Objectives

- To practice simplification techniques "in reverse," gaining comfort with algebraic manipulation
- To build a sense of equivalence, indicated by identity of graphs

Synonyms and "Function Fictionary"

Introduction

The TI-Navigator™ system can provide an environment for students to make sense of ideas of "equivalence" in Algebra. This is a fundamental concept behind simplifying expressions, a core topic in the curriculum.

The graphical representation of a function can anchor students' understanding of what they are doing when they simplify. For the purposes of the study of Algebra, two expressions can be considered equivalent if the graphs of the functions defined by these expressions overlap completely. Some common problems that students face in Algebra stem from a failure to grasp fully the mathematical meaning of the set of manipulation techniques and algorithms that they are learning. With the image of the unchanging graph in mind, students can think of simplifying expressions as "renaming" the same mathematical object ("finding synonyms").

The activities described here suggest ways in which you can guide your class through these concepts and their use in Algebra. A grounded understanding of these concepts is particularly valuable for students who struggle with the simplification algorithms themselves. Although there can be a temptation to focus on "drill" with such students, the time spent in understanding the nature of equivalence is time well spent.

Prerequisites and Materials

Prerequisites: The preparatory steps below help to smooth the way from basic algebra skills and an understanding of numerical equality to the concept of equivalent variable expressions.

Materials: The Activity Settings file, `Synonyms.act` located on the CD.

Setting Up the Activity

From the TI-Navigator™ teacher computer home screen, open the Activity Center, 🌐. Then choose **File>Load>Load Activity Settings** to load the activity settings file, `Synonyms.act`.

Preparation: Equivalent Numerical Expressions

To introduce the idea that there are different names for the same mathematical entity, you can begin by working with <u>numbers</u> rather than variable expressions. You might choose to do this without the network, but you could also use the TI-Navigator system's Quick Poll functionality, 📦. For example, you could pose an **Open Response** poll, prompting students to provide "Names for Half a Dollar."

You can transition to a prompt that uses arithmetic operations in numeric expressions. For example, ask for "Names for 12." You can then add constraints, such as that students must use numeric expressions containing three or more operations.

With these polls, the **Poll Summary** tab will provide a spot check that your students have entered expressions that are in fact equivalent to the target number (e.g., 12). Then, you can view specific student contributions by moving to the **Poll Details** tab. This will allow you to draw attention to interesting and inventive ways that your students have found to build the target number.

We also recommend that you issue these polls with the **Resubmit** option checked. That way, your students will be able to change their answer if they mistakenly contribute an expression that is not equivalent to 12. Furthermore, when students see the numerical expressions that their classmates have contributed, they can contribute more and more exciting ones.

Transitioning to Variable Expressions

Once your students have exercised the notion of equivalence in numeric expressions, you may wish to get them thinking about what it would mean for two <u>rules</u> or two <u>processes</u> to be equivalent.

One concept-grounding introduction that a research partner of ours has found effective is to hold a conversation on what it would mean for two soda machines to be "the same," "interchangeable" or "equivalent". Student thinking on this question can be guided towards a consideration of "inputs and outputs" and the formulation of a rule relating the two (two soda machines are equivalent in the most essential sense if they both give you the same output in exchange for the same input). Making a graph of this rule—number of

cents graphed against number of cans of soda—leads to the idea that equivalent rules have identical graphs.

"The Same as 4X"

When your students have some basic comfort with the relationship between equivalent expressions and identical graphs, bring them into the Activity Center to explore. In the discussion below, we use the "target" expression of 4X as an example. To set up this target, switch to the **Graph-Equation** tab, enter the equation into the **Y=** entry field, and click **Add**.

Configuration Options

In "Setting Up the Activity", above, you have already configured the Activity Center for students to contribute equations by loading the `Synonyms.act` settings file. Below, we describe the configuration settings contained in that file:

- <u>Number of equations per student (`Synonyms.act` value = 3).</u> The setting of 1 encourages students to contribute quickly. However, you might choose to challenge students to stretch their thinking by asking for some larger number of equivalent expressions.
- <u>Let students view graphs of equations (value = yes).</u> We suggest that you allow students to view the graphs of their equations before submitting. This will encourage them to be more adventurous in their submissions and it will allow them to check their thinking before exposing it publicly.
- <u>Send graph contents as background (value = yes).</u> This will allow students to match the graphs of their functions with the target Y=4X when they preview on their calculators.
- <u>Let students resubmit equations (value = yes).</u> If students are allowed to resubmit, they can explore the space of equivalent expressions more fully.

Running the Activity

Present your students with the challenge:

Give me 3 different functions that are equivalent to Y=4X

We suggest that you run the activity with **Show Student Names** unchecked, so that students who make mistaken submissions can correct themselves anonymously.

Start the activity, ▶, and watch as the students' solutions come in. If you pause the activity, ❚❚, you can highlight and discuss student expressions that use particularly interesting operations or approaches. Resume the activity, ❚❚, and allow students to build from their first answers toward more "risky" or inventive expressions that are equivalent to 4X.

Variations

By changing the prompts you give or the configuration options of this activity you can lead your students to different kinds of learning experiences. For instance:

1. Constrained Challenges. To exercise students' simplification skills further (or, more accurately, their unsimplification skills!) try imposing different constraints or conditions on the solutions. For example, "Give me a function that is the same as 4X and that has 5 terms".

2. Sequential Performances. In building toward the concept of equivalence, you can ask students to submit their equations one by one, by raising their hands and talking their solutions out. In this variation, you will want to make sure your students have the chance to check their work (through preview and comparison with the target graph as a background image), before they submit. In this variation, you can expect student contributions to exhibit a playful "one-upsmanship" as the activity goes on.

3. "Function Fictionary." You can do this variation with individual student work, but we recommend trying it in small groups (of four). Group members work together on paper to create four equations, three of which are equivalent to the target and one of which is not equivalent. They should try to create a false equation that they think will trick their fellow students. Once each group has decided on its equations, the activity begins. Switch to the **Equation** tab, so that the graphical representations of the equations that come up from the class are not visible. Ask the first group to submit its four equations (if you are using groups of four, you can have each member submit one of the equations). Next, start a Quick Poll, 📦, using the **Multiple Choice A-D** format. Assign a letter to each of the four equations submitted by the first group. Tell the students not in the first group to choose which of the equations they think is not equivalent to the target. When the responses are in, reveal which expression is not equivalent to the target. The group's score is the number of incorrect answers. After the poll has been completed, you can switch to the **Graph-Equation** tab in the Activity Center to discuss the equations, identify the non-equivalent one, and work out why it is not equivalent. Then, hide the first group's equations, switch back to the **Equation** tab, and ask the second group to submit their equations. The activity proceeds one by one through all the groups, and the group who tricks the most students is the winner!