EXPLORATIONS™

Activity 18

Altitude, Median, and Angle Bisector of a Triangle

Objectives

- To investigate the special properties of an altitude, a median, and an angle bisector
- To reinforce the differences between an altitude, a median, and an angle bisector

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Introduction

Altitudes, medians, and angle bisectors are common structures used to study triangles in geometry. In this activity, you will construct these special segments (lines) of a triangle using their definitions. You will look at how an altitude, a median, and an angle bisector divide the area of a triangle.

This activity makes use of the following definitions:

Altitude of a triangle — a segment or line drawn from a vertex perpendicular to the opposite side (or an extension of that side) of the triangle. A triangle has three altitudes.

Median of a triangle — a segment drawn from a vertex to the midpoint of the opposite side of the triangle. A triangle has three medians.

Angle bisector — a line (or ray) that passes through the vertex of an angle to form two congruent angles.

Construction

Construct an altitude, a median, and an angle bisector of a triangle.

Draw an acute triangle, $\triangle ABC$, in the center of the screen.

Construct the altitude $\overline{AD}$ by constructing a line perpendicular to side $\overline{BC}$ that passes through and is defined by point A.

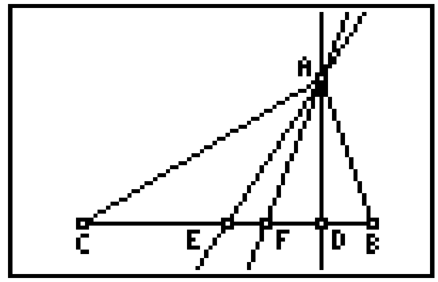
Construct and label point D, the intersection of the altitude with side $\overline{BC}$.

Construct and label the midpoint (point E) of side $\overline{BC}$.

Construct the median $\overleftrightarrow{AE}$.

Construct the angle bisector of $\angle BAC$.

Construct and label the intersection of the angle bisector with side $\overline{BC}$ point F.



Exploration

Show $\overleftrightarrow{AD}$ and hide $\overleftrightarrow{AE}$ and $\overleftrightarrow{AF}$. Explore the altitude $\overline{AD}$ by dragging the sides and vertices of $\triangle ABC$. Use various measurement tools (**Distance and Length**, **Angle**, **Area**) and observe any relationships that exist between $\triangle ABC$, $\triangle ADB$, $\triangle ADC$ and their parts. Be sure to investigate these relationships when $\triangle ABC$ is a special triangle (acute, right, obtuse, scalene, isosceles, and equilateral).

Show $\overleftrightarrow{AE}$ and hide $\overleftrightarrow{AD}$ and $\overleftrightarrow{AF}$. Explore the median $\overline{AE}$ by dragging the sides and vertices of $\triangle ABC$. Use various measurement tools and observe any relationships that exist between $\triangle ABC$, $\triangle AEB$, $\triangle AEC$ and their parts. Be sure to investigate these relationships when $\triangle ABC$ is a special triangle.

Show $\overleftrightarrow{AF}$ and hide $\overleftrightarrow{AD}$ and $\overleftrightarrow{AE}$. Explore the angle bisector $\overleftrightarrow{AF}$ by dragging the sides and vertices of $\triangle ABC$. Use various measurement tools and observe any relationships that exist between $\triangle ABC$, $\triangle AFB$, $\triangle AFC$ and their parts. Be sure to investigate these relationships when $\triangle ABC$ is a special triangle.

Show $\overleftrightarrow{AD}$, $\overleftrightarrow{AE}$ and $\overleftrightarrow{AF}$ and explore the relationship among the altitude, median, and angle bisector constructed from the same vertex. Use various measurement tools and observe any relationships that exist among these lines, $\triangle ABC$, and the triangles formed by these lines and the side $\overline{BC}$ (for example $\triangle ADE$).

Questions and Conjectures

1. Make a conjecture about any special relationships that exist between a triangle and an **altitude** of the triangle. Be sure to identify any relationships that exist when $\triangle ABC$ has specific properties. Explain your answers and be prepared to demonstrate.

2. During the exploration of the altitude $\overleftrightarrow{AD}$, you should have seen that point D did not always exist. Explain why this happened. How could you have done this construction differently to ensure that point D was displayed for any $\triangle ABC$?

3. Make a conjecture about any special relationships that exist between a triangle and a **median** of the triangle. Be sure to identify any relationships that exist when $\triangle ABC$ has specific properties. Explain your answers and be prepared to demonstrate.

4. Make a conjecture about any special relationships that exist between a triangle and an **angle bisector** of the triangle. Be sure to identify any relationships that exist when $\triangle ABC$ has specific properties. Explain your answers and be prepared to demonstrate.

5. Make a conjecture about the relationship between an altitude, median, and angle bisector constructed from the same vertex of a triangle. Be sure to identify any relationships that exist when $\triangle ABC$ has specific properties. Explain your answers and be prepared to demonstrate.

Teacher Notes

Activity 18

Altitude, Median, and Angle Bisector of a Triangle

Objectives

- To investigate the special properties of an altitude, a median, and an angle bisector
- To reinforce the differences between an altitude, a median, and an angle bisector

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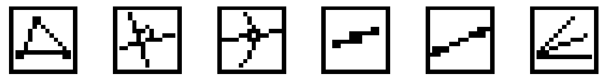


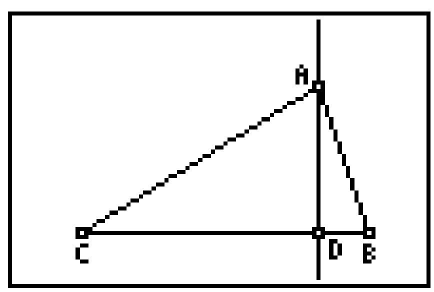
Additional Information

You may want to break your class into small groups and assign each group a different exploration. Each group can report its findings back to the class.

Answers to Questions and Conjectures

1. Make a conjecture about any special relationships that exist between a triangle and an **altitude** of the triangle. Be sure to identify any relationships that exist when $\triangle ABC$ has specific properties. Explain your answers and be prepared to demonstrate.

   Conjectures that the students might develop include, but are not limited to, the following:

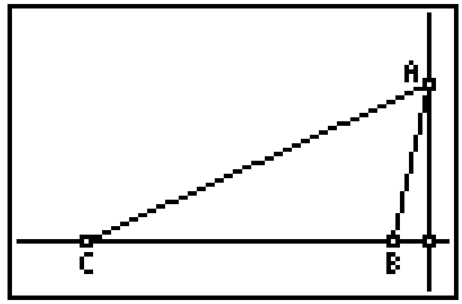


   - When vertex A moves along a path parallel to side $\overline{BC}$, the distance AD remains constant.
   - An altitude falls outside the triangle when either $\angle ABC$ or $\angle ACB$ are obtuse.
   - The altitude of a triangle is one of its sides when either $\angle ABC$ or $\angle ACB$ are right angles.
   - When $\angle BAC$ is a right angle the following are true:
     - $m\angle DAB$ is equal to $m\angle ACB$,
     - $m\angle DAC$ is equal to $m\angle ABC$, and
     - $\triangle BAC$, $\triangle BDA$ and $\triangle ADC$ are similar.
   - When the length of segment $\overline{AB}$ equals the length of segment $\overline{AC}$, $\triangle ADB$ is congruent to $\triangle ADC$.

- When the length of segment $\overline{AB}$ equals the length of segment $\overline{AC}$, the area of ΔADB is equal to the area of ΔADC.

2. During the exploration of the altitude $\overleftrightarrow{AD}$, you should have seen that point D did not always exist. Explain why this happened. How could you have done this construction differently to ensure that point D was displayed for any ΔABC?

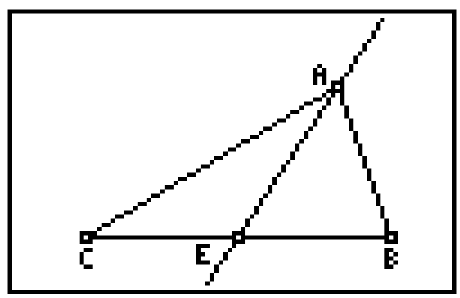
   Based on the construction, it is possible to move point A so that the line perpendicular to side $\overline{BC}$ through point A does not intersect $\overline{BC}$. This construction can be modified by having students construct a line $\overleftrightarrow{BC}$ and define the point D to be on $\overleftrightarrow{BC}$ instead of $\overline{BC}$. The students can then hide $\overleftrightarrow{BC}$ if desired.



3. Make a conjecture about any special relationships that exist between a triangle and a **median** of the triangle. Be sure to identify any relationships that exist when ΔABC has specific properties. Explain your answers and be prepared to demonstrate.

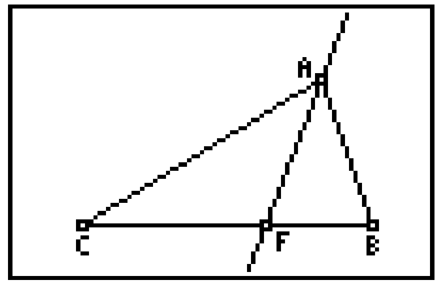
   Conjectures that the students might develop include, but are not limited to, the following:

   - The median will always divide ΔABC into two triangles, ΔAEB and ΔAEC, having equal areas.
   - When $\overline{AB}$ equals $\overline{AC}$, ΔAEB is congruent to ΔAEC.



4. Make a conjecture about any special relationships that exist between a triangle and an **angle bisector** of the triangle. Be sure to identify any relationships that exist when ΔABC has specific properties. Explain your answers and be prepared to demonstrate.

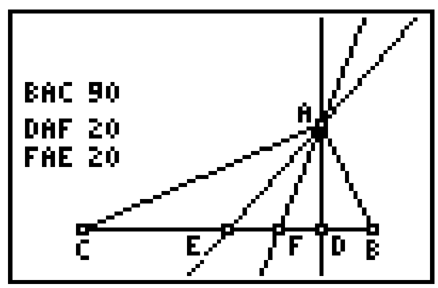
   Conjectures that the students might develop include, but are not limited to, the following:

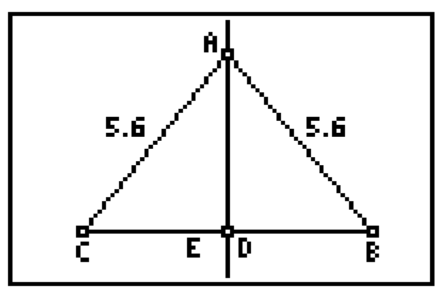
   - The ratio $\frac{BF}{FC}$ is equal to the ratio $\frac{AB}{AC}$.
   - When the length of segment $\overline{AB}$ equals the length of segment $\overline{AC}$, ΔAFB is congruent to triangle ΔAFC.
   - When the length of segment $\overline{AB}$ equals the length of segment $\overline{AC}$, the area of ΔAFB is equal to the area of ΔAFC.

5. Make a conjecture about the relationship between an altitude, median, and angle bisector constructed from the same vertex of a triangle. Be sure to identify any relationships that exist when ΔABC has specific properties. Explain your answers and be prepared to demonstrate.

   Conjectures that the students might develop include, but are not limited to, the following:

   - When $\angle BAC$ is a right angle, the angle bisector $\overleftrightarrow{AF}$ will also bisect $\angle DAE$ (the angle formed by the altitude and the median from point A).

   - When the length of segment $\overline{AB}$ equals the length of segment $\overline{AC}$, points D, E and F coincide.