Shopping for Parabolas

Student Activity – Teacher Notes

7 8 **9 10** 11 12

TI-Nspire™ Investigation Student 50 min

Teacher Notes:

Engineering represents an amazing opportunity to see applications of mathematics and architecture it's beauty. To help inspire students load up a set of PowerPoint slides with work from architect Santiago Calatrava. Santiago qualified as an architect from the Polytechnic University of Valencia (PhD) and also as an engineering from the Swiss Federal Institute of Technology in Zurich, it's no wonder that his creations are so inspiring!

While Santiago's work is featured overseas, we can look closer to home for inspiration including the Eastland's shopping centre where an entire atrium is flooded with light via a family of parabolas! This investigation is limited to simply finding the questions individually and repetitively, however a deeper dive is possible as the parabolas are technically all the same. The difference in equations is purely the perspective of the image. It is however important to note that the dilation factor remains approximately constant as it is not impacted by the progressive 'enlargement' of each parabola.

Australian Curriculum Standards

AC9M9A02

Simplify algebraic expressions, expand binomial products and factorise monic quadratic expressions.

AC9M9A04

Identify and graph quadratic functions, solve quadratic equations graphically and numerically, and solve monic quadratic equations with integer roots algebraically, using graphing software and digital tools as appropriate.

AC9M9A06

Investigate transformations of the parabola $y = x^2$ in the Cartesian plane using digital tools to determine the relationship between graphical and algebraic representations of quadratic functions, including the completed square form, for example: $y = x^2 \to \ y = \frac{1}{3}x^2$ (vertical compression) …

AC9M10A05

Experiment with functions and relations using digital tools, making and testing conjectures and generalising emerging patterns.

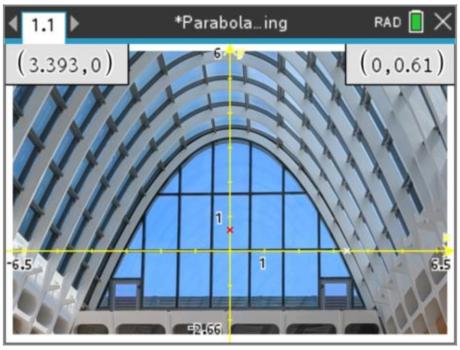
Author: P. Fox

Introduction

Some architectural, engineering and advertising curves feature parabolic forms, the arches at Eastlands shopping centre were designed as parabolic. The long atrium is illuminated via glass panels resting on top of this long series of parabolic arches. The arches continue around a corner adding further mathematical beauty.

In this activity, a short passage of these arches will be modelled with appropriate equations.

Set up



Open the TI-Nspire document: "Parabola Shopping".

A picture of the atrium in one section of the shopping centre has been inserted in the Graphs Application.

The axes provide a numerical reference frame on which models (equations) will be generated. The scale and location of the cartesian plane have been set.

The x axis is approximately 14m above ground level.

Two points have already been created and the coordinates placed in the top left and right of screen.

> You can change the colour and style of the point to make it easier to see.
> - Changing the attributes of the point allows you change its shape/appearance.
> - Change the colour depending on where the point is located.

Modelling Arch 1:

The atrium consists of a huge family of parabolas. The images shows one section of the atrium. Each parabola in this section is the same size, however, they appear smaller and smaller, this is due to perspective.

Question: 1 Equation 1 – Difference of perfect squares

a) Move the point along the x – axis to locate where the first arch (end of atrium) intersects the axis and record the abscissa (x coordinate) to the left and right.

Answer: Answers will vary ≈ -3.3 & 3.2

b) For the purposes of this first equation we assume that the arch is symmetrically oriented around the y – axis. Use the two values from the previous questions to write an appropriate quadratic equation that passes 'close' to these two points. The equation should be of the form: $y=-(x-b)(x+b)$

Answer: Students can calculate the absolute average: $y=-(x-3.25)(x+3.25)$.

c) Determine the current y intercept for your equation.

Answer: Answers will vary ≈ $3.25^2 \approx 10.56$.

d) Use the point on the y axis to locate the required coordinate for where the parabola should cross the y axis.

Answer: Answers will vary ≈ 3.43

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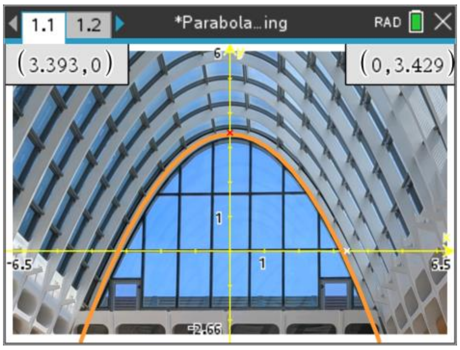
e) Use your previous two results to finalise the equation for the arch in the form: $y = a(x-m)(x+n)$.

Answer: $y = -0.324(x-3.25)(x+3.25)$

Sample Graph:

Comments: The graph is a very good fit.



f) Using the current scale the first floor (where the arches terminate) would be represented by the line: y = -10. Determine the width of the arches at the first floor.

Answer: $solve(-10 = -0.324(x-3.25)(x+3.25), x)$

x ≈ +/- 6.5. Separation distance ≈ 13m

Measured at the venue ≈ 14m.

Question 2 - Modelling Arches 2 & 3:

a) Use the same technique from Question 1 to determine an equation for the second arch.

Answer: x axis intercepts ≈ -3.7 & 3.7. y axis intercept ≈ 4.1. Dilation: 4.1 ÷ -3.7^2 ≈ -0.3

$y = -0.3(x-3.7)(x+3.7)$

b) Comment on how well the graph models the arch.

Answer: The model is still good.

c) Explain why the width of the arches (at the first floor) is different for the second arch.

Answer: The second arch predicts the separation distance ≈ 14m. It isn't 'more' accurate, the difference is due to perspective! As the arches get closer and closer to the foreground they appear bigger so too the distances between the arches and the first floor level.

d) Use the same technique from Question 1 to determine an equation for the third arch.

Answer: x axis intercepts ≈ --4.4 & 4.2. y axis intercept ≈ 4.9. Dilation: 4.9 ÷ -4.3^2 ≈ -0.27

$y = -0.26(x-4.25)(x+4.25)$

e) Comment on how well the graph models the arch.

Answer: The quality of the model is deteriorating as the image is slightly asymmetric. The turning point is not exactly on the y axis anymore.

Question 3 - Modelling Arch 4:

a) Identify the approximate x and y axis intercepts.

Answer: x axis intercepts ≈ -5.0 & 4.5. y axis intercept ≈ 5.6.

b) The dilation factor can still be calculated using the y axis intercept and substitution x = 0.

Answer: Dilation factor ≈ -0.25

c) Determine the equation in the form: . $y = a(x-m)(x-n)$

Answer: Equation ≈ $y = -0.25(x+5)(x-4.5)$

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Question 4 - Modelling Arch 5:

A slightly different approach is required for this arch as the top of the arch is not visible.

a) Identify the approximate x axis intercepts.

Answer: x axis intercepts ≈ -5.7 & 5.

b) Place a point somewhere on the Cartesian plane (Press P) and then measure it's coordinates.
Move the point so that it is on the arch to be modelled. Record the location. (coordinates)

Answer: Answers will vary. Example (3, 4)

c) Determine the equation in the form: . $y = a(x - m)(x - n)$

Answer: Equation ≈ $y = -0.23(x + 5.7)(x - 5)$

Question 5 - Modelling

Explain why the parabolas all have different equations when the physical parabolas (steel structures) are all the same?

Answer: Perspective! As the arches get closer and closer to the foreground they appear bigger so too the distances between the arches. There is however only a small variation in the dilation factor as this quantity is preserved while the arches are 'enlarged'.

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