Problem 1 – The Derivative of the Sine Function

Use the definition of derivative $f'(a) = \lim_{h \to 0} \frac{\sin(a+h) - \sin(a)}{h}$ to set up the limit to find the derivative of $f(x) = \sin(x)$. The identity for the sine of the sum of two angles $\sin(a+b) = \sin(a) \cdot \cos(b) + \sin(b) \cdot \cos(a)$ is needed to simplify the limit.

- What is the limit expression?

In the Y= screen, enter the equation $y = \frac{\cos(x) - 1}{x}$. In the Table Setup menu, change the initial x to –0.1 and Δx to 0.025. Now look at the table of values.

- Why is the function undefined at x = 0?

- What do you think the limit is?

Use the **Limit** command (**F3:Calc>3:Limit**) to confirm the limit of this ratio.

- What do you get? How does it compare to the answer above?

Substitute your answer and $\lim_{h \to 0} \frac{\sin(h)}{h} = 1$ into the original limit you found for the derivative of $f(x) = \sin(x)$.

- What is the limit?

Use the **Derivative** command (**F3:Calc>1:Derivative**)to confirm this.

- What is the derivative of $f(x) = \sin(x)$?

Problem 2 – The Derivative of the Cosine Function

Use the definition of derivative and set up the limit to find the derivative of $f(x) = \cos(x)$. The identity for the sine of the sum of two angles $\cos(a+b) = \cos(a) \cdot \cos(b) - \sin(b) \cdot \sin(a)$ is needed to simply the limit.

- What is the limit expression?

Substitute the limit values of $\frac{\cos(x) - 1}{x}$ and $\frac{\sin(h)}{h}$ found in Problem 1 into the limit.

- What is the value of the limit?

Use the **Limit** command to find the above limit.

- What do you get? How does it compare to your answer above?

Use the **Derivative** command to confirm your answer.

- What is the derivative of $f(x) = \cos(x)$?

Problem 3 - Derivative of the Tangent Function

To find the derivative of the tangent function, write the tangent in terms of sine and cosine.

- Use the quotient rule to find the derivative of this expression.

Simplify the result and check your answer suing the Derivative command to find the derivative of $\tan(x)$.

The derivative of tangent is usually written in terms of the reciprocal trigonometric functions, cosecant, secant, or cotangent.

- Write the derivative of the tangent in terms of one of these reciprocals.

Extension – The Derivative of $y = \sin(u)$ and $y = \cos(u)$

What happens for more complicated sine or cosine expressions? Use the **Derivative** command for the following functions.

- $f(x) = \sin(12x)$ $\quad f'(x) =$

- $g(x) = \cos(5x)$ $\quad g'(x) =$

What do you think will be the general rule for the derivative of $y = \sin(u(x))$ and $y = \cos(u(x))$.

Use the derivative command to get the result. Be sure to use $u(x)$ in the command.

- $\frac{d}{dx}\left(\sin(u(x))\right) =$

- $\frac{d}{dx}\left(\cos(u(x))\right) =$

Now use the **derivative** command for the following more complicated expressions. Be careful with your parentheses.

- $h(x) = \sin^3(4x)$ $\quad h'(x) =$

- $j(x) = \cos^7(3x)$ $\quad j'(x) =$