

Activity 4

Objectives

- Use analytic geometry to investigate the attributes of geometric figures
- Practice mathematical verification
- Use the Shading and Point-of-Intersection Trace features of the Inequality Graphing application

Perimeters, Areas, and Slopes—Oh, My!

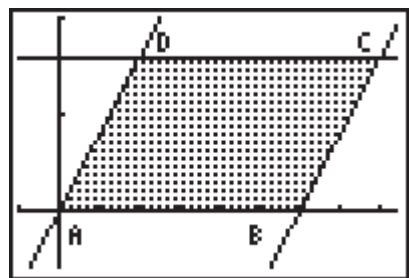
Introduction

French mathematician and philosopher René Descartes, who was credited with the linking of algebra and geometry, was a frail child who spent many childhood hours in bed staring at the ceiling of his bedroom. According to legend, it was his interest in the position of a fly on the ceiling that provided the impetus for the important mathematical link between algebra and geometry. Descartes realized that he could describe the location of the fly if he created a numbered grid on his ceiling. This discovery was the birth of analytic geometry, the basis of all applied mathematics.

In this activity, you will use the Inequality Graphing application to create geometric figures. You will then use analytic and coordinate geometry to investigate the attributes of geometric figures.

Problem

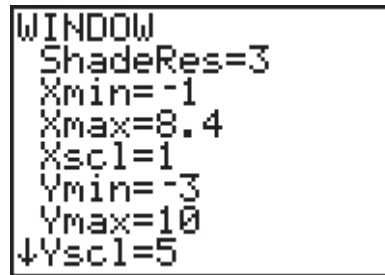
The figure in the graphic shown appears to be a parallelogram. However, appearance alone cannot serve as proof. Mathematicians require proof before they will draw any definitive conclusions. How can you use algebra to prove that quadrilateral $ABCD$ is a parallelogram?



Exploration

The first step in providing an analytic proof is to construct a frame of reference as Descartes did. You can do this by positioning quadrilateral $ABCD$ in the Cartesian coordinate plane, so named in honor of René Descartes. Place vertex A at the origin $(0, 0)$. However, you still need more information before you can accurately determine the coordinates of the other three vertices.

Take a look at the window settings that were used to create the graph. Press **WINDOW**, and then enter the values shown. (Set **Xres=1**)

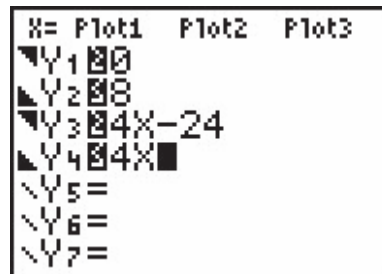


Now that a frame of reference has been established, you are ready to graph the system of inequalities that determines the quadrilateral *ABCD* and its feasible region.

1. Use the Inequality Graphing App to graph the system:

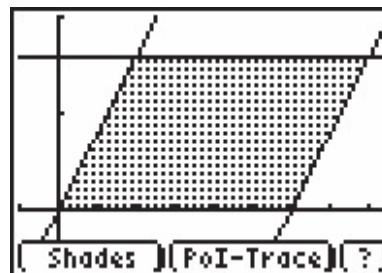
$$y \geq 0; y \leq 8; y \geq 4x - 24; y \leq 4x$$

2. Enter the inequalities above in the **Y=** editor. (Remember to press **ALPHA** and the appropriate **F** key to change the inequality signs.)



3. Press **GRAPH** to graph the system of inequalities.

4. Use the Shading feature to see the feasible region of the system of inequalities. Select **Shades** (**ALPHA** **F1**). Then select **Ineq Intersection** and press **ENTER**.

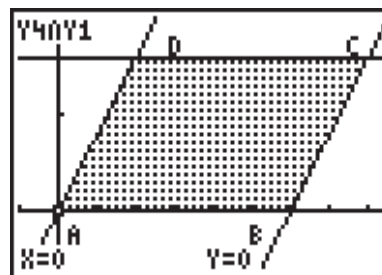


5. Use the Point-of-Intersection Trace (**Pol-Trace**) feature to find the other three vertices.

Note: Make sure **CoordOn** is highlighted in the **FORMAT Menu**.

Record the coordinates of vertices *B*, *C*, and *D*.

B: _____ *C*: _____ *D*: _____



The next step is to devise a strategy for determining whether quadrilateral *ABCD* is a parallelogram. Brainstorm some ideas with a partner or group of students. Then describe the procedure that you have decided to use.

6. Use your procedure to determine if quadrilateral $ABCD$ is a parallelogram. Provide mathematical evidence to support your conclusion.

7. Coordinate geometry can also be used to calculate other attributes, such as segment lengths, perimeters, and areas of geometric figures. Find the perimeter and area of parallelogram $ABCD$. Show your work.

Student Worksheet

Name _____

Date _____

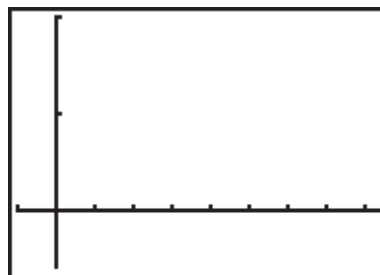
Use the window settings shown for this worksheet.
(Set $Xres=1$)

```
WINDOW
ShadeRes=3
Xmin=-1
Xmax=8.4
Xscl=1
Ymin=-3
Ymax=10
↓Yscl=5
```

1. Use the Inequality Graphing application to graph the system of inequalities:

$$y \geq 0; y \leq 5; y \leq 5x; y \leq -1.25x + 8.75$$

Sketch the graph of the feasible region on the grid provided.



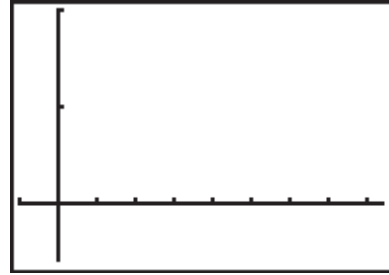
2. Find the coordinates of the vertices of the feasible region. Show that the polygon formed is a trapezoid. Label the trapezoid $RSTU$.
3. Verify that the median of trapezoid $RSTU$ equals one-half the sum of its bases.

4. Use the Inequality Graphing application to graph the following system of inequalities:

$$y \geq 0; y \leq 7; x \geq 1; x \leq 7$$

Sketch the graph of the feasible region on the grid provided.

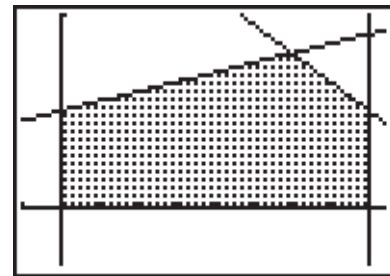
Note: Highlight the **X=** in the upper left and press **ENTER** to switch between the **Y=** and **X=** editors.



5. Find the coordinates of the vertices of the feasible region. Label the rectangle $WXYZ$. Show that the quadrilateral formed by connecting the midpoints of the rectangle is a rhombus.

6. The feasible region in the shape of the pentagon shown has vertices $(0, 0)$, $(8, 0)$, $(8, 5)$, $(6, 8)$, and $(0, 5)$.

- Label the pentagon $JKLMN$.
- Determine the system of inequalities that has been graphed to create the feasible region.



- Find the perimeter and area of the pentagon $JKLMN$.

Teacher Notes



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Objectives

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Materials

- TI-84 Plus/TI-83 Plus
- Inequality Graphing application

Teaching Time

- 60 minutes

Prerequisite Skills

- Knowledge of polygons and their attributes
- Knowledge of slope, distance, and midpoint formulas
- Ability to write linear equations in different forms given two coordinates
- Ability to graph systems of linear inequalities and determine their feasible regions

Management

There are a number of algebraic and geometric skills required to complete the Exploration and Student Worksheet items. Depending on student mastery of these skills, you may wish to monitor student progress closely and provide assistance as needed. It is important that students provide appropriate mathematical evidence when responding to verification and proof items.

Students are encouraged to work in pairs or in groups through the Exploration activity and the Student Worksheet. Allow students to share ideas and to provide assistance to one another as needed.

Notes about Exploration

The Exploration guides students through the process of proving that quadrilateral $ABCD$ is a parallelogram. Many students will benefit from a follow-up class discussion of the concepts used before proceeding to the Student Worksheet.

In order to create the screenshots used in this lesson, the **CoordOff**, **AxesOff**, and **ExprOff** settings from the **FORMAT Menu** ($\boxed{2nd}$ [FORMAT]) were selected. To remove the **Shades**, **Pol-Trace**, and **Help** options from view, press \boxed{TRACE} .

The vertex labels can be created by using the **DRAW Menu**. Press $\boxed{2\text{nd}} \boxed{[\text{DRAW}]} \mathbf{0}$ to select **0:Text**. Use the cursors to position each label.

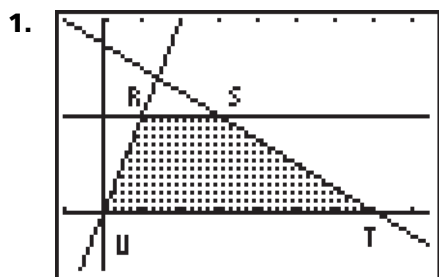
Answers to the Exploration Questions

5. $B: (6, 0); C: (8, 8); D: (2, 8)$

Answers will vary. One way is to find the slopes of the sides of the quadrilateral. If the slopes of the opposite sides are equal, then the quadrilateral is a parallelogram.

6. The slope of $\overline{AB} = \text{slope of } \overline{DC} = 0$; slope of $\overline{BC} = \text{slope of } \overline{AD} = 4$; There are two pairs of parallel sides, thus the figure is a parallelogram.
7. The perimeter = $12 + 4\sqrt{17}$ or approximately 28.5 units; area = 48 square units

Answers to Student Worksheet



2. The coordinates of the vertices are $(0, 0)$, $(7, 0)$, $(3, 5)$, and $(1, 5)$.

slope of $\overline{RS} = \text{slope of } \overline{TU} = 0$, slope of $\overline{RU} = 5$, slope of $\overline{ST} = -1.25$

\overline{RS} and \overline{TU} are parallel. \overline{RU} and \overline{ST} are not parallel. Therefore, quadrilateral $RSTU$ is a trapezoid.

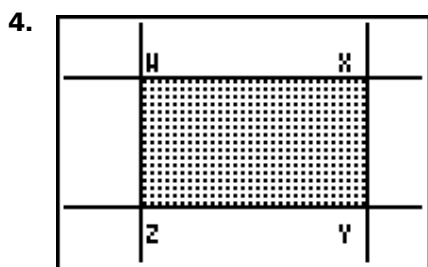
Note: If students labeled their vertices in a different order, adjustments in answers must be made.

3. The median is equal to one half of the sum of the bases of the trapezoid.

midpoint of $\overline{RU} = \left(\frac{1}{2}, \frac{5}{2}\right)$, midpoint of $\overline{ST} = \left(5, \frac{5}{2}\right)$

median length = $\sqrt{(4.5)^2 + 0^2} = 4.5$

$RS = 2$, $TU = 7$, $\frac{RS + TU}{2} = 4.5$



5. The coordinates of the vertices are $(1, 7)$, $(7, 7)$, $(7, 0)$, and $(1, 0)$.

Let E , F , G , and H represent the midpoints of \overline{WX} , \overline{XY} , \overline{YZ} , and \overline{ZW} , respectively.

Point E has coordinates of $(4, 7)$. Point F has coordinates of $(7, 3.5)$.

Point G has coordinates of $(4, 0)$. Point H has coordinates of $(1, 3.5)$.

$$\text{slope of } \overline{EH} = \text{slope of } \overline{FG} = \frac{7}{6}, \text{ slope of } \overline{EF} = \text{slope of } \overline{HG} = \frac{-7}{6}$$

$$EF = FG = \sqrt{9 + 12.25} = \sqrt{21.25}$$

A rhombus is a parallelogram that has two equal adjacent sides.

6. $x \geq 0$, $x \leq 8$, $y \geq 0$, $y \leq 0.5x + 5$, $y \leq -1.5x + 17$

perimeter = $18 + \sqrt{13} + 3\sqrt{5}$ or approximately 28.3 units; area = 52 square units