

## ACTIVITY 9

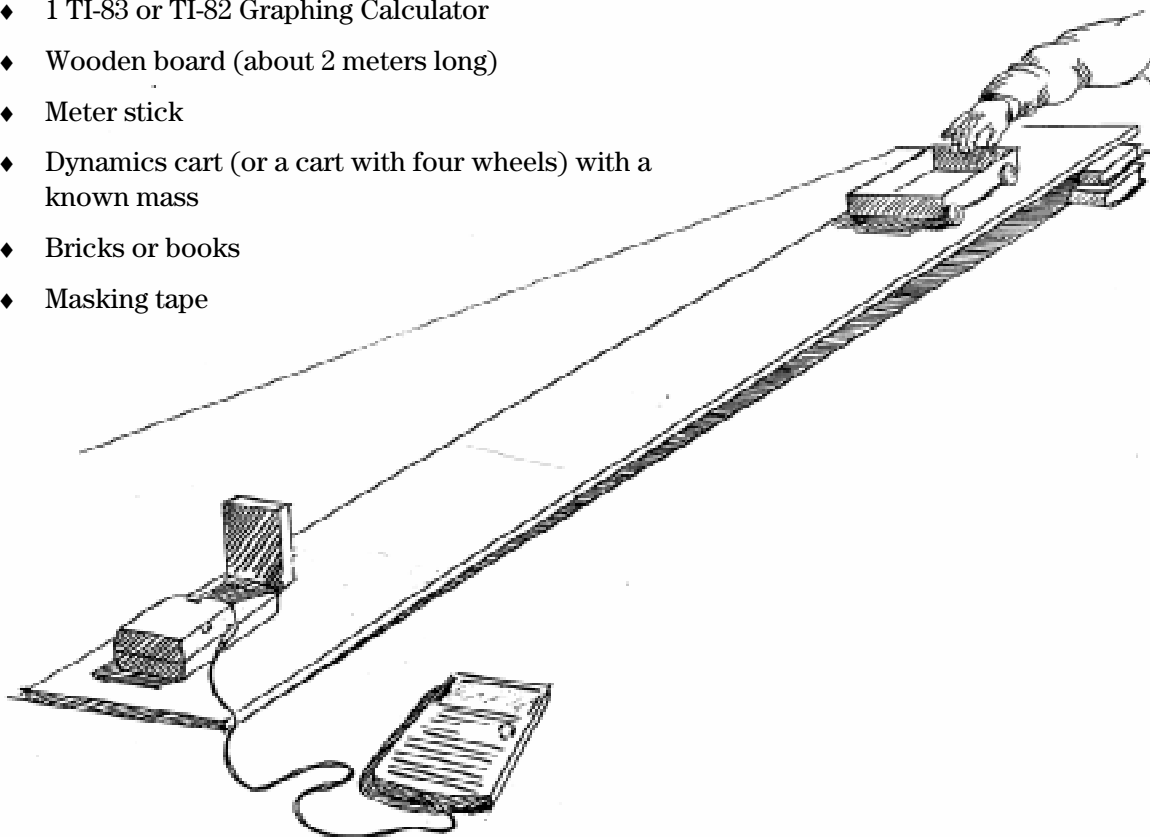
### Conserving Energy

When a cart rolls down a ramp, energy is neither gained nor lost, but rather conserved (when friction is ignored). There are two forms of energy that occur as the cart rolls down the ramp: *kinetic* and *potential*. The *potential energy* of the cart is dependent upon the mass and the distance the cart is above the ground. Stated mathematically, it is  $U = mgh$ . The *kinetic energy* is a result of the motion of the cart as well as its mass. Its mathematical statement is  $K = \frac{1}{2}mv^2$ .

In this activity, you will find both the kinetic and potential energies as the cart rolls down the ramp. You will then find the sum of the two energies, and show that this value is constant at all times.

#### You'll Need

- ◆ 1 CBR unit
- ◆ 1 TI-83 or TI-82 Graphing Calculator
- ◆ Wooden board (about 2 meters long)
- ◆ Meter stick
- ◆ Dynamics cart (or a cart with four wheels) with a known mass
- ◆ Bricks or books
- ◆ Masking tape



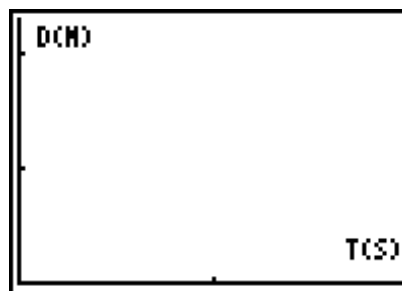
## Instructions

- Stack a few bricks or books underneath one end of the ramp.
- Set the CBR on the bottom of the ramp as shown. Measure 0.5 meter from the CBR and place a piece of masking tape on the ramp at that point.
- Run the **RANGER** program on your calculator.
- Enter the setup instructions.
  - From the **MAIN MENU** select **1:SETUP/SAMPLE** to access the setup menu.
  - Press **ENTER** until the **REALTIME** option reads **no**.
  - Press **↓** (the down arrow) to select the next line **TIME (S)** and press **ENTER** **2** **ENTER** to change the time to **2** seconds.
  - Press **↓** to select the next line. Correct or verify the settings and press **ENTER**. Repeat until the options for each line read as shown at right.
  - Press **↓** to move the cursor to the **START NOW** command. Do *not* press **ENTER**.
- Place the cart at the top of the ramp. Release the cart and simultaneously press **ENTER** on the calculator. Stop the cart before it reaches the CBR or after it stops collecting data.

MAIN MENU	▶START NOW
REALTIME:	NO
TIME (S):	2
DISPLAY:	DIST
BEGIN ON:	[ENTER]
SMOOTHING:	LIGHT
UNITS:	METERS

## Data Collection

- The graph should appear to be part of a parabola that opens downwards. If you are not satisfied with the results, press **ENTER** and select **5:REPEAT SAMPLE**.
- To remove extra data at either end of the graph, press **ENTER** to return to the **PLOT MENU**.
  - Select **4:PLOT TOOLS**. Under **PLOT TOOLS**, select **1:SELECT DOMAIN**.
  - Press **▶** until you reach the top of the parabola. Press **ENTER** to select this as the **LEFT BOUND**.
  - Press **▶** until you reach the bottom of the parabola. Press **ENTER** to select this as the **RIGHT BOUND**.
- Sketch a Distance-Time graph in the space provided.



- Record the known mass (in kilograms) of the cart as  $m$ .

$m =$  \_\_\_\_\_

- Press **ENTER** then **7:QUIT** to quit the program.

## Questions

1. Find the quadratic fit to the distance data.

**For the TI-83:** Press  $\boxed{\text{STAT}}$   $\blacktriangleright$ , select **5:QuadReg**, and then press  $\boxed{2\text{nd}}$   $\boxed{[L1]}$   $\boxed{,}$   $\boxed{2\text{nd}}$   $\boxed{[L2]}$   $\boxed{\text{ENTER}}$ .

**For the TI-82:** Press  $\boxed{\text{STAT}}$   $\blacktriangleright$ , select **6:QuadReg**, and then press  $\boxed{2\text{nd}}$   $\boxed{[L1]}$   $\boxed{,}$   $\boxed{2\text{nd}}$   $\boxed{[L2]}$   $\boxed{\text{ENTER}}$ .

Record the equation.

$$y = \underline{\hspace{10em}}$$

2. A standard equation for the motion of an object with constant acceleration is

$$s = s_0 + v_0 t - \frac{1}{2} a t^2.$$

Compare the coefficients of the two equations.

$$s_0 = \underline{\hspace{2em}} \quad v_0 = \underline{\hspace{2em}} \quad -\frac{1}{2} a = \underline{\hspace{2em}}$$

$$a = \underline{\hspace{2em}}$$

3. Run the **RANGER** program and select **4:PLOT MENU** then **2:VEL-TIME**. The graph of this data should be linear. Press  $\boxed{\text{ENTER}}$  and **7:QUIT** to quit the program. Find the linear fit to the velocity data.

**For the TI-83:** Press  $\boxed{\text{STAT}}$   $\blacktriangleright$ , select **4:LinReg (ax+b)**, and then press  $\boxed{2\text{nd}}$   $\boxed{[L1]}$   $\boxed{,}$   $\boxed{2\text{nd}}$   $\boxed{[L3]}$   $\boxed{\text{ENTER}}$ .

**For the TI-82:** Press  $\boxed{\text{STAT}}$   $\blacktriangleright$ , select **5:LinReg (ax+b)**, and then press  $\boxed{2\text{nd}}$   $\boxed{[L1]}$   $\boxed{,}$   $\boxed{2\text{nd}}$   $\boxed{[L3]}$   $\boxed{\text{ENTER}}$ .

Record the equation.

$$y = \underline{\hspace{10em}}$$

4. A standard equation for the velocity of a particle is  $v = v_0 - at$ . Compare this equation to the linear regression found above.

$$v_0 = \underline{\hspace{2em}} \quad -a = \underline{\hspace{2em}}$$

$$\text{so } a = \underline{\hspace{2em}}$$

5. How do the values of  $v_0$  and  $a$  from the distance and velocity equations compare?

6. From the home screen, store the value of the acceleration as A by entering the value of the acceleration, then pressing  $\boxed{\text{STO}\blacktriangleright}$   $\boxed{\text{ALPHA}}$   $\boxed{[A]}$   $\boxed{\text{ENTER}}$ . Store the value of the mass as M by entering the value of the mass, then pressing  $\boxed{\text{STO}\blacktriangleright}$   $\boxed{\text{ALPHA}}$   $\boxed{[M]}$   $\boxed{\text{ENTER}}$ .

7. The distance from the cart to the CBR is the hypotenuse of a right triangle. Therefore,

$$\sin(\theta) \text{ is } \frac{h}{s}.$$

Find  $h$  in terms of  $s$  and  $\sin(\theta)$ .

$$h = \underline{\hspace{4cm}}$$

8. The potential energy is  $U = mgh$ . Write an expression for  $U$  in terms of  $m$ ,  $g$ ,  $s$  and  $\sin(\theta)$ .

$$U = \underline{\hspace{4cm}}$$

9. Use the fact that  $a = g\sin(\theta)$  to write  $U$  in terms of  $m$ ,  $a$  and  $s$ .

$$U = \underline{\hspace{4cm}}$$

10. The distance data can therefore be multiplied by the mass  $m$  and the acceleration  $a$  to obtain the potential energy. From the home screen, press  $\boxed{\text{ALPHA}} \boxed{\text{M}} \boxed{\text{ALPHA}} \boxed{\text{A}} \boxed{2\text{nd}} \boxed{\text{L2}} \boxed{\text{STO}} \boxed{2\text{nd}} \boxed{\text{L4}} \boxed{\text{ENTER}}$  to store the potential energy data in **L4**.

11. Since the kinetic energy is

$$K = \frac{1}{2}mv^2,$$

from the home screen type  $\boxed{\cdot} \boxed{5} \boxed{\text{ALPHA}} \boxed{\text{M}} \boxed{2\text{nd}} \boxed{\text{L3}} \boxed{x^2} \boxed{\text{STO}} \boxed{2\text{nd}} \boxed{\text{L5}} \boxed{\text{ENTER}}$  to store the kinetic energy data in **L5**.

12. Add the two energies together by typing  $\boxed{2\text{nd}} \boxed{\text{L4}} \boxed{+} \boxed{2\text{nd}} \boxed{\text{L5}} \boxed{\text{STO}} \boxed{2\text{nd}} \boxed{\text{L6}} \boxed{\text{ENTER}}$  to store the total energy in **L6**.

- Press  $\boxed{2\text{nd}} \boxed{\text{STAT PLOT}}$  and select **1:Plot 1**. Highlight **On**, and press  $\boxed{\text{ENTER}}$ .
- Select  $\boxed{\text{L1}}$  for the **Type**, **L1** for the **Xlist**, **L6** for the **Ylist**, and the **dot** for the **Mark**.
- Press  $\boxed{\text{WINDOW}}$  and enter 0 for **Xmin**, 2 for **Xmax**, 0 for **Ymin** and 2 for **Ymax**.



13. Press **9:ZOOM** to sketch. Sketch a plot of the sum of the two energies in the space provided.

14. Press  $\boxed{2\text{nd}} \boxed{\text{DRAW}}$  and select **3:Horizontal**. Use  $\boxed{\blacktriangle}$  and  $\boxed{\blacktriangledown}$  to find the horizontal line that best fits the data. Record the value of the total energy below.

$$U + K = \underline{\hspace{4cm}}$$

**Make a Mathematical Statement**

How did the sum of the potential and kinetic energies become a constant? Some simple algebra can be used to verify the result.

1. Recall that the position was given by

$$s = s_0 + v_0 t - \frac{1}{2} a t^2$$

and that  $h = s \sin(\theta)$ . Write an equation for  $h$  as a function of  $t$ .

$$h(t) = \underline{\hspace{10em}}$$

2. Write an equation for the potential energy as a function of  $t$ .

$$U(t) = \underline{\hspace{10em}}$$

3. Since  $a = g \sin(\theta)$ , substitute  $a$  into the equation above to eliminate  $g \sin(\theta)$ .

$$U(t) = \underline{\hspace{10em}}$$

4. Recall that the velocity equation is  $v = v_0 - at$ . Use this equation to write an equation for the kinetic energy as a function of  $t$ .

$$K(t) = \underline{\hspace{10em}}$$

5. Add the two equations in 3 and 4 together.

$$U(t) + K(t) = \underline{\hspace{15em}}$$

Simplify, and record your result below.

$$\underline{\hspace{15em}}$$

6. Use this result to find the sum of the energies.

$$U + K = \underline{\hspace{10em}}$$

7. Press **[2nd] [QUIT]** to return to the home screen. Press **[2nd] [DRAW]** and select **3:Horizontal**. Enter the theoretical value found in step 6 of this section. Press **[ENTER]**. How does the theoretical value for the constant energy compare to the experimental value found in step 12 on page 48?

$$\underline{\hspace{15em}}$$

