EXPLORATIONS

Activity 6

# Class of Gold

## Objective

- To investigate Fibonacci numbers and the ratios of successive Fibonacci numbers

## Materials

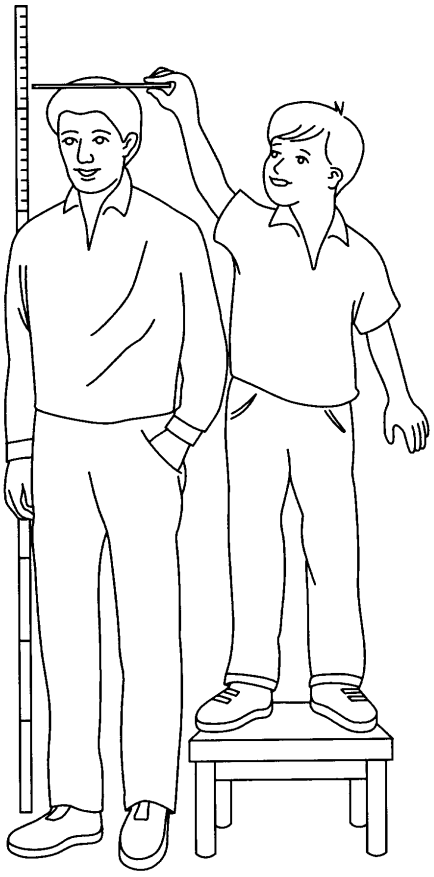
- TI-73 calculator
- Meter stick
- Student Worksheet

### *In this activity you will:*

- take measurements of two different heights
- investigate the relationship of these two heights (a person's height and the height of that person's navel)
- examine a graph (scatterplot) of these two measurements

### *You will need to be familiar with this math vocabulary:*

- ratio
- proportion
- Fibonacci numbers
- the Golden Ratio



### *Introduction*

Patterns occur in nature and in all branches of science. **Fibonacci numbers** (1, 1, 2, 3, 5, 8, 13…) appear in many places in mathematics, and ratios of successive Fibonacci numbers are found in various measurements. Some people have long bodies and short legs while others may have short bodies and long legs. There are advantages and disadvantages to both types of proportion. Can you name some?

### *Problem*

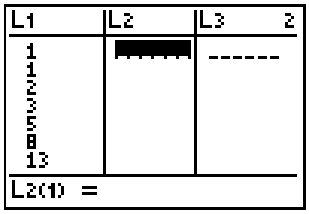
Here are the first seven terms of the Fibonacci sequence:

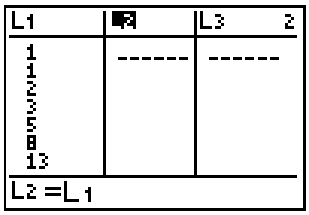
**1, 1, 2, 3, 5, 8, 13.**

On a separate piece of paper, write the next 13 terms.

## *Activity*

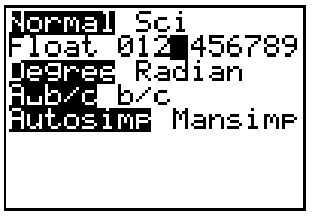
**1.** Enter terms (1-20) in **L1**. Then, enter terms 2-21 in **L2**. Use the shortcut of pasting **L1** in **L2**, deleting the first term in **L2** and adding a 21st term at the bottom of list. To do this, highlight **L2**, press [2nd] [STAT] and select **1: L1** [ENTER]. Don't forget to add the 21st term in as the 20th element in **L2**.



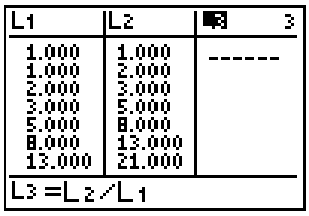




**2.** Find the ratio of successive Fibonacci numbers as decimals rounded to the nearest thousandth.

**a.** Set the [MODE] to 3 decimal places.

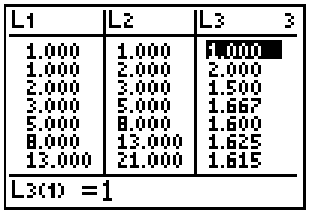



**b.** Let **L3** = **L2**/ **L1** by highlighting **L3** and entering the formula **L2**/ **L1**, and then pressing [ENTER]. (See the screen at the right.)




**c.** Scroll down **L3**. What do you notice about these ratios as you go down the list?

Do you notice that the numbers approach 1.618? This is the approximated value of the **Golden Ratio**. The Golden Ratio is prevalent in Greek architecture and is found in rectangles and in other geometric shapes.




**3.** According to some statisticians, the ratio of a person's height to the height of their navel should be close to the Golden Ratio (1.618) if the person is properly proportioned.

**a.** You and your partner will now measure each other and find these two heights to the nearest tenth of a centimeter. Then find the ratio of the two numbers. For example, if your height is 7mm marks past 161 cm, then call it 161.7 cm.

✎ Go to the Student Worksheet and record the measurements.

**b.** Give your name and your heights to the teacher to be recorded on the teacher's transparency.

✎ When your teacher displays the class data, copy it on Table 1 of your Student Worksheet.

**4.** Name a list **HT**. To do this, press [LIST] and scroll over to the first empty list. Press [2nd][TEXT], use the cursor to select **H,** press [ENTER], select **T**, press [ENTER], select **Done** [ENTER] [ENTER]. Name a list to the right of this list as **NHT** for navel height using similar keystrokes. The third list name that corresponds to Table 1 will be called **HTNHT** to stand for **HT** ÷ **NHT**.

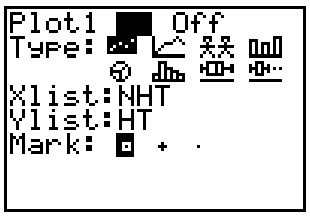
**a.** Enter class students' heights in list **HT** and navel heights in a list **NHT**. Then find the ratio of **HT**÷ **NHT**. To do this press [2nd][STAT], select **HT** [ENTER][÷][2nd][STAT] **NHT,** select [ENTER]. Record these ratios in the fourth column of Table 1.

✎ Answer questions 1 and 2 on the Student Worksheet.

**b.** Find the mean average of your classes ratios. Press [2nd][STAT] [▸] [▸] to **Math**, select **3:mean(**, then type **LHTNHT** and press [ENTER].

✎ Answer question 3 on the Student Worksheet.

**5.** You can determine relationships between two quantities like height and navel height using the TI-73. Create a Scatterplot of navel height vs. Height. The person's navel height will be on the horizontal axes and their height will be on the vertical axes. Before setting up the graph, press [2nd] [PLOT] and select **4: Plotsoff** [ENTER] to turn the plots off. You will work on Plot 1. Press [2nd] [PLOT] and select **1: Plot1…On** to turn Plot 1 on. Define Plot 1 as shown at the right.




**a.** Set an appropriate window remembering that the **X** values are the navel heights and the **Y** values are the student heights. Note that **Xmin** and **Xmax** refer to the minimum and maximum values on the horizontal axis and **Ymin** and **Ymax** refer to the vertical axis. **Xscl** and **Yscl** defines the distance between tick marks on the axis.

**b.** Before viewing the graph, press [Y=] and make sure that all the equations are turned off. Now press [GRAPH]. [TRACE] the graph and find the data point that represents your navel height and height.

✎ Using your window parameters, sketch the graph in the window provided in question 4 on the Student Worksheet. Label the axis with a few reference numbers.

**6.** Now you will place a line on the plotted data. This line would represent a graph if everyone was proportionally the same. You will use the **Manual-Fit** option of the TI-73.

**a.** From the Home screen, press [2nd][STAT] [◂] [2nd] [VARS] **2:Y-Vars** then **1:Y1** [ENTER] [ENTER]. Position the cursor at the beginning of the line you want to draw, then press [ENTER].

**b.** As you press the cursor keys, the line is drawn and the slope or steepness is adjusted. When you have matched the plotted points as desired, press [ENTER]. You can use the cursor keys to adjust the line if you are not happy with where you placed it.

**c.** When you find the best fit, press [ENTER] again.

The equation of this line has now been pasted in the [Y=] editor in **Y1**. This is an approximated equation that describes the relationship between the classes' navel heights and student heights.

✎ Answer questions 5, 6, and 7 on the Student Worksheet.

**d.** If Y= Height, X= Navel Height and Y ÷X = 1.618 then Y= _____. This would be an equation of a line that would describe all people that are proportioned according to the golden ratio. Type this equation into **Y2**. Press [GRAPH].

✎ Answer questions 8 and 9 on the Student Worksheet.

**7.** Use the mean average of the class to write an equation in **Y3**. Type in the number you found to be the mean in **Y3** and then press [x] to follow it. Press [GRAPH].

✎ Answer questions 10, 11, and 12 on the Student Worksheet.

**8.** What if the measurements were all taken in inches, would the ratio change? To investigate this, convert the measurements in **LHT** and **LNHT** to inches.

**a.** Go to **LIST**. Name 2 new lists **HIN** (height in inches) and **NIN** (navel height in inches). Go to the top of the list **HIN**, highlighting **HIN**, press [ENTER] to select the list you want to convert (**LHT**) in [2nd][STAT]. Press [2nd] [CONVERT] select **1:Length** [ENTER], select **2:cm**, then **4:inch** and [ENTER] again. Do the same thing with the list called **NHT**.

**b.** Next, take the ratio of **HIN** to **NIN** into a list named **RATIO**. Compare this **RATIO** list to the ratios in the list named **HTNHT**. Explain your findings in question 13.
***Note:*** To make the lists easier to compare paste **HTNHT** to the right of **RATIO**.

✎ Answer questions 14, 15, and 16 on the Student Worksheet.


Name_____

Date _____

## Activity 6

# Class of Gold

*Record your results on the table below. Then answer the questions about the activity.*

My height in centimeters is _____

The height of my navel is _____

My height (Ht)/ Navel height (NHt) = _____

***Table 1***

| Name | Height (cm) LHT | Navel Height (cm) LNHT | Height/ Navel Ht. LHTNHT |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

**1.** Which ratio is the "most golden"? Justify your answer with a reason.

_____

_____

**2.** Which ratio is the "least golden"? Justify your answer with a reason.

_____

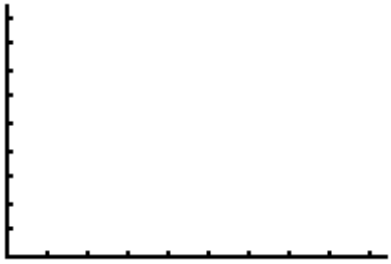
_____

**3.** Find the mean average of the **HTNHT** list. How golden is your class?

_____

_____

**4.** Sketch the graph of navel height verse student height. Make sure to label the axes and use numbers to show the values on number lines.



**5.** In general, as navel height increases, what happens to student height?

_____

_____

**6.** Write the equation of the line that has been pasted in the Y= editor.

_____

_____

**7.** The number in front of the x in the equation is defined as the ratio of change in y over change in x. How does this number compare to the Golden ratio?

_____

_____

**8.** If Y= Height, X= Navel Height and Y ÷X = 1.618 then Y= _____

**9.** Are there any similarities between **Y1** and **Y2**? Explain.

_____

_____

**10.** Are there any similarities between **Y1**, **Y2**, and **Y3**? Explain.

_____

_____

_____

**11.** Based on the relationship of a person that fits a truly golden proportion, which equation would you use to find their height?

_____

**12.** Find the height of a truly "golden" person whose navel height is 100 cm.

_____

**13.** How does the ratio of height to navel height in centimeters compare to the ratio of height to navel height in inches?

_____

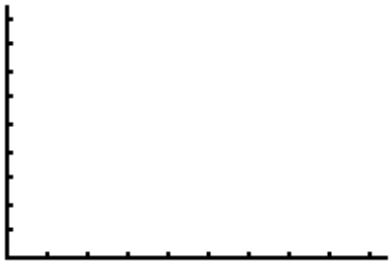
_____

**14.** Write a paragraph on how you could predict your height in three years if your navel height increases by 10 centimeters.

_____

_____

**15.** Sketch a graph of the lines of **Y1**, **Y2**, and **Y3**.



**16.** If the golden ratio (1.618) represents a person whose height is perfectly proportionally to their leg length discuss the possible ratios of people with long legs and short bodies and short legs and long bodies.

_____

_____

_____

_____

_____

# Teacher Notes

## Activity 6

# Class of Gold

## Math Strand

- Patterns and functions
- Algebraic reasoning

## Materials

- TI-73 calculator
- Meter sticks
- Rulers
- Student Worksheet (page 51)
- Teacher transparency (page 51)

Students will investigate Fibonacci numbers and the ratios of successive Fibonacci numbers. They will take measurements of their height and their navel heights and investigate the relationship between the two. Finally, they will examine a scatterplot and a linear relationship fitted to the data points.

### *Vocabulary*

| | |
|---|---|
| ratio | The comparison of two numbers by division. |
| proportion | An equation that involves equal ratios. |
| Fibonacci numbers | A pattern of numbers generated by adding two successive numbers to obtain the next. {1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597…} |
| The Golden Ratio | $\frac{\sqrt{5}+1}{2}$ which is approximately equal to 1.618. |

### *Classroom Management*

After measurements are taken, this activity could be totally teacher-directed, or the students could do the investigation in pairs or small groups. However, you may want to introduce the activity with the opening paragraph. Middle school students can be sensitive to their body proportions. You may want to have the students discuss how important it is that everyone is unique as well as the advantages and disadvantages of both proportions described in the opening paragraph. When pairing students to take measurements, you may want to pair girls with girls and boys with boys. When measuring the height of their navels have the students hold a ruler perpendicular to the navel so the other student does not have to find it.

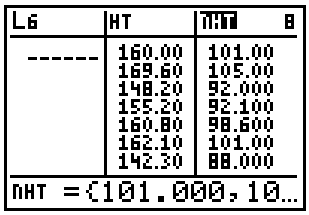
## Activity

1. For instructions on accessing a list, see Appendix A. You may want to make sure that the students have turned off stat plots, cleared any equations out of the equation editor, and executed the SetUp Editor before they begin this activity. Press [2nd][PLOT] **4: PlotsOff** [ENTER] [Y=] and scroll down and press [CLEAR] to remove equations. To execute the Setup Editor, press [2nd][CATALOG] and scroll down to **SetUpEditor**, then press [ENTER].

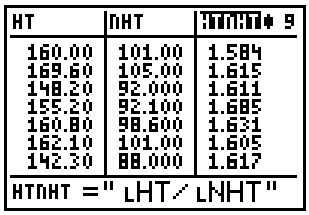
2. For more information on using formulas in lists, see Appendix C. If students use the formula **L3 = L2/L1** and get a **dim mismatch** error, then they probably forgot to enter the 21st Fibonacci number as the 20th element in **L2**, or **L1** and **L2** do not contain the same number of elements.

3. Have a transparency of Table 1 available for students to record their heights.

4. For more information on naming lists, see Appendix B. The screen at the right shows sample data of heights and navel heights.

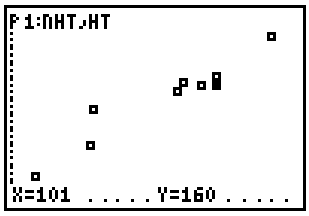



   To find the ratio of **HT** to **NHT**, you may want to have the students put the formula in quotes. This will allow students to view the formula at the top of the list and will automatically change the ratio if you find someone measured inaccurately and you go in and change a measurement in one of the lists in the formula.

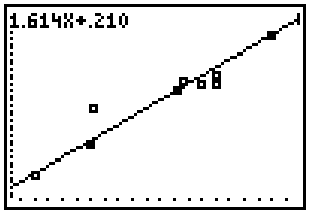


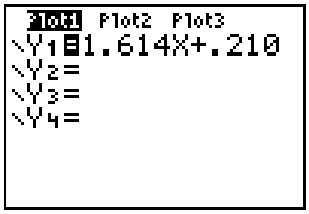

5. Students may need help setting an appropriate window depending on their experience. You may want them to use [ZOOM] **7:ZoomStat**, but it is very important that students learn how to determine range and scaling when drawing graphs. (If they need help, have them begin by entering the smallest value from **NHT** for **Xmin** and the largest value from **NHT** for **Xmax.** Do the same for **Ymin** and **Ymax,** using the values from **HT**.)

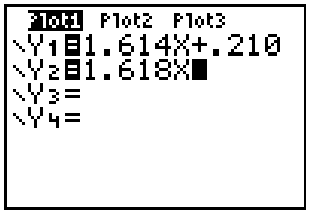
   Sample data of a scatterplot is at the right.

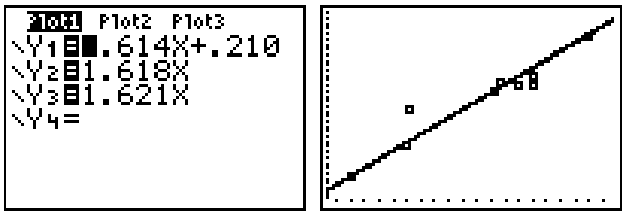



6. Sample data of **Y1**, **Y2**, and **Y3**. The last screen shot appears to be one line but it is all three.

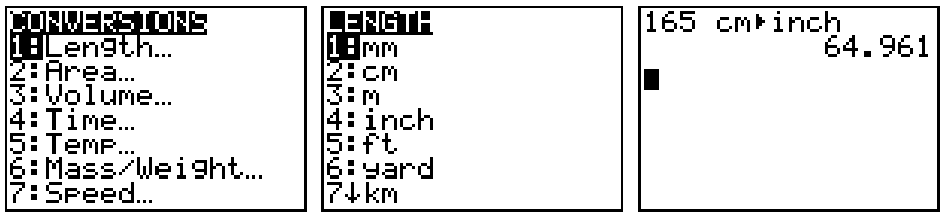





7. If **Conversion** function has never been used, you may want to have the students convert their measurements in centimeters to inches (or vice-versa). They will do this on the Home screen.




## *Answers to Student Worksheet*

1. Answers will vary. It should be the ratio closest to 1.618.
2. Answers will vary. It should be the ratio the furthest from 1.618.
3. Answers will vary. It should be close to 1.6.
4. See student sketch.
5. It increases.
6. Answers will vary.
7. It should be fairly close to 1.618.
8. Y=1.618 X
9. Answers will vary. They should look close to the same line.
10. Answers will vary. They should all look close to the same line.
11. Y=1.618 X
12. 100 cm x 1.618 = 161.8 cm
13. It is the same.
14. Add 10 cm to present navel height and multiply it by the person's ratio.
15. See sketch.
16. A ratio greater than 1.618 would indicate the person has shorter legs and a longer body. A ratio less than 1.618 would indicate the person has longer legs and a shorter body.

## *Going Further*

Have students investigate where Fibonacci numbers and the golden ratio are found in nature and in science.

Depending on the level of student, you may want to have them solve this quadratic equation to derive the algebraic representation of the golden ratio. The ratio *of 1 to a positive number is equal to the ratio of the number to 1 minus the number.*

$$\frac{1}{x} = \frac{x}{1-x}$$

$$1 - x = x^2$$

$$x^2 + x - 1 = 0$$

Using the quadratic formula you find x = $\frac{\sqrt{5}+1}{2}$