John Unson, Gates Chili HS, Rochester, NY Time required: 30 minutes

Activity Overview

This lesson is intended to allow students to investigate the segment relationships when 2 secants are drawn to a circle from a common external point. Pages include a statement of the theorem, a dynamic geometry demonstration, several problems that apply the theorem, and a 2-column geometric proof of the theorem.

Teacher Preparation

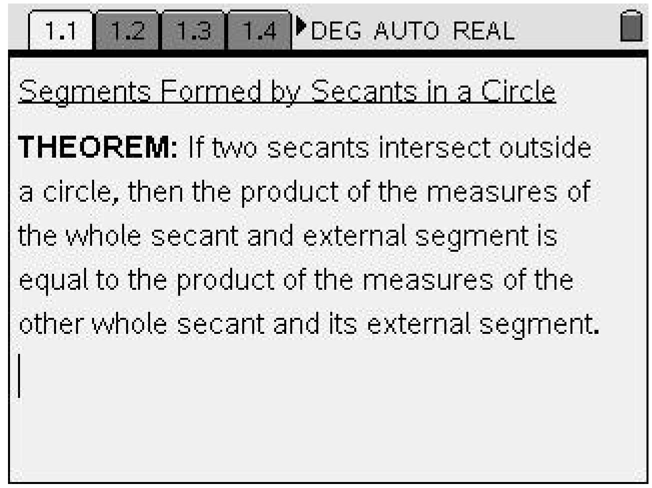
This lesson is created for use in a middle school or high school geometry class.

- Similar triangles have corresponding sides that are proportionate.
- External segments of secants, along with the whole secants themselves, will have equal products. This is the premise of this lesson.

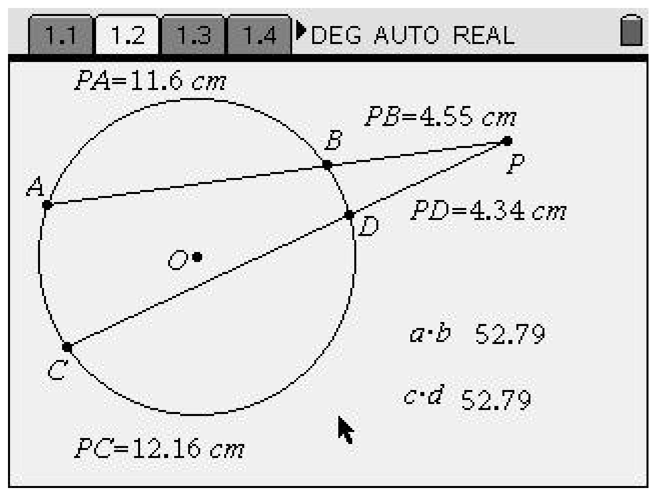
Classroom Management

- This lesson is intended to allow students to investigate the secant-segment relationships using the TI-Nspire Geometry Application.
- Students need to read carefully due to the fact that sometimes they are given (or asked to find) segment measures (both internal & external), and at other times the lengths of the entire secants.
- The student worksheet contains additional diagrams that allow the student to work on each of the problems, as well as the geometric proof.

On page 1.1, the theorem is stated for the students. Most students will experience greater success working with these problems if they always deal with external segment measures. In the case that they are given internal segment lengths, they should subtract from the whole secant length to obtain the external length, if possible.



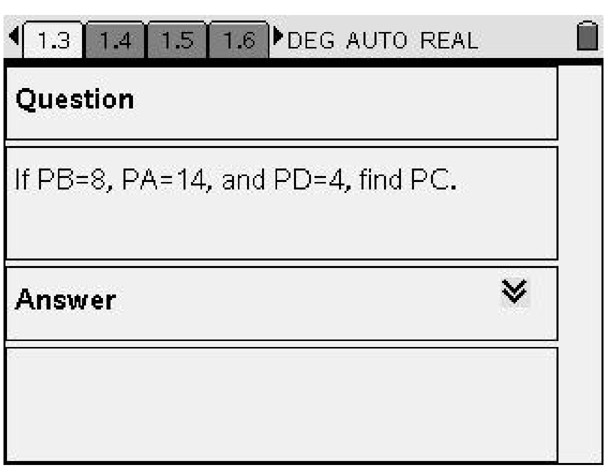
On page 1.2, the Geometry Application allows the student to manipulate the measures of the segments by dragging the endpoints of the secants, or point P, to different locations. As the measures of the external segments and whole secants change, so do the products, and the student is expected to be able to confirm that these products remain equal to each other.



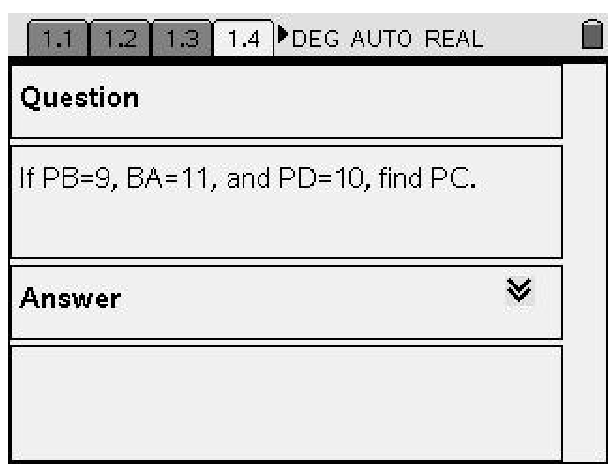
The questions on pages 1.3-1.6 each require use of the theorem and correspond to the diagrams provided on the student worksheet.

Answers:

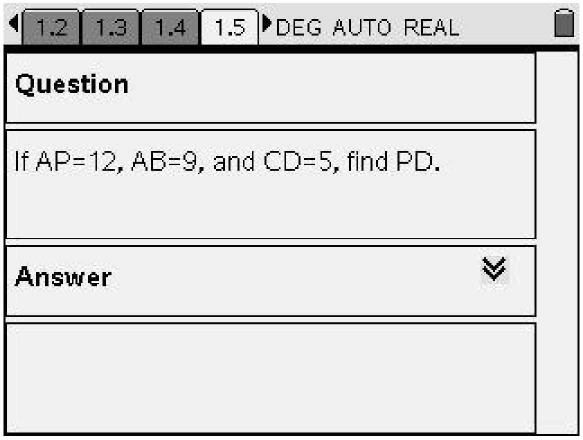
1.3 – PC = 28



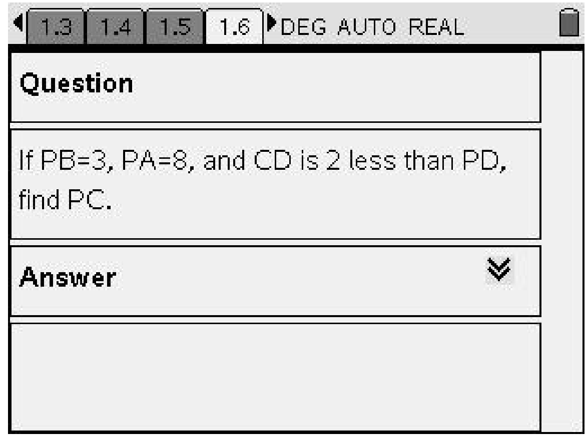
1.4 – PC = 18



1.5 – PD = 4



1.6 – PC = 6



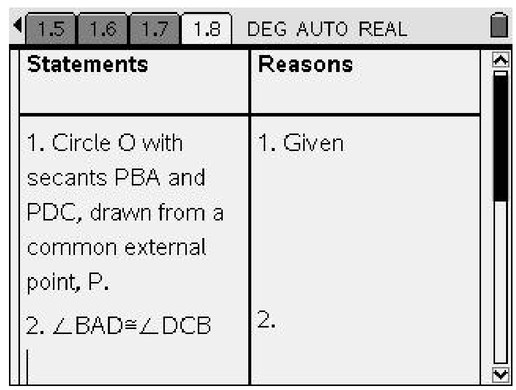
Page 1.7 instructs the student to complete the geometric proof that follows.

Page 1.8 illustrates a 6 step geometric proof of the theorem.

It is necessary to draw additional chords (AD & BC) in order to clearly identify the inscribed angles and triangles ADP & CBP referred to in the proof.

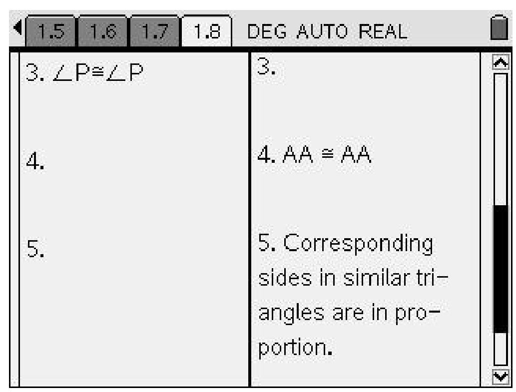
The missing items are:

Reason #2 – Two angles inscribed in the same arc are equal.



Reason #3 – Reflexive property.

Statement #4 – ∠ADP ~ ∠CBP.



Statement #5 – $\frac{PA}{PC} = \frac{PD}{PB}$.

Reason #6 – The product of the means is equal to the product of the extremes.