Graphs of Polynomial Functions – ID: 10222

By Michele Patrick

Time required
45 minutes

Activity Overview

In this activity, students graph polynomial functions with both positive and negative leading coefficients to determine what characteristics of the function determine the end behavior of the function.

The activity begins by having students compare functions to introduce the concept of end behavior. Then they graph cubics and quartics, noting the respective end behaviors for positive and negative leading coefficients. Finally, they compare quadratics to quartics and cubics to quintics to discover that the degree of the polynomial also plays a role in end behavior. They conclude the activity by summarizing their findings.

Concepts

- *End behavior*
- *Degree of a polynomial, leading coefficient of a polynomial*
- *Positive and negative infinity*

Teacher Preparation

This activity is designed to be used in a Precalculus classroom. It can also be used in an Algebra 2 classroom.

- *Students should be familiar with the classifications of polynomials and how to determine the degree of a polynomial. Students should also be comfortable adjusting window settings in the Graphs & Geometry application to obtain a complete graph.*
- *The screenshots on pages 2–5 demonstrate expected student results. Refer to the screenshots on pages 6 and 7 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the student .tns file and student worksheet, go to education.ti.com/exchange and enter "10222" in the quick search box.***

Classroom Management

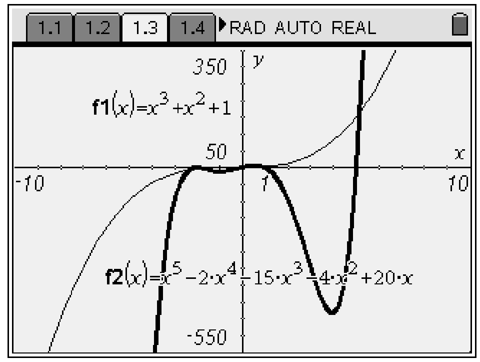
- *This activity is intended to be mainly* ***teacher-led****, with breaks for individual student work. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds.*
- *The student worksheet PreCalcAct34_PolyGraphs_worksheet_EN helps guide students through the activity and provides a place to record their answers.*

TI-Nspire™ Applications

Graphs & Geometry, Notes

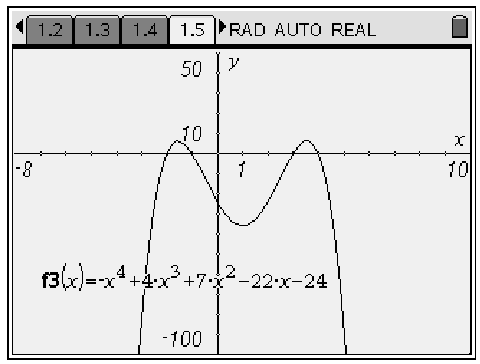
Problem 1 – Making comparisons

Two functions **f1**(*x*) and **f2**(*x*) are given on the student worksheet and on page 1.2. Have students graph these functions on page 1.3. They may find it easier to view the graphs separately if they adjust its line weight using the **Attributes** tool from the Actions menu.

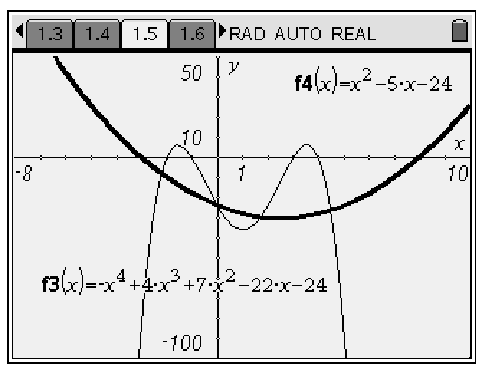


Students should adjust the **Window Settings** as needed to view a complete graph (all relative maximums/minimums, implied end behavior, etc.) Students should sketch the graphs on their worksheets and tell how they are alike. This can be done informally at this point—with students' responses being similar to the following:
the left "arms" point down, the right "arms" point up

Have students graph **f3**(*x*) on page 1.5 and discuss how this graph compares to the graphs of **f1**(*x*) and **f2**(*x*). Here, students should find that both arms point down, or in the same direction.

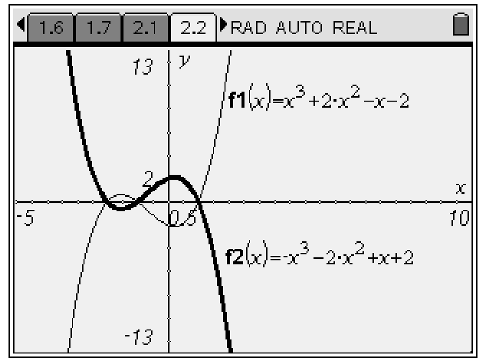


Have students graph **f4**(*x*) on page 1.7 and tell how it is similar to the graph of **f3**(*x*). Here, students should notice that both arms in the same direction, except for this function, that direction is *up*.



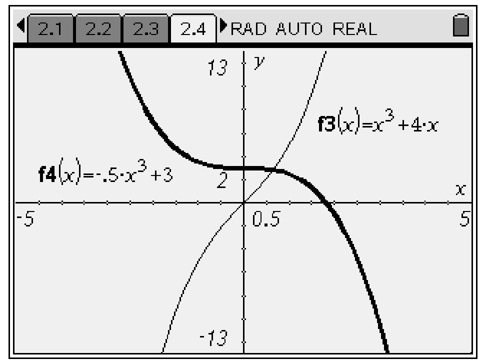
At this point, students may be making comparisons about any number of characteristics, though someone should mention something about how the graphs "begin" and "end." That is, how the graphs behave as *x* gets smaller (approaches negative infinity) and larger (approaches positive infinity). Define and discuss *end behavior*. Have students look back at the end behavior of the graphs and use phrases such as, "As *x* approaches positive infinity, the graph approaches negative infinity."

Problem 2 – Cubic functions



Now, students will begin to explore specific types of functions. Have them graph the two cubic functions **f1**(x) and **f2**(x) from their worksheets on page 2.2. They should then sketch the shapes of the graphs on their worksheets.

(***Note:*** These graphs are actually reflections across the x-axis, but that is not relevant to this activity.)



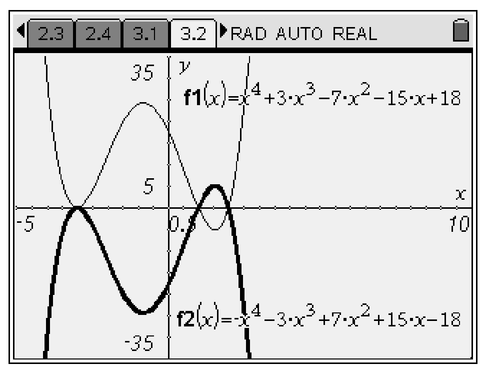
On page 2.4, allow students to graph any two cubic functions, as long as the leading coefficients have opposite signs.

Encourage students to use coefficients other than 1 and to experiment with non-leading coefficients being 0. Students usually enjoy trying to make interesting or "weird-looking" graphs.

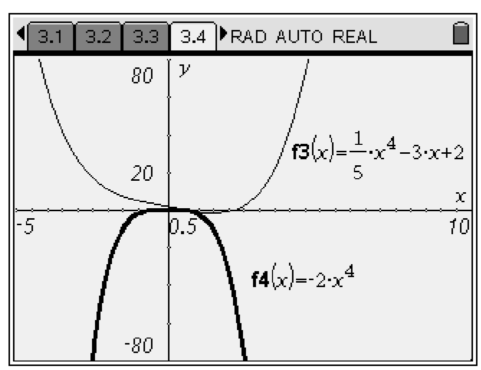
When they are finished, have them make a conjecture about the graphs of cubic functions.

(If the leading coefficient is positive, the left arm points down and the right arm points up; if it is negative, the left arm points up and the right arm points down.)

Problem 3 – Quartic functions



Next, students should graph the two quartic functions **f1**(x) and **f2**(x) from their worksheets on page 3.2. As before, they should sketch the shapes of these graphs on their worksheets.



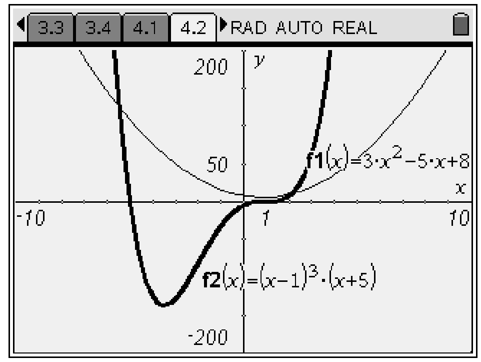
On page 3.4, students will graph any two quartic functions for which the leading coefficients have opposite signs. Again, encourage students to try different coefficients to obtain interesting graphs.

When they are finished, have them make a conjecture about the graphs of cubic functions.

(If the leading coefficient is positive, both arms point up; if it is negative, the both arms point down.)

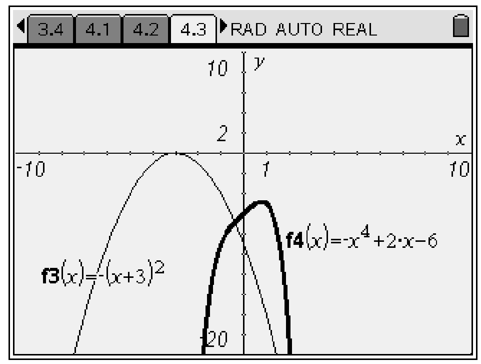
Problem 4 – Quadratic and quartic functions

This problem has students begin their exploration of the effect of the *degree* of the polynomial (in conjunction with the sign of the leading coefficient) on its end behavior. On page 4.2, have students graph any quadratic and any quartic function, both with *positive* leading coefficients.



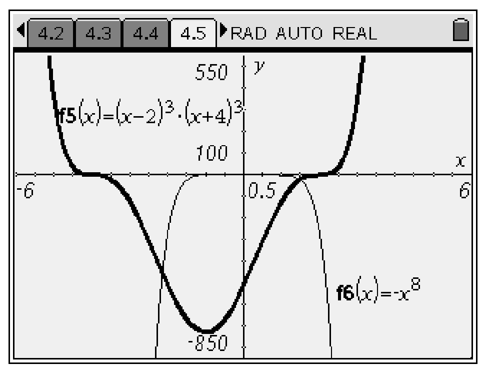
Students should find that both arms of each point up.

On page 4.3, have students graph any quadratic and any quartic function, both with *negative* leading coefficients.



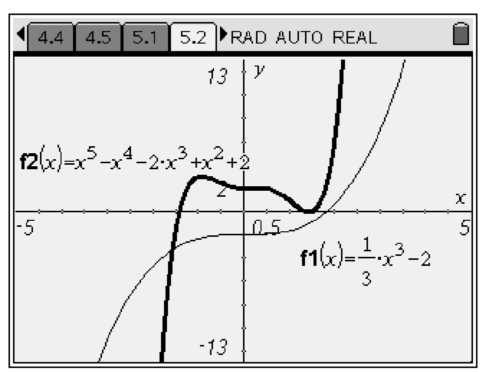
Students should find that both arms of each point down.

For the two functions given on page 4.4, ask students to predict the end behavior of their graphs. (Be sure they can explain their reasoning!) Then have them test their predictions by graphing the functions on page 4.5.



Problem 5 – Cubic and quintic functions

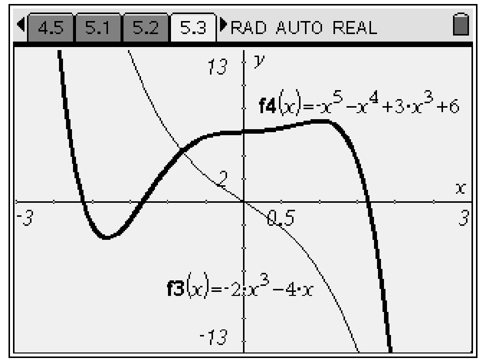
Now, students will explore polynomials of odd degree. On page 5.2, have students graph any cubic and any quintic function, both with *positive* leading coefficients.



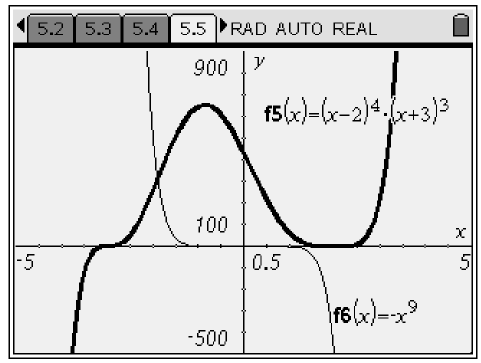
Students should find that the left arms point down and the right arms point up.

On page 5.3, have students graph any cubic and any quintic function, both with *negative* leading coefficients.

Students should find that left arms point up and the right arms point down.

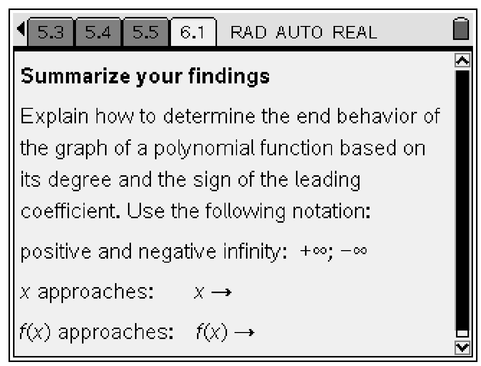


For the two functions given on page 5.4, ask students to again predict the end behavior of the graphs. They can test their predictions by graphing the functions on page 5.5.



Problem 6 – Summary

To conclude the activity, have students look over their graphs and the corresponding functions of those graphs and summarize their findings for the end behavior of the graph of a polynomial function. They should consider cases of whether the leading term is of an even or odd degree and if the coefficient of this term is positive or negative. Encourage students to use mathematical notation.

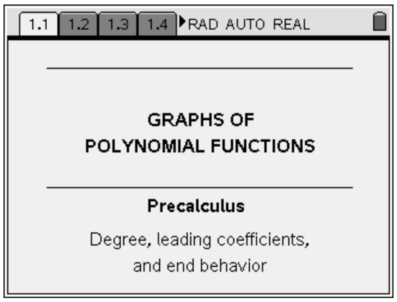


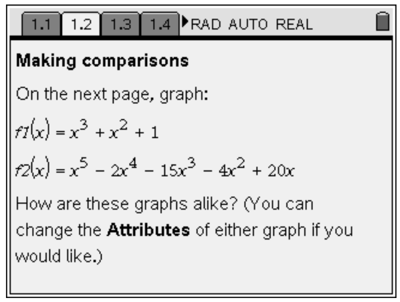
Generalizations:

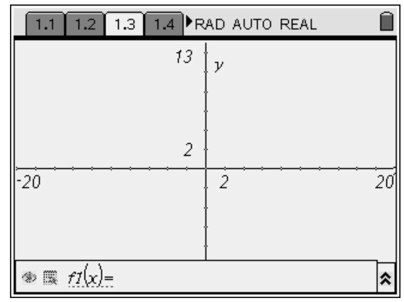
- For a polynomial of *odd* degree and a *positive* leading coefficient,
  $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$
- For a polynomial of *odd* degree and a *negative* leading coefficient,
  $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$
- For a polynomial of *even* degree and a *positive* leading coefficient,
  $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$
- For a polynomial of *even* degree and a *negative* leading coefficient,
  $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$

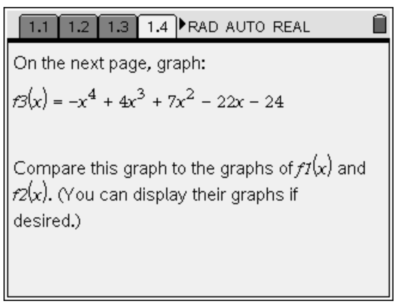
Graphs of Polynomial Functions – ID: 10222

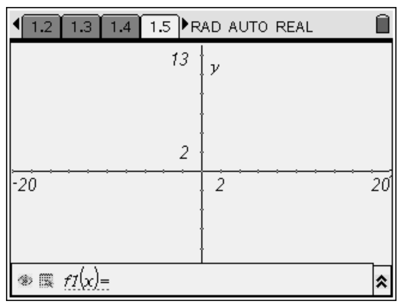
(Student)TI-Nspire File: *PreCalcAct34_PolyGraphs_EN.tns*

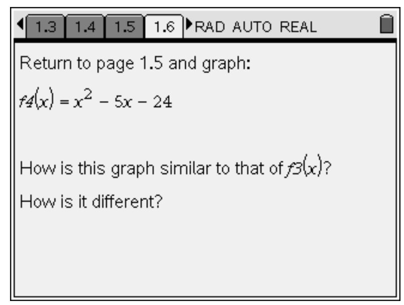


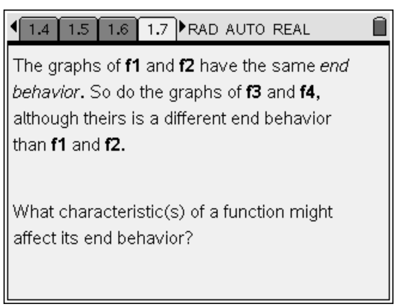


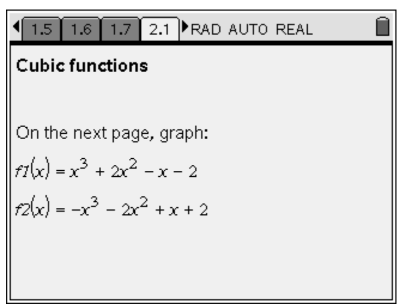


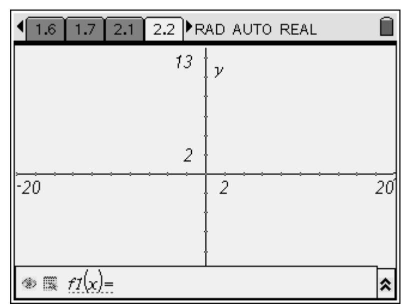


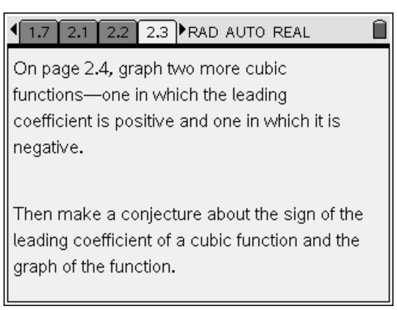


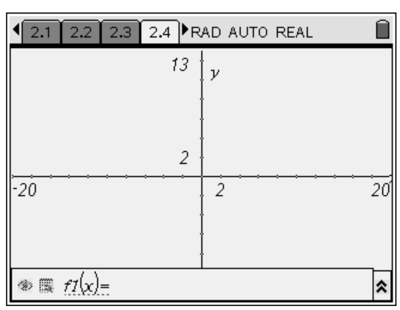


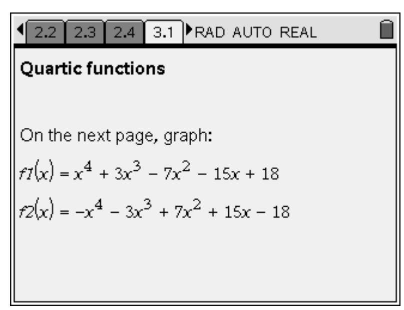


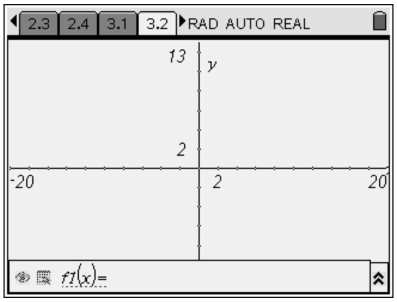


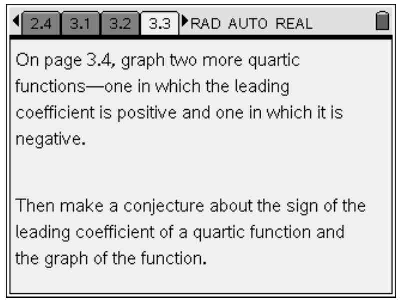


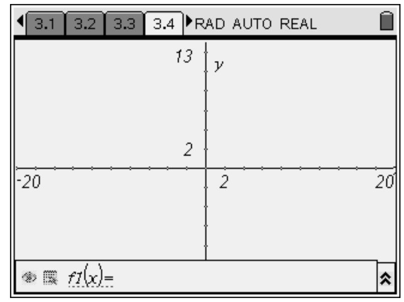


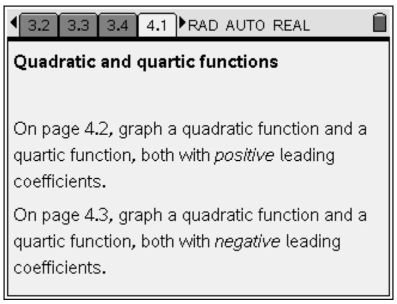


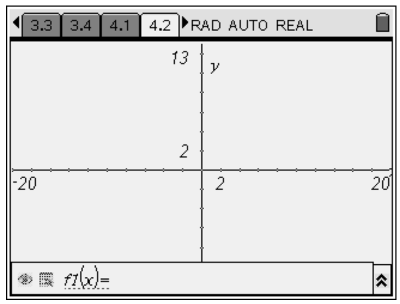


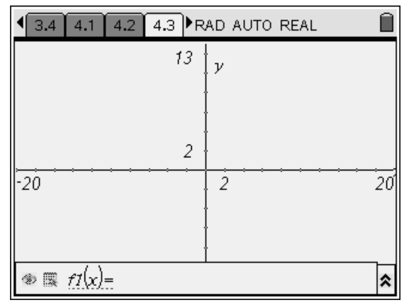

2.3 2.4 3.1 3.2 RAD AUTO REAL
13
y
2
-20
2
20
f1(x)=

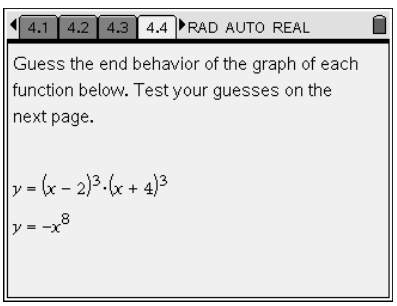

2.4 3.1 3.2 3.3 RAD AUTO REAL
On page 3.4, graph two more quartic functions—one in which the leading coefficient is positive and one in which it is negative.
Then make a conjecture about the sign of the leading coefficient of a quartic function and the graph of the function.

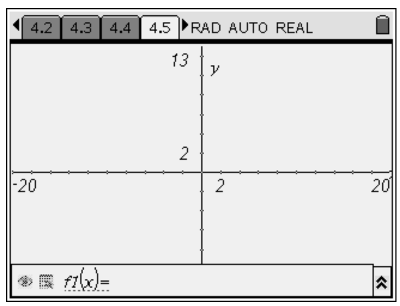

3.1 3.2 3.3 3.4 RAD AUTO REAL
13
y
2
-20
2
20
f1(x)=

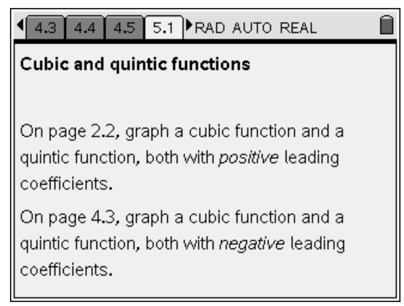

3.2 3.3 3.4 4.1 RAD AUTO REAL
Quadratic and quartic functions
On page 4.2, graph a quadratic function and a quartic function, both with positive leading coefficients.
On page 4.3, graph a quadratic function and a quartic function, both with negative leading coefficients.

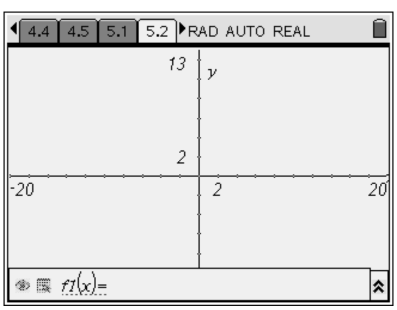

3.3 3.4 4.1 4.2 RAD AUTO REAL
13
y
2
-20
2
20
f1(x)=

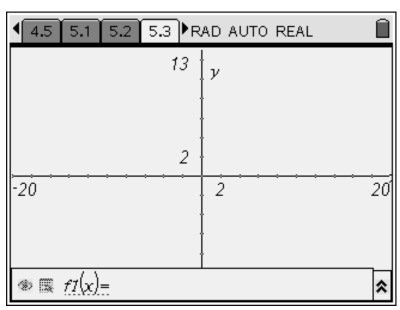

3.4 4.1 4.2 4.3 RAD AUTO REAL
13
y
2
-20
2
20
f1(x)=

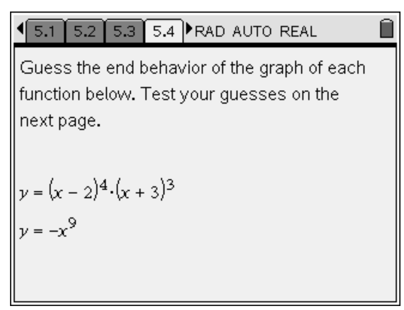

4.1 4.2 4.3 4.4 RAD AUTO REAL
Guess the end behavior of the graph of each function below. Test your guesses on the next page.
$y = (x-2)^3 \cdot (x+4)^3$
$y = -x^8$

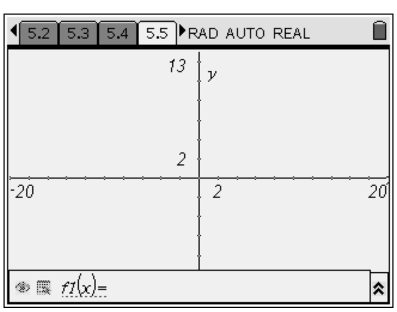

4.2 4.3 4.4 4.5 RAD AUTO REAL
13
y
2
-20
2
20
f1(x)=

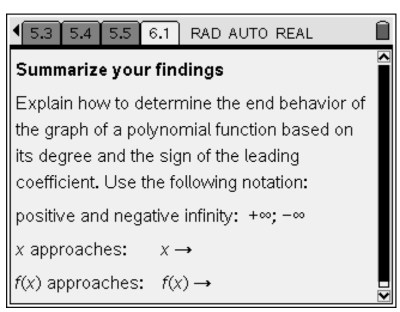

4.3 4.4 4.5 5.1 RAD AUTO REAL
Cubic and quintic functions
On page 2.2, graph a cubic function and a quintic function, both with positive leading coefficients.
On page 4.3, graph a cubic function and a quintic function, both with negative leading coefficients.


4.4 4.5 5.1 5.2 RAD AUTO REAL
13
y
2
-20
2
20
f1(x)=


4.5 5.1 5.2 5.3 RAD AUTO REAL
13
y
2
-20
2
20
f1(x)=


5.1 5.2 5.3 5.4 RAD AUTO REAL
Guess the end behavior of the graph of each function below. Test your guesses on the next page.
$y = (x-2)^4 \cdot (x+3)^3$
$y = -x^9$


5.2 5.3 5.4 5.5 RAD AUTO REAL
13
y
2
-20
2
20
f1(x)=


5.3 5.4 5.5 6.1 RAD AUTO REAL
Summarize your findings
Explain how to determine the end behavior of the graph of a polynomial function based on its degree and the sign of the leading coefficient. Use the following notation:
positive and negative infinity: $+\infty$; $-\infty$
x approaches: $x \to$
f(x) approaches: $f(x) \to$

