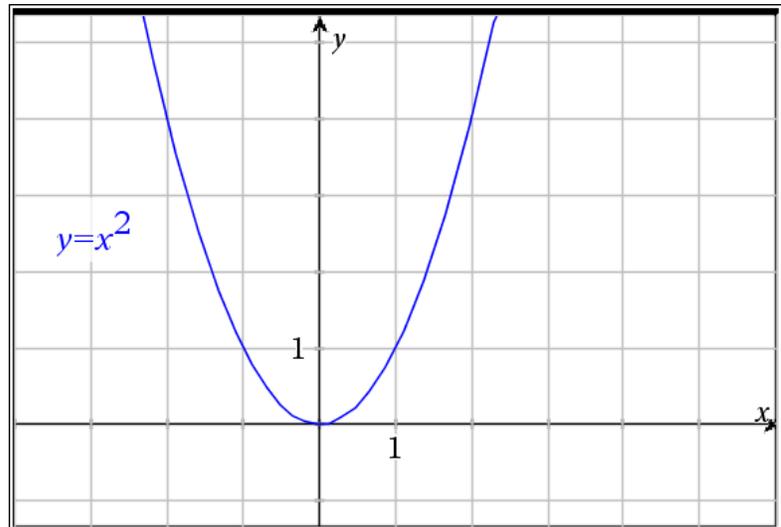


Part 1 – Draw A Tangent Line by Hand

On the graph to the right, draw a line tangent to $y = x^2$ in the first quadrant.

- Approximate the slope of the line. Show your work.



- Write the equation of your line.

Part 2 – Draw and Explore Tangent Line with Technology

Press  and select **New Document**. (Unless you want to save what you are working on, arrow over to “No” and press .)

Select **Add Graphs** by using arrows and pressing .

Graph the equation $y = x^2$ in **f1** by pressing   .

Draw a tangent line on the graph in the first quadrant. Select **MENU > Geometry > Points & Lines > Tangent**. Then use the TouchPad to move the cursor and click  the curve.

Zoom in and observe the behavior. Press **MENU > Window/Zoom > Zoom – In**. A magnifying glass with a plus sign will appear. Position this over the point of tangency and press . You have zoomed in by a factor of 2. Press enter a few more times over that point.

- Write your observation of how your tangent line and the graph $f1(x) = x^2$ compare when examined close up.
- **Conjecture** – Will this type of behavior occur for all other functions? Explain your reasoning.

You may want to try it for another function. You can choose your own or try $f2(x) = \sin(x)$ on another *Graphs* page. Press   (), or  I, to insert another page.

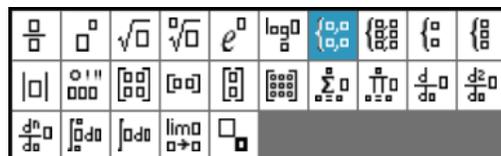
Local Linearity Discovery

Part 3 – Graph a piecewise function to explore local linearity

A function is said to be linear over an interval (i.e. locally linear over a small interval) if the slope is constant. Let's discover if all functions have a constant slope when they are examined in a small enough interval.

To graph $y = \begin{cases} x^2, & x < 2 \\ 2x, & x \geq 2 \end{cases}$, first insert another *Graph* page.

The piecewise function template can be found by pressing $\left[\frac{\square}{\square}\right]$. Arrow over to select the 2-piece piecewise function template and press $\left[\text{enter}\right]$. Then, press $\left[\text{X}\right] \left[x^2\right]$ $\left[\text{tab}\right] \left[\text{X}\right] \left[\text{ctrl}\right] \left[=\right] \left[\text{enter}\right] \left[2\right] \left[\text{tab}\right] \left[2\right] \left[\text{X}\right] \left[\text{tab}\right] \left[\text{X}\right] \left[\text{ctrl}\right] \left[=\right] \left[\text{enter}\right] \left[2\right] \left[\text{enter}\right]$.



Discover if all functions have the property of local linearity by zooming in on the point (2, 4). This point is called a *cusp*.

- Explain your observations. Use words like “slope” and “local linearity” to explain if, in the neighborhood of (2, 4), the function becomes one straight line. (You can use **MENU > Window/Zoom > Window Settings** to ensure that your zoomed-in window contains (2, 4).)

Part 4 – Graph another piecewise function

To explore if all piecewise functions lack the property of local linearity, on a new *Graphs* page

zoom in on (2, 4) of the function $f(x) = \begin{cases} x^2 & , x < 2 \\ 4x - 4, & x \geq 2 \end{cases}$.

- Does this function appear to be locally linear in the neighborhood of (2, 4)? Compare and contrast this function to the one graphed and explored in Problem 3.

Part 5 – Conclusion

You know the definition of slope is $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$. For the function $f(x)$, this can be written

as $\frac{\Delta f}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$. If you were finding the slope of function in the interval of a

repeatedly zoomed in graph, describe what happens to $\Delta x = x_2 - x_1$.