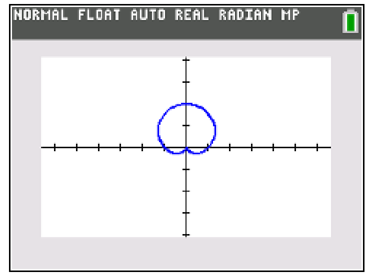
Limaçon Curves
Student Activity

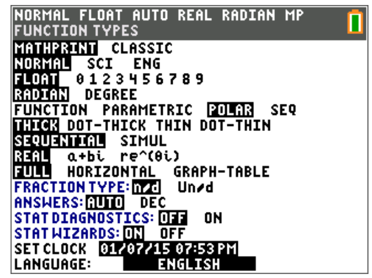
Name ________________

Class ________________

In this activity, you will investigate the effect of changing the values of *a* and *b* in the polar equations $r = a \pm b * sin(\theta)$ and $r = a \pm b * cos(\theta)$, where $a > 0$ and $b > 0$. You will also explore the relationship between the polar curve $r = a \pm b * sin(\theta)$ (or $r = a \pm b * cos(\theta)$) and the sinusoidal function $f(x) = a + b * sin(x)$ (or $f(x) = a \pm b * cos(x)$).



To set your calculator to Polar mode, press mode and select **POLAR** as shown to the right. Also set your graphing calculator to Radian mode by selecting **RADIAN** on this screen as well.
To graph a polar equation on your graphing calculator, press y= and enter your equation. The X,T,θ,n key produces θ in your equation when you are in Polar mode.



1. Graph the following by editing $r1$ to observe each graph. Press zoom and select 4: ZDecimal.

   $i)\ r1 = 1 + 1 * sin(\theta)$ $\quad ii)\ r1 = 1 - 1 * sin(\theta)$

   $iii)\ r1 = 2 + 2 * sin(\theta)$ $\quad iv)\ r1 = 2 - 2 * sin(\theta)$

   $v)\ r1 = 3 + 3 * sin(\theta)$ $\quad vi)\ r1 = 3 - 3 * sin(\theta)$

   Why do you think these graphs are called cardioids?

2. What similarities do you notice about the equations of the six graphs?

3. How do the addition and subtraction signs affect the graphs?

4. Graph the following by editing $r1$ to observe each graph. Press zoom and select 4: ZDecimal.

   $i)\ r1 = 1 + 1 * cos(\theta)$ $\quad ii)\ r1 = 1 - 1 * cos(\theta)$

   $iii)\ r1 = 2 + 2 * cos(\theta)$ $\quad iv)\ r1 = 2 - 2 * cos(\theta)$

   $v)\ r1 = 3 + 3 * cos(\theta)$ $\quad vi)\ r1 = 3 - 3 * cos(\theta)$

   How are the equations different from those in **Problem 1**? How does this difference affect the graph?

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Limaçons have different shapes depending on the ratio $\frac{a}{b}$. We have already seen the cardioid graph that is the result when $a = b$ (or $\frac{a}{b} = 1$).

5. Graph the following by editing $r1$ to observe each graph. Complete the table with the values of a, b, and $\frac{a}{b}$ as you observe each graph.

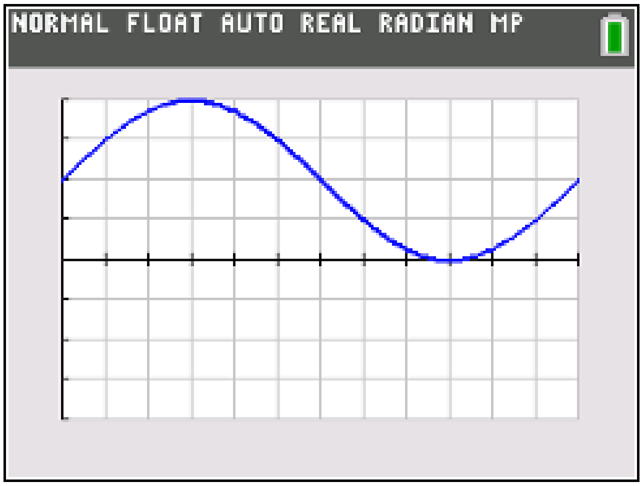
Limaçon	a	b	$\frac{a}{b}$
$i)\ r1 = 1 + 3 * cos(\theta)$			
$ii)\ r1 = 1 + 2 * cos(\theta)$			
$iii)\ r1 = 3 + 2 * cos(\theta)$			
$iv)\ r1 = 2 + 1 * cos(\theta)$			
$v)\ r1 = 3 + 1 * cos(\theta)$			

6. If the ratio $\frac{a}{b} < 1$, the limaçon has a special feature. Describe the shape of the limaçon.

7. One of the polar curves in the table above has a ratio which satisfies $1 < \frac{a}{b} < 2$. Write an equation of another polar curve for which $1 < \frac{a}{b} < 2$. Graph your limaçon and describe the shape of the limaçon.

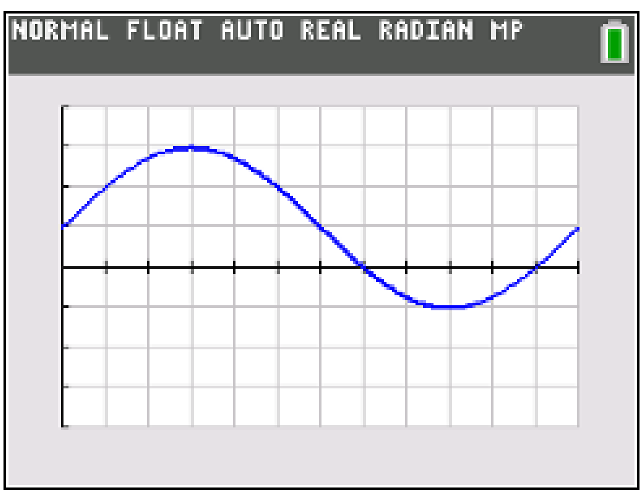
8. Write an equation of a limaçon in which the ratio $\frac{a}{b} > 2$. Graph your limaçon and describe the shape of the limaçon.

9. The graph of the sinusoidal function $f(x) = 2 + 2 * sin(x)$ is shown below. The *x*-scale for the gridlines is $\pi/6$.



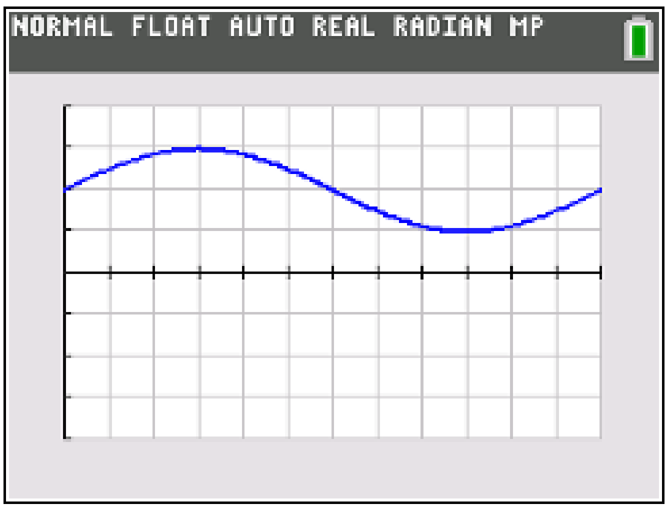
Graph the limaçon given by $r1 = 2 + 2 * sin(\theta)$. Press $\boxed{\text{2nd}}\boxed{\text{zoom}}$ to access format. In the first row, use the right arrow to highlight **PolarGC** and press $\boxed{\text{enter}}$. Press $\boxed{\text{trace}}$ and then the right arrow to move your cursor. Observe the change in the *r* and θ values.
On the interval from *x* = 0 to *x* = 2π of the sinusoidal function, the maximum occurs at $x = \frac{\pi}{2}$ and the minimum occurs at $x = \frac{3\pi}{2}$.
How do the y –values at these two points correspond to the r –values on the cardioid?

10. The graph of the sinusoidal function $f(x) = 1 + 2 * sin(x)$ is shown below. The *x*-scale for the gridlines is $\pi/6$.



Graph the limaçon given by $r1 = 1 + 2 * sin(\theta)$. Press trace and then the right arrow to move your cursor. Observe the change in the *r* and θ values. Explain why the polar curve $r = 1 + 2sin(\theta)$ has an inner loop in the interval $\frac{7\pi}{6} \leq \theta \leq \frac{11\pi}{6}$.

11. The graph of the sinusoidal function $f(x) = 2 + 1 * sin(x)$ is shown below. The *x*-scale for the gridlines is $\pi/6$.



Graph the limaçon given by $r1 = 2 + 1 * sin(\theta)$. Press trace and then the right arrow to move your cursor. Observe the change in the *r* and θ values. Explain why the polar curve does not contain the point located at the pole.