Who Would You Work For?

Expanding the Notion of Function Representation

By- Tammy Crittenden

Dana Alshikhali

Activity Overview

1. In this activity, students will investigate various types of functions.
2. Students will also find sequences in a function.

Concepts

NCTM Standards

In grades 9-12 all students should-

- generalize patterns using explicitly defined and recursively defined functions;
- understand relations and functions and select, convert flexibly among, and use various representations for them;
- analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
- understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on more-complicated symbolic expressions;
- understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;
- interpret representations of functions of two variables.
- use symbolic algebra to represent and explain mathematical relationships;
- draw reasonable conclusions about a situation being modeled.
- approximate and interpret rates of change from graphical and numerical data.

TN Algebra 1 Standards

Course Level Expectation:

3102.3.1 Use algebraic thinking to analyze and generalize patterns.

3102.3.6 Understand and use relations and functions in various representations to solve contextual problems.

Checks for Understanding:

3102.3.1 Recognize and extend arithmetic and geometric sequences.

3102.3.2 Explore patterns including Pascal's Triangle and Fibonacci sequence.

3102.3.3 Justify correct results of algebraic procedures using extension of properties of real numbers algebraic expressions.

3102.3.17 Recognize "families" of functions.

3102.3.25 Find function values using *f(x)* notation or graphs.

State Performance Indicators:

3102.3.1 Express a generalization of a pattern in various representations including algebraic and function notation.

3102.3.6 Interpret various relations in multiple.

Teacher Preparation

Before the activity, the teacher should preload the activity file TINspire2_Tammy&Dana on to the student handheld devices. This can be done via Connect-to-class software, the Navigator System, or by using link cables with the handheld.

Classroom management tips

1. This activity can be done by the students using hand-outs and the TINspire handhelds.
2. This activity is intended to be completed in small groups or individually as determined by teacher.

TINspire Applications

- ✓ Notes
- ✓ Lists & Spreadsheets
- ✓ Data & Statistics
- ✓ Calculator Page

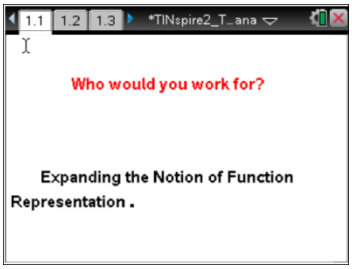
Step-by-step direction

Steps:

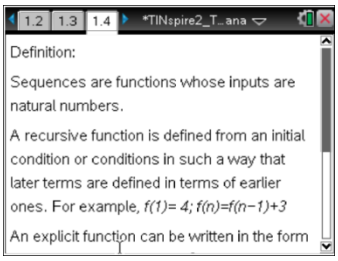
1. From the home screen, choose My Documents and navigate to the appropriate folder containing the .tns file

   TINspire_Tammy& Dana. Highlight the file and press. choose whether or not to save changes to any previous document.

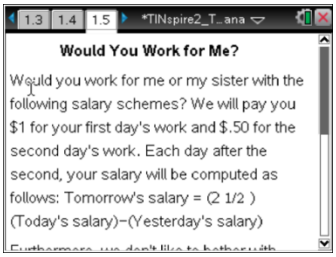
2. Page 1.1 is the title page for the activity. press ctrl and ⬚ right or left arrow to navigate through the document. Pages 1.2 and 1.3 list the standards addressed in this activity.



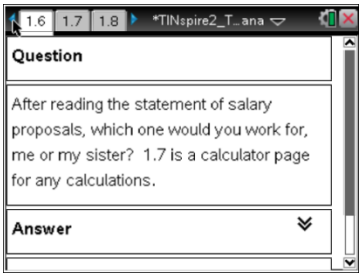
3. 1.4 gives the definitions for the lesson that we are to complete.



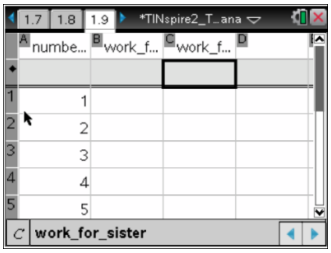
4. 1.5 introduces the first of three problems in the activity.



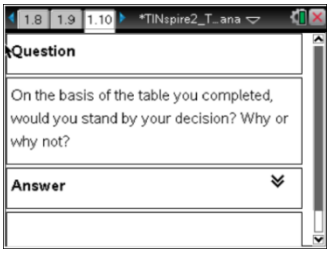
5. Students can answer the various questions within the document by using the ⬚ key until they get to the answer section of the page. 1.7 is a calculator page for any calculations. 1.8 instructs the students to complete the chart on the following slide.



6. Students may complete the spreadsheet one cell at a time or those with spreadsheet skills can use the gray formula cells to complete the spreadsheet.

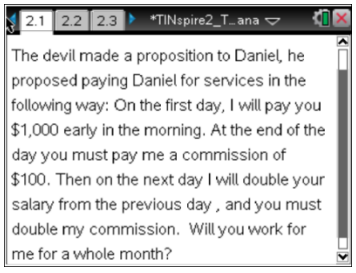


7. On 1.10-1.16, students will answer questions on the table from 1.9. some are real world questions asking the students to think about

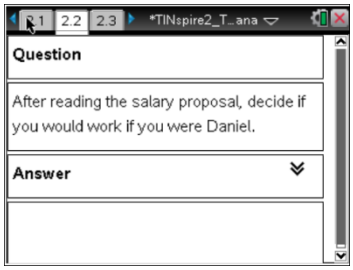


a situation that might occur similar to the problem presented.

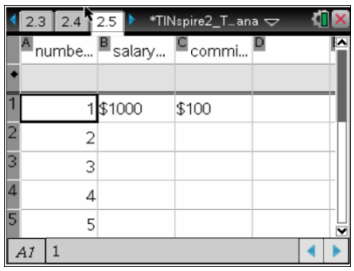
8. 2.1 is the beginning of problem 2.



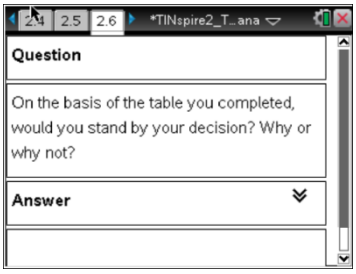
9. 2.2 asks the students to make a decision before getting started on the actual problem.
   2.3 and 2.4 are about a calculator page inserted for any
   2.4 calculations needed.



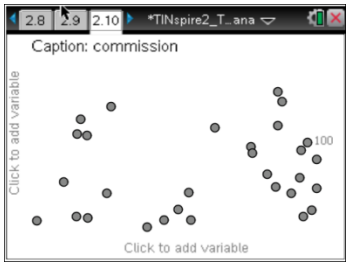
10. 2.5 is a spreadsheet to fill in with equations or to manually fill in with information from calculations.



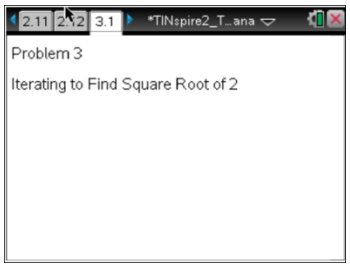
11. On 2.6-2.9, students are answering questions from the chart on 2.5.



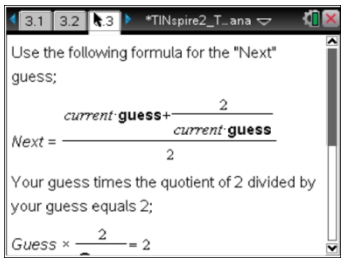
12. 2.10, a graph for the students to complete. By clicking on sides to determine the captions, you will need to use the menu key while on the graph and select plot properties, add Y variable to get all of the information on the graph. A box will appear to select the variables from.
   2.11-2.12 are questions that the graph will help the students complete.



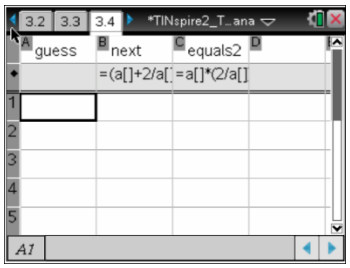
13. 3.1 starts problem 3, Iterating to Find Square Root of 2.
    3.2 asks the students to start by choosing a number greater than $\sqrt{2}$.



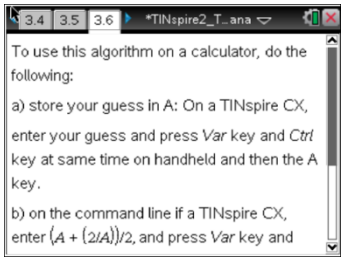
14. 3.3 will give the students the formula to use to find the next guess.



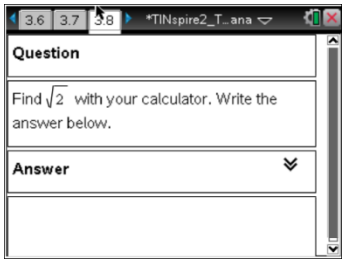
15. 3.4 is the table for the students to insert their guess. 3.5 asks for the students next guess.



16. 3.6 helps the students with instructions on how to use the calculator to find iteration. The student will need to enter the guess on the calculator and store it by pressing the var ctrl keys at the same time and then pressing the A key on the handheld.
    3.7 is for calculations.



17. 3.8-3.9 are for use with the calculator. Students enter the results found by doing the steps from 3.6.



18. 3.10 is a graph to be used with 3.11-3.14. These questions are discussion and extension questions that will have the students analyzing the results found. The students need to add the variables to left side and bottom to see the results.

Assessment and evaluation

- ✓ The teacher can collect the student files using the Navigator System.
- ✓ Sample answers to questions in .tns file:

  Q: After reading the statement of salary proposals, which one would you work for, me or my sister?

  A: Students will probably be divided over whether or not they will work under either salary scheme.

  Q: On the basis of the table you completed, would you stand by your decision? Why or why not?

  A: The answer to standing by the decision will depend on what the student chose.

  Q: From the table of daily salaries that you earned from me, what does the sequence become after six days? Do you recognize it?

  A: From day 6 0n, it is apparent that the sequence is a geometric sequence of powers of 2.

  Q: From the table of daily salaries that you earned from my sister, what does the sequence become after seven days? Do you recognize it?

  A: From day 7 on, it is apparent that the sequence is a negative geometric sequence in which the next term, *n+1*, is equal to –(*2n + $0.10*).

  Q: In my sister's salary scheme, what is the meaning of the salary on day 4? On day 5?

  A: The salary for the day 4 is 0, indicating that there is no salary. The salary for day 5 is negative, which could be interpreted to mean that you pay my sister to work.

  Q: Is either salary scheme really realistic?

  A: Neither salary scheme is realistic.

  Q: If you graphed the data for either salary scheme, what type of graph would you expect to obtain? Why?

  A: After completing the table, students would probably expect to get exponential graphs. Each graph is discrete.

  Q: Describe some real-life situation in which one might encounter such schemes as those in the problem.

  A: Students' answers may vary, but chain letters or pyramid schemes for making money may be in the discussion.

  Q: After reading the salary proposal, decide if you would work if you were Daniel.

  A: The answer will depend on the student. The outcome is not apparent in the beginning

  Q: On the basis of the table you completed, would you stand by your decision? Why or why not?

  A: The answer depends on what the student decided early on. The student definitely will not want to work long with this commission.

  Q: If you would not work for a month, for how many days would you work?

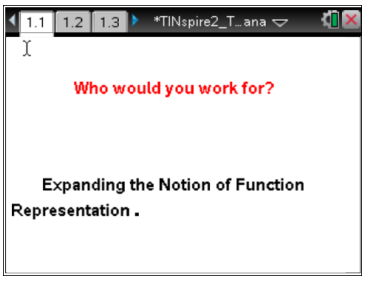
  A: Working nine days is reasonable with the given scheme.

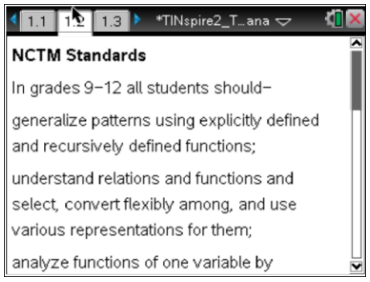
  Q: From reading the problem what type of curve would you expect your salary data to generate?

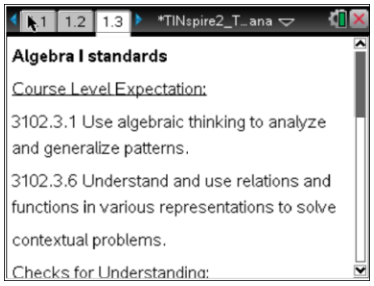
A: Most will expect an exponential curve.
Q: what type of curve would you expect the commission data to generate?
A: Most students will expect exponential curve.
Q: Is the salary scheme realistic? Why or why not?
A: The salary scheme is not realistic.
Q: What type of graph would you expect to obtain**?**
A: Most will expect the graph to be an exponential curve. It is not at all clear that they will expect the curve that they get.
Q: In your own words, compare the graphs of the salary data and the commission data.
A: The graphs are both of the exponential type. The commission curve grows faster than the salary curve.
Q: How accurate was the iterative algorithm you used?
A: The algorithm is very accurate.
Q: Is there a way to represent this iterative process graphically in order to see that it really does find √(2)?
A: Yes, There are several ways to do this. One is to use a web plot. Another is to plot the points as a sequence and compare the curve of the sequence to the line $y = \sqrt{2}$.
Q: Is there an algebraic argument for convincing someone that the iterative process used really does keep getting closer to √(2)? State your argument.
A: Yes. If the guess is correct, the solution to the equation is $\sqrt{2}$. If the guess is exactly equal to $\sqrt{2}$, then $\frac{2}{guess} = guess$, but if the guess is less than $\sqrt{2}$, then $\frac{2}{guess} > \sqrt{2}$, and vice versa. The average of *guess and 2/guess* lies between those values and hence is closer to $\sqrt{2}$. Each iteration gets progressively closer to $\sqrt{2}$.
Q: How many iterations would be required to get accuracy to one hundred decimal places in the estimate of √(2)?
A: With seven iterations, well over one hundred-decimal-place accuracy is achieved. (The exact number of places depends on the original guess.)
Q: If the process converges to $\sqrt{2}$, how "fast" does it converge? That is, how does the error change from iteration to iteration?
A: The process is extremely accurate. The error changes very little after the first five iterations.

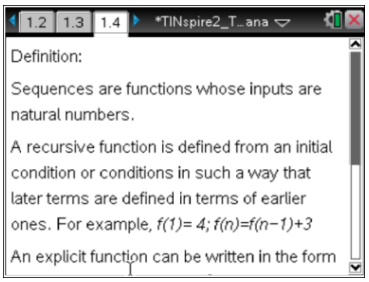
Student TI-Nspire Document

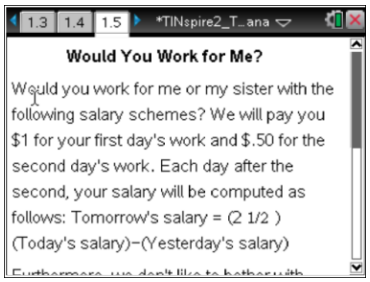
Expanding the Notion of Function Representation

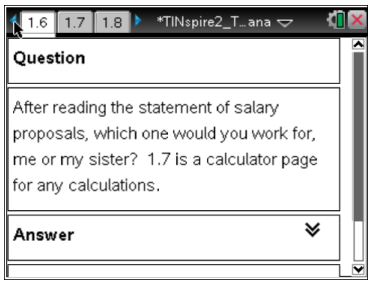






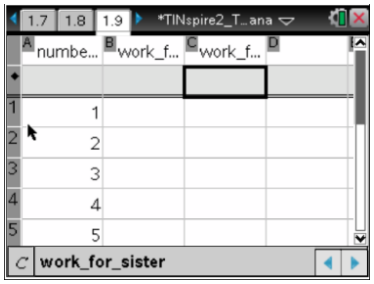

1.2 1.3 1.4 *TINspire2_T...ana
Definition:
Sequences are functions whose inputs are natural numbers.
A recursive function is defined from an initial condition or conditions in such a way that later terms are defined in terms of earlier ones. For example, f(1)= 4; f(n)=f(n−1)+3
An explicit function can be written in the form

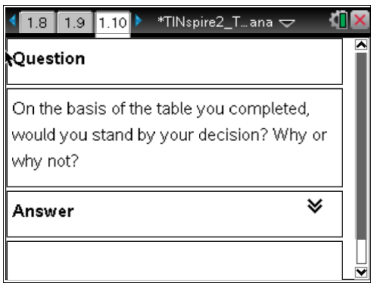

1.3 1.4 1.5 *TINspire2_T...ana
Would You Work for Me?
Would you work for me or my sister with the following salary schemes? We will pay you $1 for your first day's work and $.50 for the second day's work. Each day after the second, your salary will be computed as follows: Tomorrow's salary = (2 1/2) (Today's salary)−(Yesterday's salary)

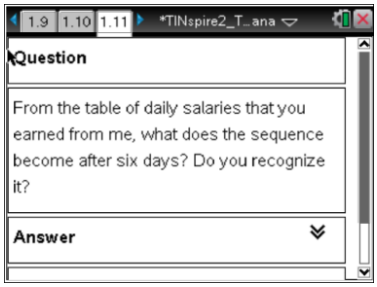

1.6 1.7 1.8 *TINspire2_T...ana
Question
After reading the statement of salary proposals, which one would you work for, me or my sister? 1.7 is a calculator page for any calculations.
Answer


1.5 1.6 1.7 *TINspire2_T...ana
0/99


1.6 1.7 1.8 *TINspire2_T...ana
Complete the chart on the following page.


1.7 1.8 1.9 *TINspire2_T...ana
numbe... work_f... work_f...
1
2
3
4
5
work_for_sister


1.8 1.9 1.10 *TINspire2_T...ana
Question
On the basis of the table you completed, would you stand by your decision? Why or why not?
Answer


1.9 1.10 1.11 *TINspire2_T...ana
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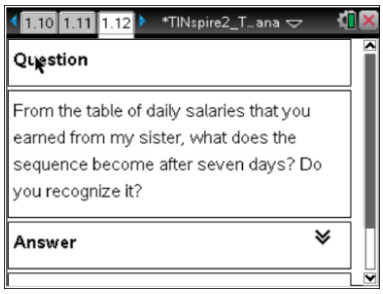
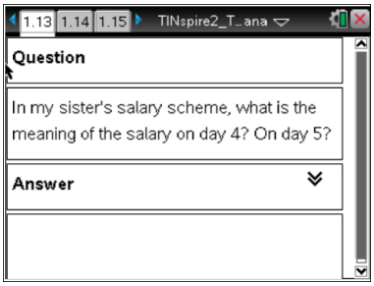
Answer


1.10 1.11 1.12 *TINspire2_T...ana
Question
From the table of daily salaries that you earned from my sister, what does the sequence become after seven days? Do you recognize it?
Answer


1.13 1.14 1.15 TINspire2_T...ana
Question
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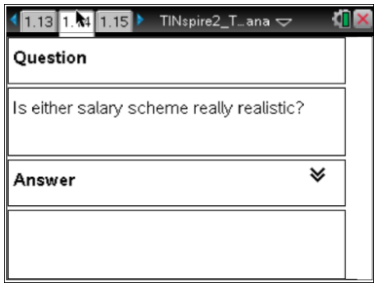
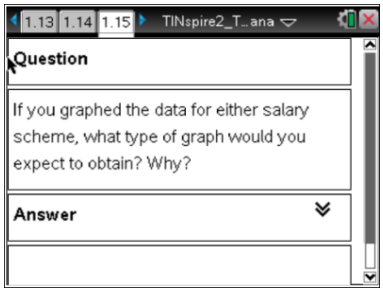
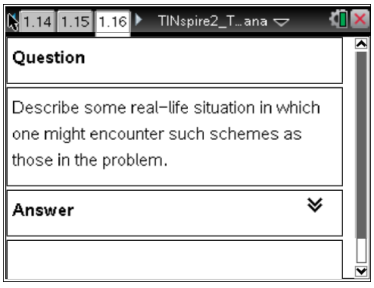
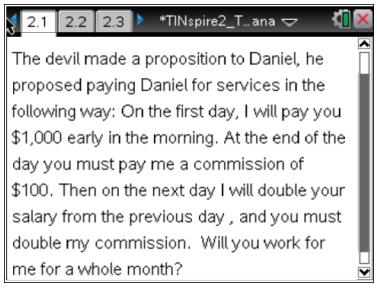
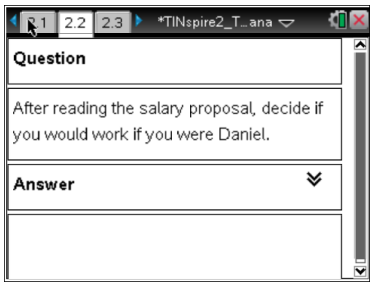
Answer


1.13 1.14 1.15 TINspire2_T...ana
Question
Is either salary scheme really realistic?
Answer


1.13 1.14 1.15 TINspire2_T...ana
Question
If you graphed the data for either salary scheme, what type of graph would you expect to obtain? Why?
Answer


1.14 1.15 1.16 TINspire2_T...ana
Question
Describe some real−life situation in which one might encounter such schemes as those in the problem.
Answer


2.1 2.2 2.3 *TINspire2_T...ana
The devil made a proposition to Daniel, he proposed paying Daniel for services in the following way: On the first day, I will pay you $1,000 early in the morning. At the end of the day you must pay me a commission of $100. Then on the next day I will double your salary from the previous day , and you must double my commission. Will you work for me for a whole month?


2.1 2.2 2.3 *TINspire2_T...ana
Question
After reading the salary proposal, decide if you would work if you were Daniel.
Answer

2.1 2.2 2.3 *TINspire2_T...ana

Complete the graph on page 2.5.

Page 2.4 is a calculator page for any calculations needed.

2.2 2.3 2.4 *TINspire2_T...ana

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2.3 2.4 2.5 *TINspire2_T...ana

	A numbe...	B salary...	C commi...	D
1	1	$1000	$100	
2	2			
3	3			
4	4			
5	5			

A1 1

2.4 2.5 2.6 *TINspire2_T...ana

Question

On the basis of the table you completed, would you stand by your decision? Why or why not?

Answer

2.5 2.6 2.7 *TINspire2_T...ana

Question

If you would not work for a month, for how many days would you work?

Answer

2.6 2.7 2.8 *TINspire2_T...ana

Question

From reading the problem what type of curve would you expect your salary data to generate?

what type of curve would you expect the commission data to generate?

Answer

2.7 2.8 2.9 *TINspire2_T...ana

Question

Is the salary scheme realistic? Why or why not?

Answer

2.8 2.9 2.10 *TINspire2_T...ana

Caption: commission

Click to add variable

100

Click to add variable

2.9 2.10 2.11 *TINspire2_T...ana

Question

What type of graph would you expect to obtain?

Answer

2.10 2.11 2.12 *TINspire2_T...ana

Question

In your own words, compare the graphs of the salary data and the commission data.

Answer

2.11 2.12 3.1 *TINspire2_T...ana

Problem 3

Iterating to Find Square Root of 2

2.12 3.1 3.2 *TINspire2_T...ana

Question

Pick some number that you know is greater than $\sqrt{2}$ as the first guess in determining this root.

Answer

3.1 3.2 3.3 *TINspire2_T...ana

Use the following formula for the "Next" guess;

$$Next = \frac{current\cdot guess + \frac{2}{current\cdot guess}}{2}$$

Your guess times the quotient of 2 divided by your guess equals 2;

$$Guess \times \frac{2}{Guess} = 2$$

3.2 3.3 3.4 *TINspire2_T...ana

	A guess	B next	C equals2	D
		=(a[]+2/a[])	=a[]*(2/a[])	
1				
2				
3				
4				
5				

A1

3.3 3.4 3.5 *TINspire2_T...ana

Question

What is your second guess?

Answer

3.4 3.5 3.6 *TINspire2_T...ana

To use this algorithm on a calculator, do the following:

a) store your guess in A: On a TINspire CX, enter your guess and press *Var* key and *Ctrl* key at same time on handheld and then the A key.

b) on the command line if a TINspire CX, enter $(A + (2/A))/2$, and press *Var* key and

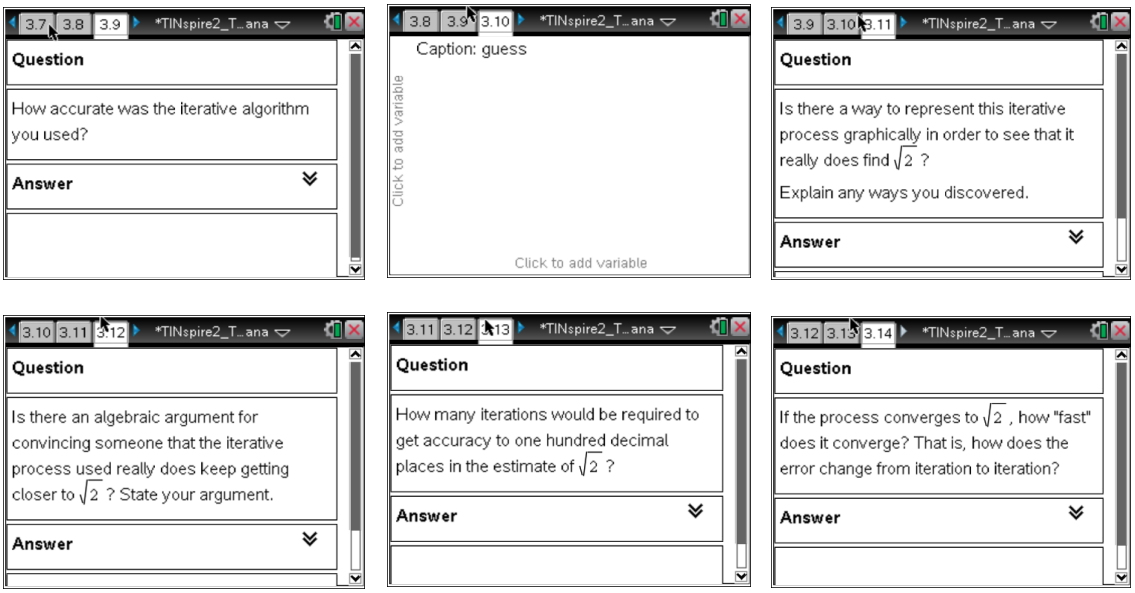
3.5 3.6 3.7 *TINspire2_T...ana

3.6 3.7 3.8 *TINspire2_T...ana

Question

Find $\sqrt{2}$ with your calculator. Write the answer below.

Answer



This activity was adapted from *Navigating Through Algebra* in Grades 9-12, *Chapter 3, Expanding the Notion of Function Representation*, a publication from the National Council of Teachers of Mathematics Navigation Series.