(9,12)				
(7,5)				
(6,20)				
(8,16)				
(15, 9)				
(11, 12)				
(36, 48)				
(16,14)				

1. How is the product of the two numbers (**a** and **b**) related to the product of the GCD and the LCM of the two numbers?

____

2. Justify your conclusion.
Hint: You may want to examine the **prime factorization.**

a x b GCD x LCD

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____

3. Suppose you have a four-function calculator that will only add, subtract, multiply and divide. You know that the GCD of 40 and 48 is 8 and you need to know the LCM. Explain the keystrokes you could use on the calculator to find it.

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4. James and Andy are training for a bike race. James can go around the park on the bike path in 20 minutes and Andy can go the same distance in 16 minutes. If they start at the same time, when will they be side-by-side again? Write a mathematical expression you could use to answer this question.

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____

____

5. Use Oliver's method to find the LCM of 18 and 72.

Step 1: Write as fraction ________

Step 2: Simplify_______________
How do you simplify fractions?

____

Step 3: Multiply the original fraction by the reciprocal of the simplified fraction to get the LCM

____

6. Use Oliver's method to find the LCM of 9 and 12. Show the process.

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7. Write an equation that describes the relationship between **a**, **b**, GCD, and LCM that you saw in Table 1.

____

8. How can you change the equation to make it fit Oliver's method?

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Teacher Notes

Activity 4

Oliver's Method

Math Strand

- Number sense
- Algebraic reasoning

Materials

- TI-73 calculators
- Student Worksheets (page 35)

Students will find and justify the relationship that the product of two numbers is equal to the product of the LCM and the GCF.

Vocabulary

Greatest Common Factor(GCF) or Greatest Common Divisor(GCD)	The greatest factor or divisor common to a set of two or more numbers
Least Common Multiple (LCM)	The least of the nonzero common multiples of two or more numbers
prime number	A number greater than 1 that has exactly two factors, 1 and itself.
prime factorization	The process of writing a composite number as the product of prime factors.
relatively prime	Two or more numbers whose GCF is 1.
multiplicative inverse or reciprocal	The product of a number and its multiplicative inverse is 1.

Classroom Management

Students may work independently on Table 1 and discuss their findings in groups of 3 to 4. Working in their groups, they will answer the questions on the Student Worksheet. Students should have prior experience in finding GCF and LCM by the traditional methods of listing and prime factorization. This activity is an example of a student who developed a unique way of performing this common middle school skill. The justification of this algorithm may provide a missing link of the connection between the GCF, LCM, and the product of the two numbers.

Activity

The directions and keystrokes on the student activity pages are complete.

Answers to Student Worksheet

1. They are equal.

2. Prime Factorization.

a x b	GCD x LCM
16 x 20	4 x 80
4 x 4 x 4 x 5	2 x 2 x 4 x 20
2 x 2 x 2 x 2 x 2 x 2 x 5	2 x 2 x 2 x 2 x 5 x 2 x 2
320 = 2 x 2 x 2 x 2 x 2 x 2 x 5	320 = 2 x 2 x 2 x 2 x 5 x 2 x 2

3. 40 x 48 ÷ 8

4. 80 minutes or 1 hour 20 minutes; The mathematical expression is 16 x 20 ÷ 4 = 80 minutes or 1 hour 20 minutes.

5. Use Oliver's method to find the LCM of 18 and 72.

a. Step 1: Write as a fraction $\frac{18}{72}$.

b. Step 2: Simplify: $\frac{1}{4}$ How do you simplify fractions? Divide by GCF.

c. Step 3: Multiply original fraction by reciprocal of simplified fraction to get the LCM.

$$\frac{18 \times 4 = 72}{72 \times 1 = 72}$$

6. $\frac{9}{12}$: $\frac{3}{4}$: $\frac{9 \times 4 = 36}{12 \times 3 = 36}$

7. a x b = GCF x LCM

8. a ÷ GCF x b = LCM or b ÷ GCF x a = LCM