Name ________________________

Class ________________________

Functions that are continuous and differentiable at a center can be approximated by polynomials. One such way of doing this is to generate a Maclaurin polynomial for a function.

An approximating polynomial is to be expanded about the center *c* which is in the domain of a function *f*. If this *c* has the same value in a polynomial *P* and function *f* then *P*(*c*) = *f*(*c*). Graphically, *P*(*c*) = *f*(*c*) means that the graph of *P* passes through the point (*c*, *f*(*c*)).

A Maclaurin polynomial is a polynomial that is based upon a function's derivatives at *c* = 0.

Specifically, the *n*th Maclaurin polynomial is defined as

$$P_n(x) = \frac{f(0)}{0!}x^0 + \frac{f'(0)}{1!}x^1 + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^{(n)}(0)}{n!}x^n$$

$$P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^{(n)}(0)}{n!}x^n$$

Problem 1 – Maclaurin polynomial for *f*(*x*) = sin(*x*)

In generating the third degree Maclaurin Polynomial for *f*(*x*) = sin(*x*), we compute

$$P_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}(x)^2 + \frac{f'''(0)}{3!}(x)^3$$

$f(0)$ = _____ $f''(0)$ = _____

$f'(0)$ = _____ $f'''(0)$ = _____

Then substitute the values into the Maclaurin polynomial. This results in:

$P_3(x) =$

In Y1 Taylor function, ***taylor(sin(x), x, degree)***. Students change the value of degree to view different graphs of polynomials approximated by sin(*x*).

1. What do you notice when the degree is 1 and 2? Why do you think this is?

2. What do you notice when the degree is 3 and 4? Why do you think this is?

Problem 2 – Maclaurin polynomial for $f(x) = e^x$

1. Write $P_1(x)$, $P_2(x)$, and $P_3(x)$ for $f(x) = e^x$

2. Graph $f(x)$, $P_1(x)$, $P_2(x)$, and $P_3(x)$

   What do you notice?

Problem 3 – Maclaurin polynomial for $f(x) = \cos(x)$

1. Find P_8(x) for $f(x) = \cos(x)$.

2. What do you notice about the value of each derivative after 0 has been substituted?

3. What do you notice about the approximated polynomial?

4. Write two expressions to describe you findings in the previous question when differentiating cos(x) in terms of n.

5. Graph P_8(x) and $f(x) = \cos(x)$.

   What do you notice?