Integers! It All Adds Up!
ID: 13309

Time required
45 minutes

Activity Overview
In this activity, students will use the number line to investigate how to add integers and to justify and support the rules that they develop. Students will use the Number Line application of the TI-73 calculator.

Topic: Integer Addition
- *Explore addition of integers using a number line.*
- *Develop strategies for adding integers.*

Teacher Preparation and Notes
- *Distribute the Student Activity pages. Have students work in pairs using one calculator between them. Roles that can be assigned are that of reader/recorder and Number Line "Operator."*
- *The Number Line application should be loaded onto students' calculator before beginning the activity, if it is not already there. To download the application, go to education.ti.com.*
- *In the Number Line App, the first option under* ZOOM *is* ***ZoomFit****. This activity does not make use of this option since the intent is to have students do mental arithmetic as much as possible. Students will certainly see* ***ZoomFit*** *and test its capabilities, letting you know that option is much faster than setting the viewing window! You may wish to postpone the use of* ***ZoomFit*** *for this activity to encourage the development of mental arithmetic skills and number sense.*
- *TI-Navigator is not required for this activity, but an extension is given for those teachers that would like to use it.*
- ***To download the student worksheet and Learning Check document, go to education.ti.com/exchange and enter "13309" in the quick search box.***

Associated Materials
- *MGWeek02_IntegerAdd_worksheet_TI73.doc*
- *MGWeek02_LrnChk_TI73.edc*
- *Number Line app*

Suggested Related Activities
To download any activity listed, go to <u>education.ti.com/exchange</u> *and enter the number in the quick search box.*
- *Integer Subtraction – What's the Difference? (TI-73 Explorer) — 4481*
- *Multiplication of Integers (TI-73 Explorer) — 4482*

Introduction

Integers (the collection of whole numbers, the opposites of the nonzero whole numbers, and 0) are quite familiar to people today, but at one time they were not accepted as true numbers in the mathematical community. Many mathematicians referred to them as "absurd," "ugly," or "fictitious" numbers even though integers would arise as solutions to problems. For example, an East Indian text from about 1150 A.D. indicates a solution to an equation as L5 but then makes fun of such a useless response!

In 1545, the Italian mathematician Girolamo Cardano, published the rules governing operations with negative numbers in his text *Ars Magna* (*Great Art*).

Problem 1 – Adding positive integers

1. Have students access the Number Line application by pressing $\boxed{\text{APPS}}$ and selecting the NUMLINE application.

Press $\boxed{\text{ENTER}}$ twice to get to the NUM/FRAC LINE menu and select option **1:Number Line**.

If the standard or default window doesn't appear with the line indicator at 0 and a view of the number line from –20 to 20, have students either press $\boxed{\text{WINDOW}}$ and change the **Min**, **Max**, **Scale**, and **Start** values (default values are –20, 20, 2 and 0, respectively) OR press $\boxed{\text{ZOOM}}$ and select option **2:ZStandard**.

Note: If no values are displayed on the Number Line other than **Min** and **Max,** check to be sure that **LabelOn** is selected in the WINDOW settings. If previous work is displayed, press $\boxed{\text{CLEAR}}$ **1:Yes** to clear the window.

2. Students can initially investigate something familiar to them on the Number Line by adding positive integers or whole numbers, and then explaining how addition of two or more numbers can be represented on the number line.

When students enter 13 + 8, the screen will show 0 + 13 + 8. The starting value will always appear and they do not need to enter + 13 + 8. As students begin to type in numbers, the application automatically assumes addition.

If students make errors while typing in the entry, they can use the arrow keys to backspace and type over the error, or use the $\boxed{\text{DEL}}$ key. If they have already pressed $\boxed{\text{ENTER}}$ and find an error, they should press $\boxed{\text{ENTER}}$ again, press $\boxed{\triangleleft}$ or $\boxed{\triangleright}$ to move to the error and correct it, then press $\boxed{\text{ENTER}}$ or $\boxed{\text{GRAPH}}$.
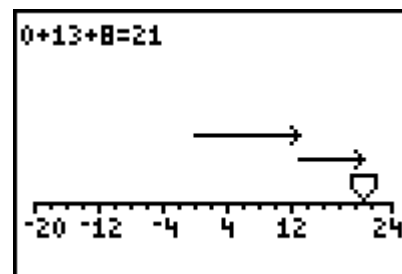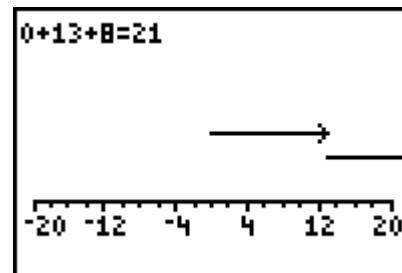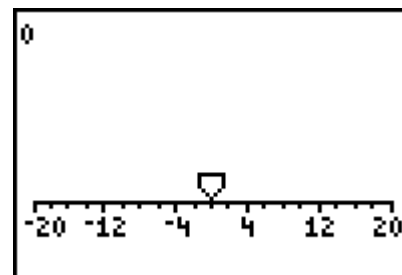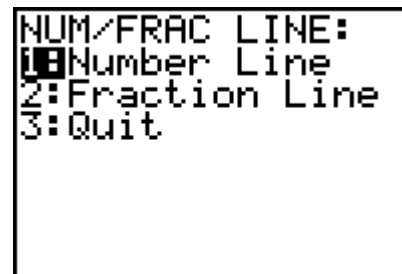
3. Ask students to explain how the "number rays" or vectors represent the operation of addition. Introduce students to the term *vector* and how it is used throughout the activity. The responses should have some reference to the length of the vectors being the same as the absolute value of the integer. The vectors should be pointed in the positive direction on the number line, with one vector beginning at the end of the other, and the sum being at the endpoint (arrow head) of the final vector.

4. Ask students to add 15 to the previous sum of 21 and determine an appropriate **Max** for the viewing window before they press GRAPH. Encourage students to share their mental arithmetic strategies. They only need an estimate to determine a new **Max**, but many will try to find the exact sum. A sample strategy might be 20 + 10 is 30 and 5 + 1 is 6 so the new sum is 36. If you view many "36's" for the maximum value as you roam the room, you know students were searching for the exact sum. You might want to talk about why the max could be 38, 39, 40, etc.
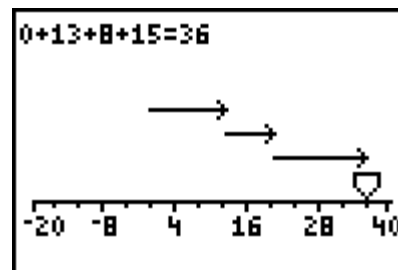
Problem 2 – Adding negative integers

5. Students clear the window and draw a sketch of their prediction for 0 + –12. Sketches should show one vector, starting from zero and ending at –12. Make sure that students know the difference between the ⊟ and ⊟ keys. (The ⊟ key allows you to enter negative values or to "take the opposite of" the current entry. The ⊟ key is used for the operation of subtraction.) The TI-73 displays the negative sign as a raised character.

6. Students add –8 to –12. Ask how the vectors show addition of negative integers. An explanation similar to the one for positive integers could be used, but the vectors are pointed in the negative or opposite direction.

7. Students add –5 to –20 and select a new **Min**. They should notice that the continued sum of negative integers will produce vectors that appear further to the left, away from zero. As long as you are adding negative integers, the vectors will not change direction. Addition of negative integers is similar to adding positive integers in that you continue moving in the same direction on the number line as you add. A difference is the direction in which you move on the line; one in a positive direction, the other in the negative direction.

    You might want to talk about the idea that although the absolute value of the sum is increasing, the sum is getting smaller. A movement towards the left on the number line will produce a number smaller than the starting point (and movement towards the right on the number line will produce a number greater than the starting point). Students usually believe this idea on the positive side of the number line, but struggle with it on the negative side.

You could make "real world" comparisons to the following:

- temperature: Which is colder, 10° below zero or 20° below zero?
- money: Would you rather be $10 in debt or $50 in debt?
- sports: Would you rather have a 5-yard loss or 15-yard loss in a football play?

Each example may help illustrate the idea that negative integers closer to zero have a greater (or more preferred) value than those that are further away, or, as you move to the left, the integer value gets smaller.

If students have had some previous experience with integer arithmetic, some might say that the vector could point in the positive direction because "a negative and a negative equals a positive." They are recalling multiplication rules without meaning. Repeatedly adding –5, for example, would model $n \times$ **–5**, which would be a negative value. They can use that thinking when they discuss multiplication.

Problem 3 – Adding negative and positive integers

8. Here you can observe from the student sketches whether students understand that addition of a negative number moves one in the negative direction. Instead of zero being the starting point, students have moved first to 15. From there, they will add –9, which should move them 9 spaces to the left.

9. Encourage students not to change their sketches in step 8 if they don't exactly match what is produced by the application. Have them discuss the differences and why they are acceptable or not. The sketches provide important information for you to use in determining their understanding of the addition.
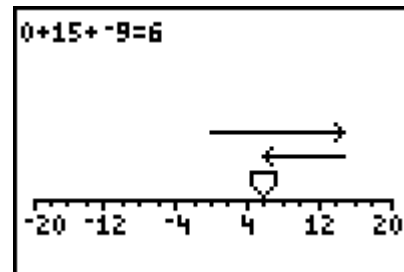
10. Students are asked if –19 added to the previous sum would produce a sum that is still in the viewing window. This question focuses on number sense. Students might think "If I'm on the right side of zero, it would take adding a number less than –20 to get me out of the viewing window. Negative 19 is bigger than –20 so I shouldn't be out of the window range." Here is another opportunity to encourage appropriate thinking. Students might say "I need to add a number bigger than –20 to move out of the viewing window" when they need to think *smaller* than –20.

11. The feeling students might get when adding positive and negative integers is one of "changing directions," "going back and forth," or being "pulled in opposite directions" on the Number Line. Remind students to use only the ◄ and ► keys when "taking a ride." Using the ▼ and ▲ keys produces a different kind of "ride."

12. Through their own examples, students should see the commutative property at work and determine that order doesn't matter. Some are surprised when this happens. "How can you start off in a positive direction one time and a negative direction the other and still end up at the same place?" The calculator allows them to explore many examples to help see why this is the case.

Problem 4 – Opposites

13. Students may write that to find opposites, you simply "turn around and go back as far as you came" or "if you're pulled 13 away from zero, the opposite pulls you 13 spaces back."

Having them think about always returning to zero should help them understand that the sum of two opposites is zero. They can "build" several sets of opposites without having to press CLEAR because the starting point is always zero when adding the opposites.
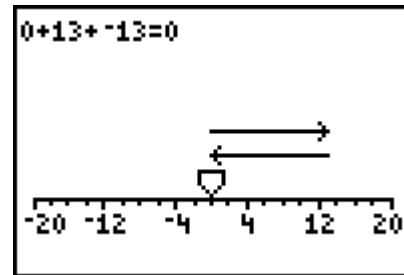
Problem 5 – Wrap Up

14. Students may write that for the sum of positive integers, you always move in a positive direction or to the right on the number line. When adding negative integers, you always move in the negative direction or to the left. Adding positives and negatives moves you in both directions. Some students may comment that adding positives and negatives is like subtraction. It is, but students must pay attention to the sign of the integer which is greater in absolute value in order to determine the sign of the sum. Having the visual image of the number line may help many students understand why the sum of one set of positive and negative integers can be positive while another sum can be negative.

Additional Problems

15. Encourage students to share their strategies either orally or perhaps as a journal entry. Use writing activities with caution when sharing mental strategies. Having students write their thoughts often discourages mental arithmetic since "I have to write it down anyway!"

TI-Navigator Extension

1. After Question 1, use **Screen Capture** to make sure that everyone has the correct window settings. If they do, then all of the screenshots should look exactly the same.

2. After Question 4, send a **Quick Poll** to see what value students choose for their **Max** value.

3. After Question 8, use **Quick Poll** to gather the classes' guesses of whether the sum of 15 + –9 will be positive or negative. This will allow students to discuss how they've applied the knowledge of the positive and negative vectors in Problems 1 and 2 to estimate the sum.

4. After Question 12, use **Screen Capture** to view the students' extra practice of adding positive and negative integers. Discuss with students to see if they learned anything that they did not from Problems 1 and 2.

5. After students have finished the activity, send the Learning Check file *MGWeek02_LrnChk_TI73.edc*. This will test what students have learned.