NUMB3RS Activity: Where is the Bullet?
Episode: "Convergence"

Topic: Projectile Motion **Grade Level**: 10 - 12
Objective: Use parametric equations to model projectile motion.
Time: 20 - 30 minutes
Materials: graphing calculator

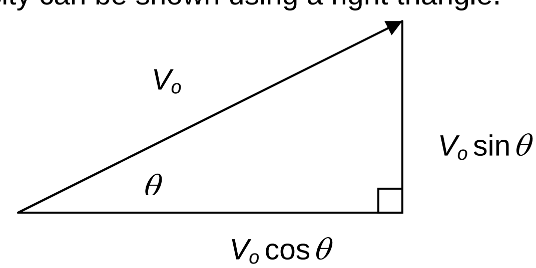
Introduction

When an object is thrown or shot, it usually follows a parabolic path. Ask students to sketch the path of a baseball after it has been hit, from the time it leaves the end of the bat until it lands on the ground. Discuss some factors that influence the path of the ball including

- initial launch height of the ball, h_o.
- initial velocity of the ball given by the bat, V_o.
- angle at which the ball was launched, θ.
- effect of gravity, $-\frac{16\,ft}{\sec^2}$.

Ask students to pick a point on the graph, which shows the location of the ball at any given elapsed time *t*. Point out that the location has both a horizontal and vertical component. The ball has traveled a horizontal distance *x,* compared to the initial starting point, and it has traveled a vertical distance *y,* compared to its initial launch height.

How far the ball has traveled horizontally depends on the initial velocity of the ball, elapsed time in the air, and the angle at which it was launched. The initial velocity of the ball is a vector, where the horizontal and vertical velocities affect the horizontal and vertical distances. The initial velocity can be shown using a right triangle.



Horizontal Distance *x*
$x = (V_o \cos(\theta))t$

How far the ball has traveled vertically depends on the initial start height, the effect of gravity which pulls the ball down vertically, the launch angle, the initial velocity, and time in the air. Explain the variables for the equations representing horizontal and vertical distances traveled.

Vertical Distance *y*
$y = -16t^2 + (V_o \sin(\theta))t + h_o$

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These equations, which represent the horizontal and vertical distance traveled by the ball, are called parametric equations. A graphing calculator can be used to simulate the actual path of the ball by graphing the parametric equations.

Example Suppose a baseball player hit a ball two feet above the ground at a 45º angle with an initial velocity of 125 feet per second on a windless day.
The path of the ball in flight can be modeled by the following parametric equations:

$$x = (125\cos(45))t$$
$$y = -16t^2 + (125\sin(45))t + 2$$

, where t is measured in seconds, and x and y are measured in feet. The equations can be graphed using a TI graphing calculator. From the graph you can determine the distance the ball traveled, the maximum height of the ball, and the time the ball is in the air.

Discuss with Students

***NUMB3RS* Example** In "Convergence", FBI Agents Colby and David are trying to locate a bullet that was fired as a warning shot over two carjack victim's heads. They turn to Charlie for advice about where to look for the bullet. Charlie uses parametric equations and a TI-84 Plus to simulate the trajectory of the bullet. After taking all of the factors into account, Colby and David are able to locate the missing bullet.

Student page answers: 1. *(455cos(10))t = horizontal distance traveled due to being shot from the gun;* $-16t^2$ *= effect of gravity; (455sin(10))t = vertical distance traveled due to being shot from the gun; 5 = height of gun when it was shot* ***2.*** *at t = 0 seconds, the bullet was 5 feet above the ground* **3.** *at t = 0.1 second, the bullet is about 45 feet away and is 12.7 feet above the ground. It misses the victims by over 6 feet.* **4.** *at t = 2.5 seconds, the bullet is about 102.5 feet above the ground.* **5.** *5 seconds* **6.** *about 2,240 feet*

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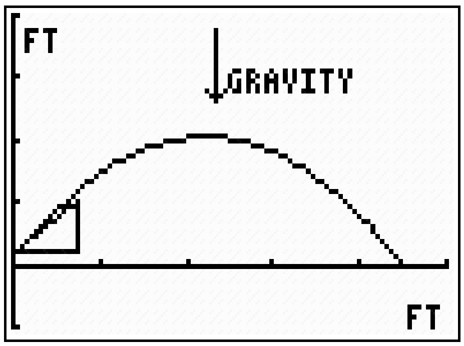
Name: ______________________________ Date: ________________

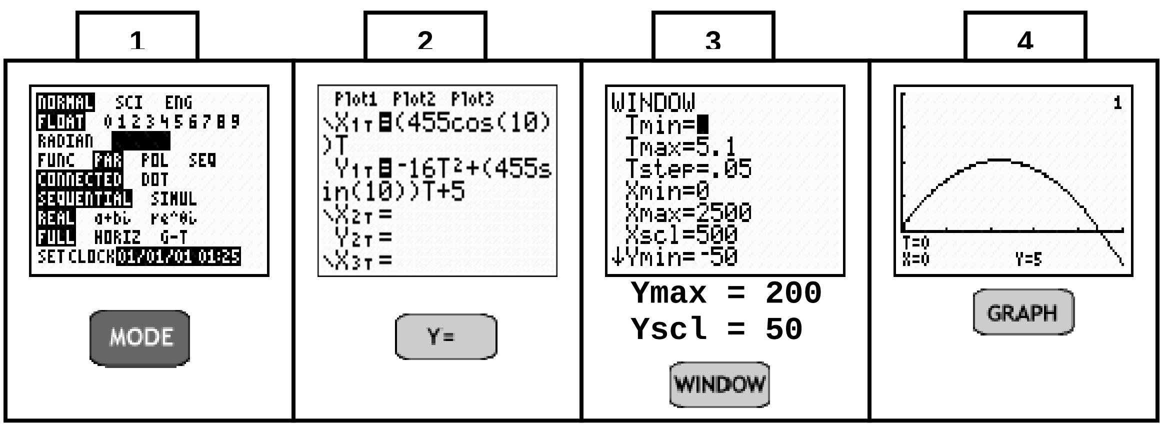
NUMB3RS Activity: Where is the Bullet?

David and Colby are trying to find the location of a bullet that was fired from a 9mm pistol. From witness reports, they estimate that the gun was five feet off the ground when it was shot. The shooter was standing in a large, level field and fired one shot over the victim's heads to scare them. David and Colby estimate that the angle of elevation of the gun when it was fired was 10 degrees. They know that a 9mm pistol can launch a bullet with an initial velocity of 455 feet per second (after air resistance has been taken into account). They have drawn the following sketch to model the situation:



Press the MODE button on your calculator and match your mode screen to the first screen shot below.

Press the Y= button, and match your Y= screen to the second shot below. (When you press the **X** key in parametric mode, the letter **T** will appear.)



1. Describe how each term in the parametric equations relates to the bullet's trajectory. ____
____
____

Press the WINDOW key and match to screen shot 3.
Press the GRAPH key and when the graph is finished, press the TRACE button so that your graph looks like screen shot 4.

2. Explain the meaning of the numbers you see on the screen with the graph.

____

____

3. If the victims were standing 45 feet away from the shooter when he shot the gun, about how far did the bullet go over the victims' heads? (Assume that the victims were on average, 6 feet tall.) ___________________________________

4. Approximately what was the maximum altitude reached by the bullet?

____

5. About how long was the bullet in the air? ______________________________

6. About how many feet did the bullet travel in the horizontal direction? ________

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The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

Other factors such as wind speed and obstacles in the path of the object should be considered when modeling trajectory problems. Consider the situations below and how they would affect the parametric equations used to model them.

For the Student

- Use a TI-84 Plus graphing calculator to find how far the baseball traveled in the example on page one.
- If the home run fence was 8 feet high and located 475 feet in the outfield, would this hit be a home run?
- How much shorter would the hit be if there were a wind blowing directly into the batter at 10 miles per hour? (hint: convert the wind speed to feet per second)
- How does changing the angle at which the ball was launched affect the horizontal distance the ball travels?
- It is possible to convert the parametric equation to a function by solving the first equation for t and then substituting the result into the second equation. Verify that the function produces the same graph as the parametric equation. What information was lost in the conversion?

Additional Resources

http://education.ti.com/guides
This site contains a .pdf file for the TI-84 Plus guidebook that gives examples and instructions on how to graph parametric equations. See Chapter 4, page 133.

http://education.ti.com/exchange
Search for "parametric". This site has several activities and examples that use parametric equations.

http://www.baseball-almanac.com/feats/art_hr.shtml
For perspective, consider the computerized measuring system used in every major league park since 1995. One of the longest home runs hit was by Cecil Fielder. Playing for the Detroit Tigers, Fielder hit a 502-foot home run at Milwaukee's County Stadium on September 14, 1991.

Related Topic

Describe how to graph a function using parametric equations. How could you graph the inverse of a function? Find the general parametric equations needed to graph the function $y = f(x)$ and its inverse.

Research how parametric equations can be used to graph conic sections. Find the general parametric equations for a circle, ellipse, parabola, and hyperbola.

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