The Pythagorean Theorem: Prove it!!

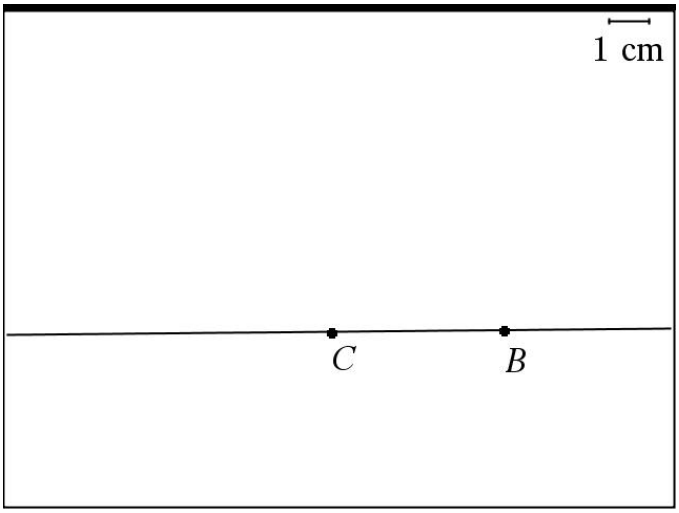
$$a^2 + b^2 = c^2$$

The following development of the relationships in a right triangle and the proof of the Pythagorean Theorem that follows were attributed to President James A. Garfield in 1876.

Step 1 - Construct A horizontal line. Label the point already on the line point B.

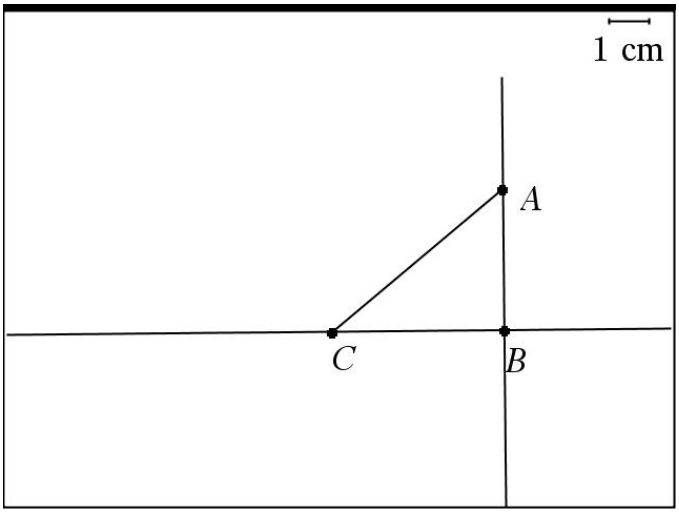
Step 2 – Place a point on the line and label the point C.

Step 1and Step 2 Sample Screen Shot



Step 3 – Construct a perpendicular line through line BC at point B. Place a point on the perpendicular line and label it point A. Draw a line segment from point C to point A, creating the triangle ABC.

Step 3 Sample Screen Shot



Step 4 – Construct a circle with center at point C and radius AB.

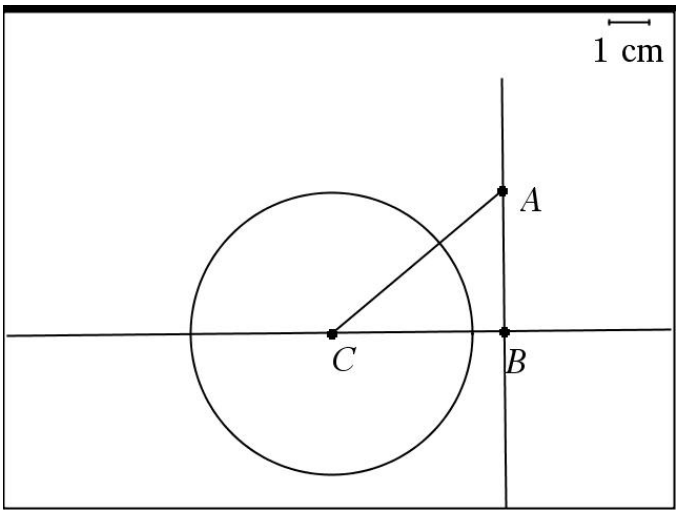
From here the students will need to use the compass tool to draw the circle. They will need to do the following:

1. Under the construction menu choose compass

2. Choose point A and then point B, Move the circle so that the center is on point C.

3. They will now have the circle they want.

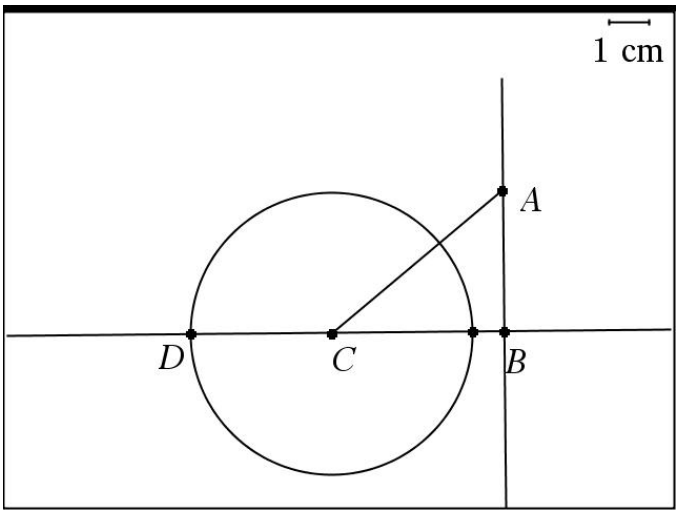
Step 4 Screen Shot



Step 5 Screen Shot

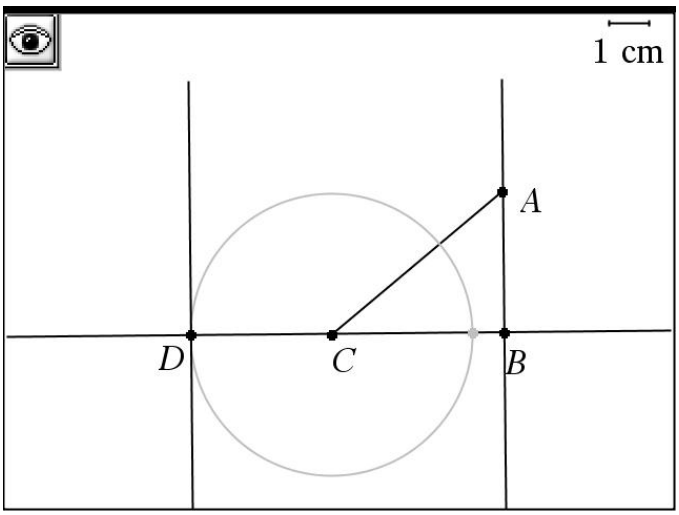
Step 5 – Find the intersection of the circle and line BC, label this point D.

The students should use the point of intersection that is to the left of point C.



Step 6 Screen Shot

Step 6 – Construct a perpendicular line through point D. At this point also have the students hide the circle as shown in the screen shot.



Step 7 – Construct a Circle with center D and radius BC.

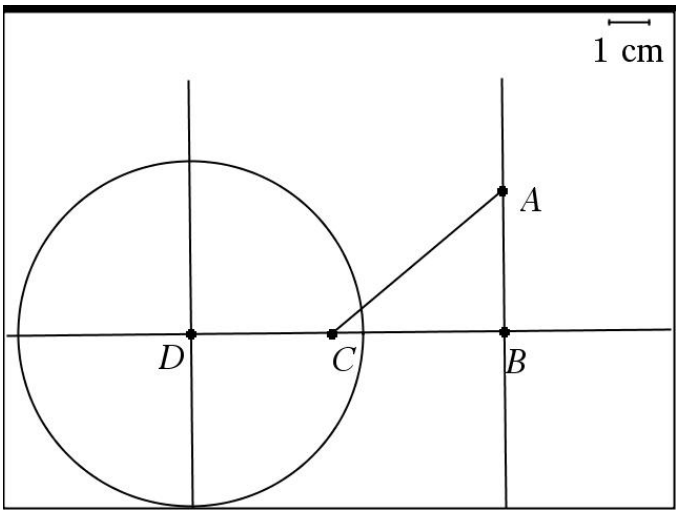
From here the students will need to use the compass tool again to draw the circle. They will need to do the following:

1. Under the construction menu choose compass

2. Choose point A and then point B, Move the circle so that the center is on point C.

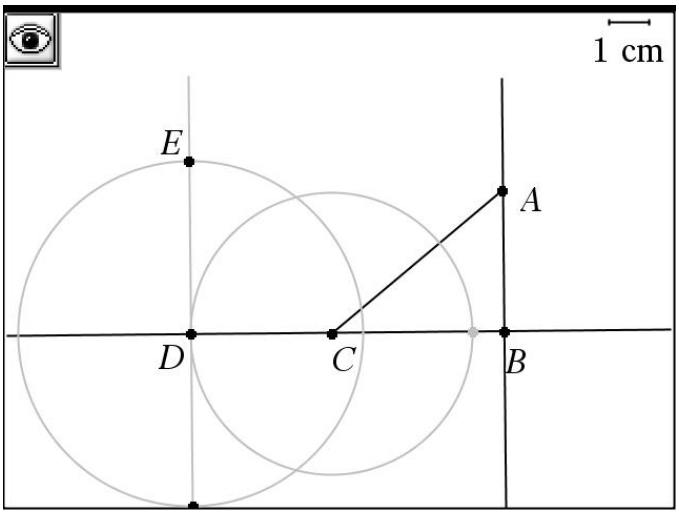
3. They will now have the circle they want.

Step 7 Screen Shot



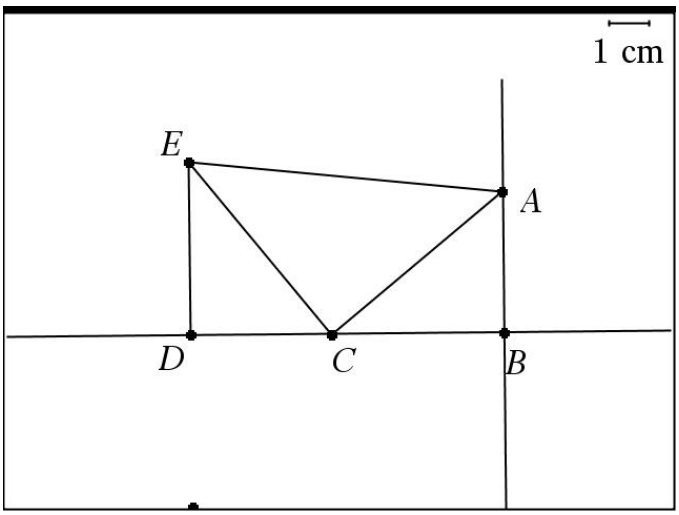
Step 8 – Find the point of intersection of this circle and the vertical line through point D. Label this point E and then hide the circle.

Step 8 Screen Shot



Step 9 – Construct a Line Segment from point E to point D. Also construct a Line Segment from Point E to point C. Also construct a segment from point E to Point A

Step 9 Screen Shot



Step 10 – On page 1.5 the students will complete a two column proof showing that segment EC is congruent to Segment AC.

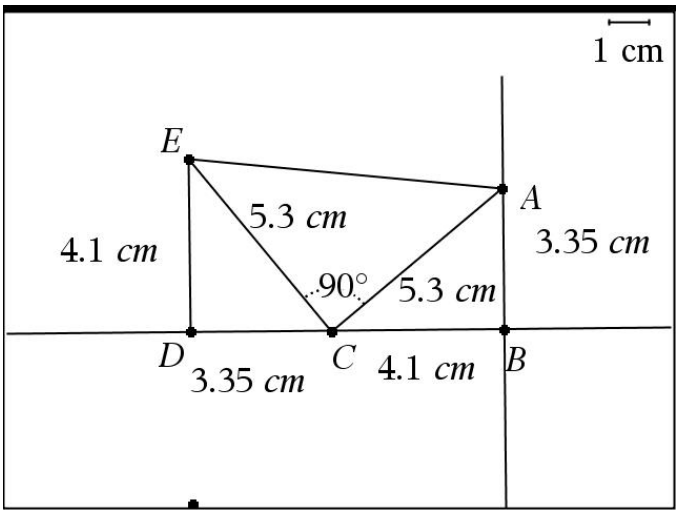
Step 10 Screen Shot

Statements	Reasons

Step 11 – Once the students have completed their proof and I would make them bring it up to me to confirm that their proof is correct they will need to go back and measure segments AB, BC, CD, DE, EC, and AC. You may also want to have the students reduce the number of decimal points to save space on the screen.

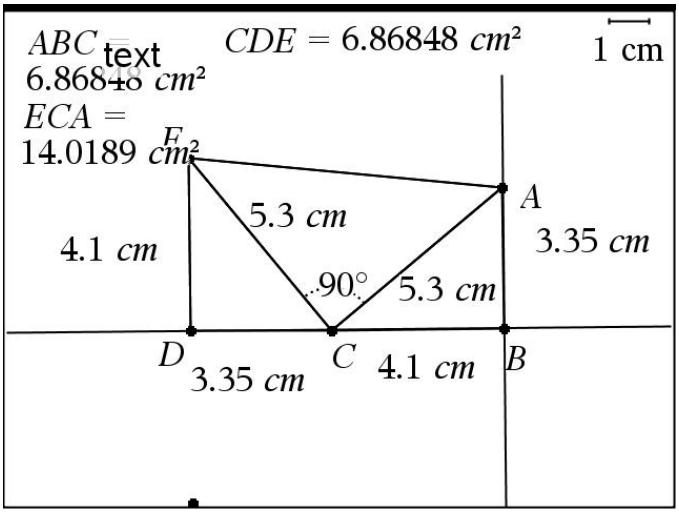
Measure Angle ECA. It appears to be 90 degrees. Can you prove that this angle must be a right angle? Have them use page 1.6 for the two column proof.

Step 11 Screen Shot



Step 12 – Use the shapes menu to construct triangles ABC, CDE and ECA. Then have them find the areas of each triangle and record on their answer sheet. There may not be space on the construction page for them to record their answers as shown in the screen shot. It makes it a little cluttered.

Step 12 Screen Shot



Step 13 – Use the polygon tool under the shapes tool to construct the quadrilateral ABDE.

Have the students answer the questions below on their student work sheet.

Can you prove that this is a trapezoid? Which sides are the parallel bases of the trapezoid? Which side is the height? Have the students use page 1.7 to complete their proof.

Step 14 – The students will need to measure the area of the trapezoid ABCE. Although it is not shown, have them add the areas of the three triangles ABC, CDE, and ECA.

They need to show ABDE = ABC + CDE + ECA

Step 15 – Have the students drag any of the points A, B, or C to alter the construction. Have them remeasure their areas and record their results on their student worksheet. Is your equation (ABDE = ABC + CDE + ECA) holding?

Step 16 – In order to complete the proof, the students will need to label the diagram on their student worksheet as follows:

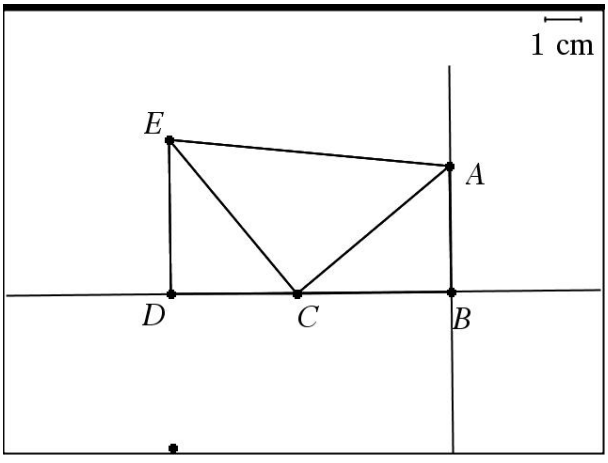
AB = DC = x

BC = ED = y

EC = AC = z

The students will need to show that $x^2 + y^2 = z^2$

In order to do this the students will need to go back and recalculate the areas of the three triangles and the trapezoid in terms of x, y and z. Once they have done that they will need to plug in the values into the equation ABDE = ABC + CDE + ECA and then simplify the equation algebraically.



Screen shots of tns file

The Pythagorean Theorem

President Garfield's Proof

Page 1

Complete the steps from the student worksheet on Page 1.3. Besure to answer all questions on the student worksheet.

Page 2

1 cm

Page 3

On the following page write a two column proof showing that

$EC \cong AC$

Once you have completed your proof go back and measure segments AB, BC, CD, DE, EC, and AC to confirm your proof.

Page 4

Statements	Reasons

Page 5-7