NUMB3RS Activity: A Matter of Half-Life and Death
Episode: “Dirty Bomb”

Topic: Exponential Decay **Grade Level:** 9 - 12
Objective: Recognize the properties of exponential decay functions through graphs and tables; become familiar with units and measures used with radiation
Materials: TI-83 Plus/TI-84 Plus graphing calculator
Time: 20 - 30 minutes

Introduction

A truck hauling radioactive waste is hijacked on its way to New Mexico. The FBI is called in to consider the possibility that the hijackers intend to use the radioactive material to make a “dirty bomb”. This activity is based on the discussion between Charlie and Larry regarding the properties of Cesium-137, a radioactive isotope that can result from the fission of plutonium and uranium, a common source of energy used to produce nuclear power. The activity uses a variation of the conventional function for exponential decay, in which the values of the function are determined using the appropriate time unit. This bypasses the extra step of determining the number of half-lives and then converting to the desired time unit.

Discuss with Students

The typical function dealing with exponential decay is $y = a(1/2)^x$, in which a is the initial amount, x is the number of half-lives, and y is the final amount. This activity deals with Caesium-137, which has a half-life of 30.23 years. Following exposure, or if ingested, it has a *biological* half-life of approximately 100 days (the length of time required for the body to eliminate half of the radiation). This activity uses the model $y = a(1/2)^{(x/h)}$, where h is the length of one half-life, and x is the time unit of the half-life.

The teacher may want to preview the types of units used in this activity. Here is a concise summary:

Half-life: time required for half of the nuclei in a substance to decay

Biological half-life: time required for the body to eliminate half of the radiation acquired from exposure

Becquerel (Bq): unit for measuring radioactivity, representing the decay of one nucleus per second (this unit replaced an older one called a *curie*, 3.7×10^{10} decays per second)

Joule (J): the amount of energy required to lift 1kg to a height of 10cm

Gray (Gy): unit for absorbed dose of radiation, defined as 1J/kg, for measuring physical effects of radiation exposure, such as burns

Sievert (Sv): the same definition as a Gray, but used to measure biological effects

Student Page Answers:
1. *158.88 grams* **2.** *About 100 years (at 99.99 years, the amount of 50.498 g, the first value that will round to 50).* **3.** *About 269 years (at 268.65 years, the amount is 1.0498 g, the first value that rounds to 1.0 g).* **4.** *With an initial amount of 1 and half-life 100, it would take 326 days, where the actual amount is 10.4%)* **5.** *Using 1971 as year 0, 1984 is year 13, and the amount was 54.93 TBq* **6.** *1987 was year 16, and the amount was 51.28TBq.* **7.** *2006 is year 35, so the amount is currently 32.42TBq.* **8.** *(3 Gy ÷ 4.56 Gy/hr) ≈ 0.66hr, or about 40 minutes.*

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Name: ________________________________ Date: ________________

NUMB3RS Activity: A Matter of Half-Life and Death

A truck hauling radioactive waste is hijacked on its way to New Mexico. The FBI is called in to consider the possibility that the hijackers intend to use the radioactive material to make a "dirty bomb." A dirty bomb, also called a Radiological Dispersal Device (RDD), uses conventional explosives like dynamite, to spread radioactive material over a large area. Charlie and Larry discuss the properties of the material and its impact if used to make a bomb.

The material on the hijacked truck is Caesium-137 (Cs-137, or ^{137}Cs) – a radioactive isotope usually formed by nuclear fission, such as is used to produce nuclear power. It is the principal radiation source at the site of the Chernobyl disaster, which occurred in Russia in 1986. Its half-life is 30.23 years, where a half-life is the amount of time for half of the nuclei of the substance to decay.

Charlie and Larry estimate that the amount of pure ^{137}Cs on the truck is about 500 grams. If made into a dirty bomb, there is enough to contaminate an area of nearly 30 km^2. For ^{137}Cs's decay, use the model $y = a(.5)^{(x/h)}$, in which *a* represents the initial amount, *h* is the length of a half-life in a specified time unit, *x* is the time, and *y* is the resulting amount. Enter the equation on your [Y=] screen (using *x*). Use the [WINDOW] shown to draw the [GRAPH] of the first half-life. Note that after one half-life, exactly half of the original ^{137}Cs remains.

WINDOW
Xmin=0
Xmax=30.23
Xscl=1
Ymin=0
Ymax=500
Yscl=10
Xres=1

Now examine the table of values for this function. Use [2nd] [TBLSET] with TblStart =0, and ΔTbl=10. Use [2nd] [TABLE] to view the table as at the right. Experiment with different values of TblStart and ΔTbl, as well as the [WINDOW] for the graph, then switch between the graph and table to answer the following questions (express answers to #2 and #3 to the nearest hundredth of a year, consistent with the way that the half-life of ^{137}Cs is given):

X	Y_1	
0	500	
10	397.55	
20	316.09	
30	251.32	
40	199.83	
50	158.88	
60	126.33	

X=0

1. How much of the original 500 g of ^{137}Cs will remain after exactly 50 years?
2. How long will it take so that 10% (to the nearest gram) of the original 500 g remains?
3. Suppose the "safe" amount in this sample of radioactive waste is 1.0 gram. How many years does this take for this amount to remain?

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In 1984, a hospital in Goiânia, Brazil was abandoned. One of the items left behind was a small canister of ^{137}Cs (originally made in 1971), used for radiation therapy. In 1987, two people found and managed to damage the canister, exposing themselves and many others to the radiation. As a result, four people, including a child, died; another 20 were hospitalized. Nearly 250 people received measurable doses of radioactivity. The incident is considered one of the worst accidents ever involving the mishandling of radioactive material. (For the details of the "Goiânia Accident," see "Extensions".)

The *biological* half-life of ^{137}Cs after ingestion is approximately 100 days (the time required for half the amount of radiation to leave the body).

4. Modify your equation from above to determine how long a person would have to survive (to the nearest whole day) following exposure such that 10% (to the nearest whole percent) of the radiation remains (use 1 as the initial amount).

Radioactivity is measure in *becquerels* (Bq) and represents the decay of one nucleus per second. The canister in Brazil contained 74 TBq (T - "tera", or 10^{12}) in 1971.

5. Using the half-life of ^{137}Cs (not the biological half-life), how much radioactivity was in the canister when it was abandoned in 1984?

6. How much radioactivity was there when it was found in 1987?

7. How much radioactivity is there now (2006)?

A *joule* (J) is approximately the amount of energy needed to lift 1kg of mass to a height of 10cm. The unit used to measure *absorbed dose* of radiation is the *gray* (Gy). It is defined as 1 J/kg, or the amount of energy absorbed by one kilogram of matter. Grays are used to measure physical effects (like burns) from radiation exposure. A similar unit, called a *sievert* (Sv), has the same definition but is used to measure the biological effects on different tissues (like cancer in muscles or bones).

The dose rate of the Goiânia canister at a distance of one meter was 4.56 Gy/hr.

8. There is a good chance (>95%) of surviving a dose of less than 3 Gy, thereby making it a "safe" dose. What is the maximum time of "safe" exposure?

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The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

The Goiânia Accident in 1987 is considered one of the worst incidents in the history of the Atomic Age, caused by the mishandling and improper disposal of nuclear material. Many variables determine the final outcome of exposure to radiation, including whether there is one single exposure, several exposures, low exposure over a period of time, what parts of the body are exposed, etc. In addition, the size, age, and general health of exposed individuals are also a consideration.

From a mathematical standpoint, creating models using these variables can provide students with valuable insights into the complexities and dangers of dealing with radioactive substances. Concerns regarding such topics as nuclear power generation and terrorism (the "dirty bomb") are important and merit investigation. The first resource below contains much detailed information that could serve as the springboard for student research projects in chemistry, physics, and mathematical modeling.

Additional Resources

For a detailed summary of the Goiânia Accident prepared by two university professors the year after the accident, see:
http://arts.bev.net/roperldavid/GRI.htm

For a less technical summary of the Goiânia Accident, see:
http://www.nbc-med.org/SiteContent/MedRef/OnlineRef/CaseStudies/csGoiania.html

Related Topic

Caesium was used in the first-ever functional atomic clock. To learn how it worked, see:
http://www.sciencemuseum.org.uk/on-line/atomclocks/page3.asp

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