

# Maths and Music



## Student Worksheet

7 8 9 10 11 12



## Representing sound using sine waves

In this activity, we will be using TI-Nspire to represent the sound waves produced on a musical scale.

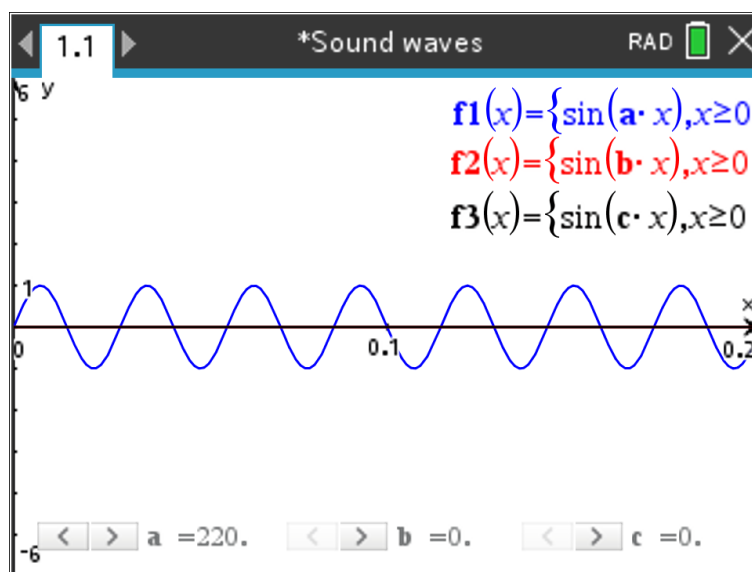
Ensure students are aware that these are not the complete equations for the notes as they have been simplified to cater for these year levels.

## The sine curve

In trigonometry, you should already be familiar with using the sine, cosine and tangent ratios to solve questions involving right angled triangles. As you continue your studies in trigonometry, you will learn about sine, cosine and tangent curves.

Sound travels in waves and when these wave pulses reach your ear drum, signals are sent to the brain which allow you recognise the sounds. Sine curves can be used to represent sound waves. In this activity, you will have the opportunity to explore how the frequency of a sine curve affects the sound generated.

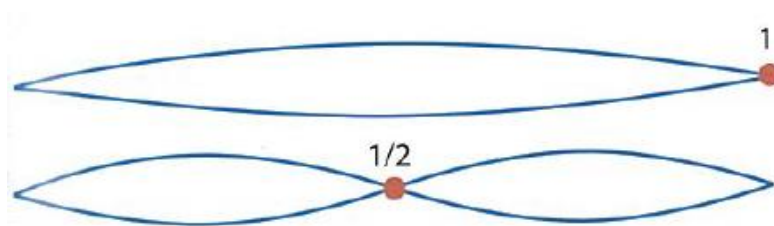
- Follow the instructions in the separate **Sound Waves** document to create a sine curve with a slider. Refer to the separate **Sound Waves** document for these instructions.
- Create a second sine curve with a slider for b.
- Create a third sine curve with a slider for c.
- Alternatively, open the **Sound Waves.tns** file.
- Click on the numbers and set the sliders so **a = 220**, **b = 0** and **c = 0**.



If student have trouble creating sliders, transfer the **Sound Waves.tns** to their handhelds.

## Octaves

Take two taut strings of the same material, one twice the length of the other, and pluck them. The short string vibrates with twice the frequency of the longer string and the resulting notes are an octave apart. The ratio of the octave, as Pythagoras discovered, is 2:1.



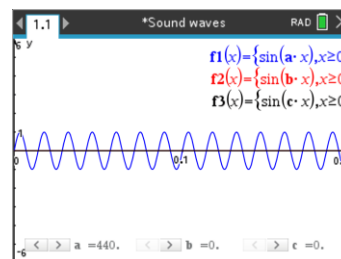
Let's explore octaves using our sliders.

### Question 1

Change the first slider to **a = 440**. What effect did this have on the sine curve?

When changing the slider value, do not use the arrows as the increment is set to 1. It will take a long time to scroll to the correct value. Tell students to click on the number to enter the new value.

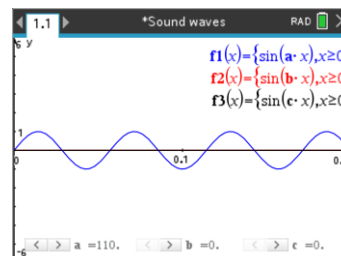
The waves get closer together.



### Question 2

Change the first slider to **a = 110**. What effect did this have on the sine curve?

The waves get further apart.

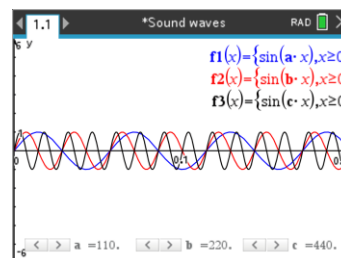


### Question 3

Change the sliders to **a = 110**, **b = 220** and **c = 440**. What can you conclude about the relationship between the frequencies and the shapes of these three sine curves?

Answers will vary but encourage students to observe the doubling factor. E.g.

- When the frequency doubles, the wave gets twice as thin.
- When the 110 curve goes through one cycle, the 220 curves goes through two and the 440 curve goes through four.



### Question 4

Experiment by changing the value(s) of the sliders to various values. What happens to the sine curve when the frequency increases? What happens when the frequency decreases?

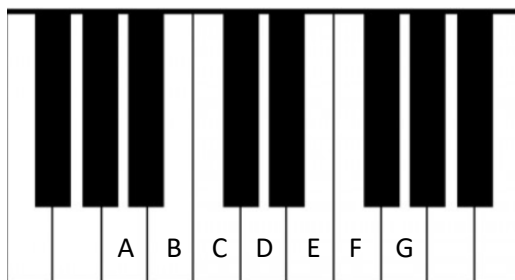
Answers will vary. When the frequency increases, the curve gets compressed. When the frequency decreases, the curve gets stretched.

## Musical scales

A musical scale is a range of notes within an octave. There are twelve notes in a conventional musical scale. The approximate frequencies for one scale of A are shown below.

Note	A	A# B $\flat$	B	C	C# D $\flat$	D	D# E $\flat$	E	F	F# G $\flat$	G	G# A $\flat$	A
Frequency	220	233	247	262	277	294	311	330	349	370	392	415	440

These notes on a piano keyboard are shown below. The highlighted notes in the table above represent the black keys.



The black note between A and B is A sharp (A#). This note can also be written as B flat (B $\flat$ ). The other sharps can also be written as flats of the next note in the scale. E.g., C# = D $\flat$ .

### Question 5

Let's look at a different octave for the A scale, one starting with a frequency of 110 and ending at 220. This means that the frequency of the notes in this octave will be half the frequency of the those in the previous octave. Complete the table below, rounding your answers to the nearest whole value.

Note	A	A# B $\flat$	B	C	C# D $\flat$	D	D# E $\flat$	E	F	F# G $\flat$	G	G# A $\flat$	A
Frequency	110	117	124	131	139	147	156	165	175	185	196	208	220

### Question 6

What do you predict will happen to the frequency of the notes for the A octave from 440 to 880?

The values in the original table will double.

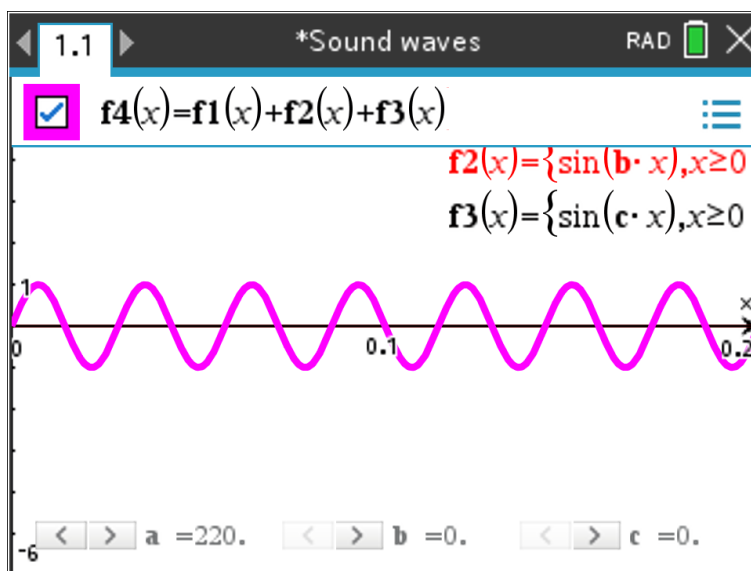
## Exploring concordant and discordant sounds

When the notes played together produce a pleasant sound, the notes are called **concordant**.

When the notes produce an unpleasant sound, the notes are called **discordant**.

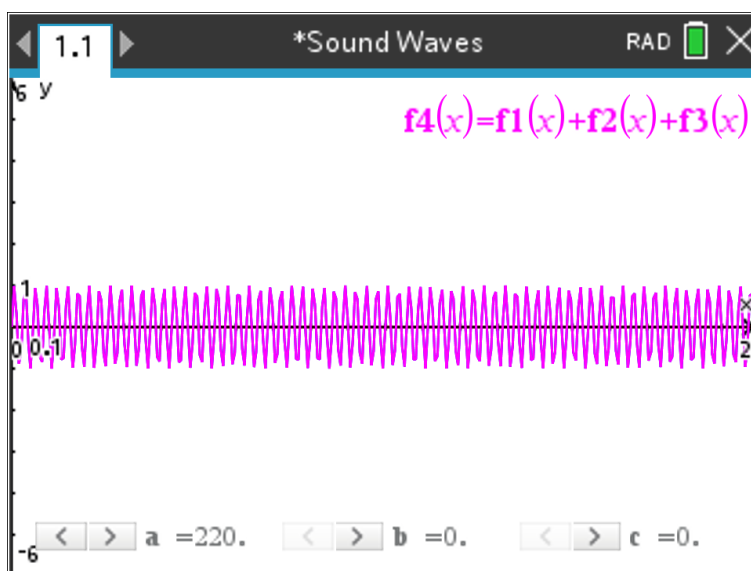
Let's explore different combinations of notes to see which are concordant and which are discordant.

Go to your **Graphs** page with the sliders and create a fourth sine curve, which is the sum of the other three curves.



If you are using the **Sound Waves.tns** file, open the **Graph Entry Line** and tick the **f4(x)** equation. To free up room on the screen, highlight the original equations, press **ctrl + menu** and select **Hide**. Be careful not to delete the equations.

To allow for the combined curve, press **menu > Window/Zoom > Window Settings** and change **XMax**: to **2**.

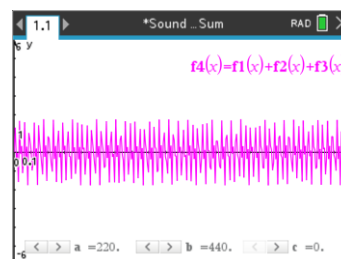


If needed, an additional file labelled **Sound Waves Sum** has been provided. In this file, the first three curves are hidden, the sum of the curves is displayed and **XMax** is set to **2**. Screen captures from this file will be used for the remaining solutions. If students have trouble creating the combined curve, transfer this file to their handhelds.

**Question 7**

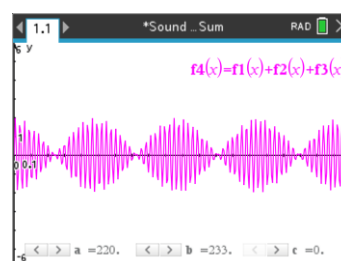
Change the sliders to **a = 220**, **b = 440** and **c = 0** as these two frequencies represent a low and high note of the A octave. What do you notice about the resultant sine curve?

The curve is very even.

**Question 8**

Change the sliders to **a = 220**, **b = 233** and **c = 0** as these frequencies represent the adjacent notes A and A#. What do you notice about the resultant sine curve?

The curve has a series of pulses.



For **question 7**, the two notes produce a smooth curve so the sound will be pleasant to the ear. These notes are **concordant**.

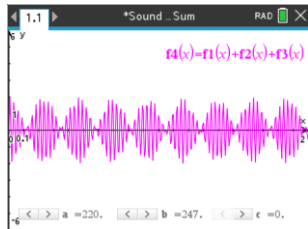
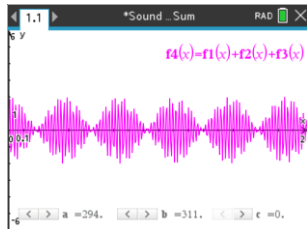
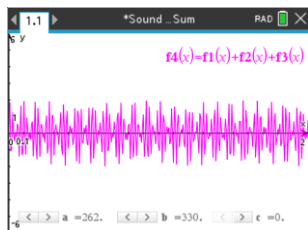
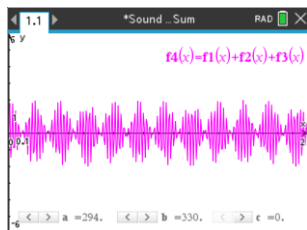
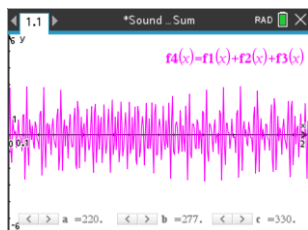
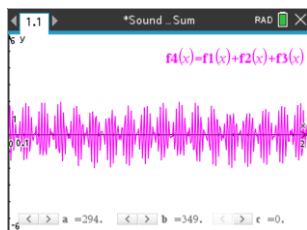
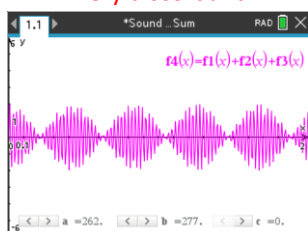
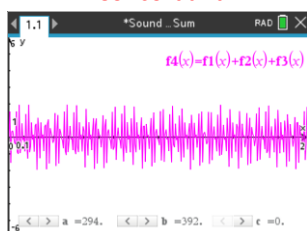
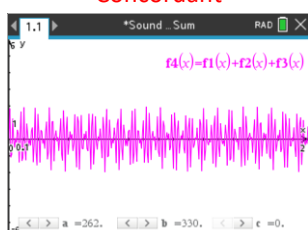
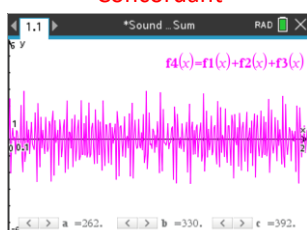
For **question 8**, the two notes produce a series of pulses so the sound will be unpleasant to the ear. These notes are **discordant**.

Below is a copy of the note frequency table from page 3.

Note	A	A# B $\flat$	B	C	C# D $\flat$	D	D# E $\flat$	E	F	F# G $\flat$	G	G# A $\flat$	A
Frequency	220	233	247	262	277	294	311	330	349	370	392	415	440

### Question 9

Set the sliders to the following frequencies to determine if these pairs or groups of notes are concordant (relatively smooth), discordant (small pulses) or very discordant (large pulses).

Notes	Result	Notes	Result
A and B	Discordant 	D and D#	Very discordant 
C and E	Concordant 	D and E	Discordant 
A, C# and E	Concordant 	D and F	Concordant* 
C and C#	Very discordant 	D and G	Concordant 
C and E	Concordant 	C, E and G	Concordant 

\*It could be argued that these notes are discordant but the pulses are small and evenly spread out.

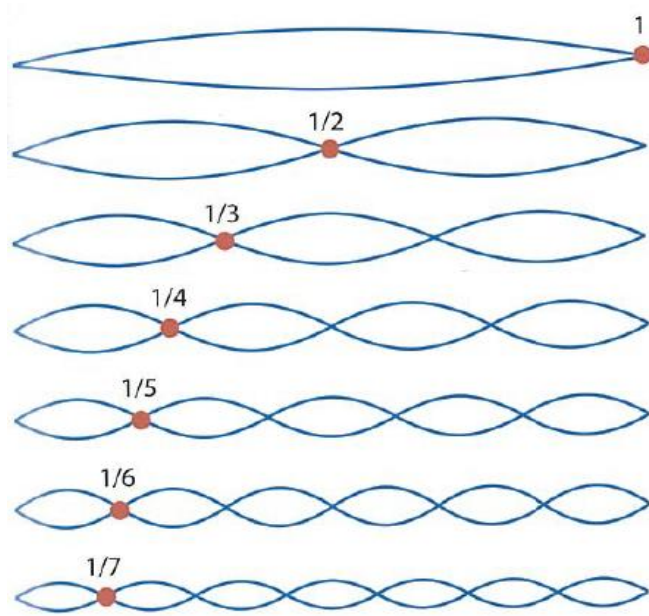
### Question 10

From question 9, which pairs or groups of notes were very discordant? Why do you think this is so?

C and C#, and D and D#. The notes are right next to each other on the scale.

## Extension 1

Looking back at plucking a string, if the ratio is a third of the original length, a ratio of 3:2 results. The interval, or difference in the notes produced, is a fifth. A ratio of 4:3 (from a quarter length string) produces an interval of a fourth. Sounding an octave, a fifth, and a fourth together produces a harmonious sound that is pleasing to the ear – a musical chord.



Set the sliders to explore what happens when combining fourth and fifth intervals. Find examples that produce harmonious chords.

Students can research fourth and fifth intervals to discover that they are concordant.

## Extension 2

There are several different types of chords such as major, minor, augmented, etc.

For the A octave, the major chord is A,C#,E, the minor chord is A,C,E and the augmented chord is A,C#,F. Try these and other chords, and comment on the waves produced.

In general, these chords will be concordant.

## Extension 3

Experiment by playing these combinations of notes and chords on a piano or organ to see if they produce pleasant or unpleasant sounds.

Set up an organ or other instruments in class and play the different combinations of notes. Determine which sound concordant and which sound discordant.