EXPLORATIONS

Chapter 1

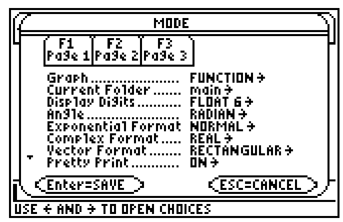
Functions, Graphs, and Limits

In this chapter, you will use the TI-89 to create graphs and tables of values for functions. You then can use this graphical and numerical information to explore limits. You also will see how to evaluate limits directly.

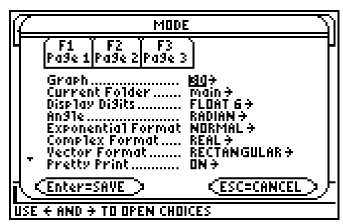
Checking the mode settings

You will need certain mode settings to do the examples in this chapter. To check your settings:

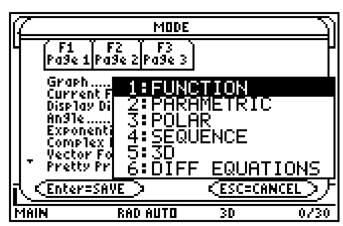
1. Press [MODE] to see Page 1 of the settings. The settings you need are shown.



   To change a setting, first press ⓤ or ⓓ to highlight that setting.

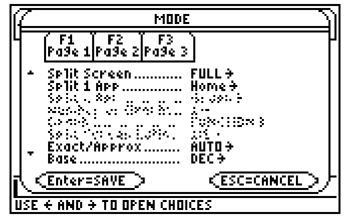


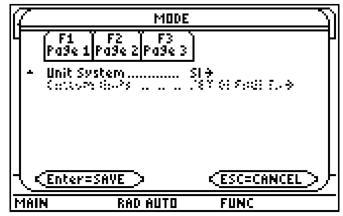
   Then press ⓡ to see the valid settings. Press the number of your choice.



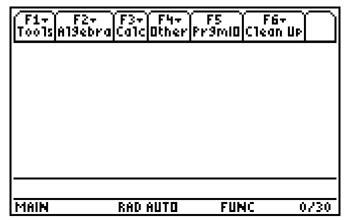
2. When the Page 1 settings are complete, press [F2] to check Page 2. Following the directions in step 1, make changes as needed. Note that certain items are dimmed. This simply indicates that they are not needed at this time. If you changed to a split screen, for example, more choices would appear.

   Use [F3] to check Page 3 in the same way.

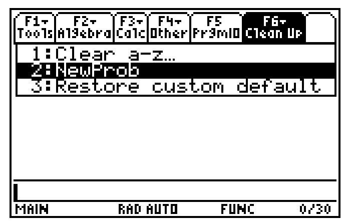




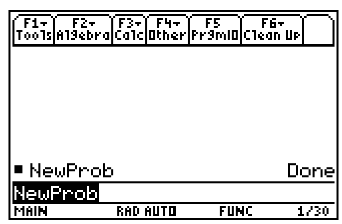
3. Press [ENTER] to save the new settings and exit from the MODE dialog box. If you did not return to the Home screen, press [HOME]. Press [F1] **Tools** and select **8:Clear Home** to clear the Home screen. If necessary, press [CLEAR] to clear the entry line.



4. Press [2nd] [F6] **Clean Up** and select **2:NewProb** to clear variables and other settings.



5. Press [ENTER].



Example 1: Graphing a rational function

In this example, you will plot a rational function and explore function values on the graph screen and by table. The function is

$$y = \frac{2x^2 + 2x - 12}{x^2 - 4}.$$

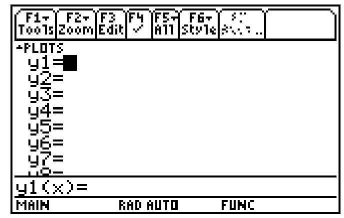
What is *y* when *x*= ⁻1? What is the x-intercept? What is *x* when *y*=1? Create a table of values for $x \in \{-4, -3, -2, -1, 0, 1, 2, 3\}$.

Solution

To find the answers, first enter the function in the Y= Editor. Use trace, zero, and intersection to compute function values. Then use the Table screen to enter the values for x.

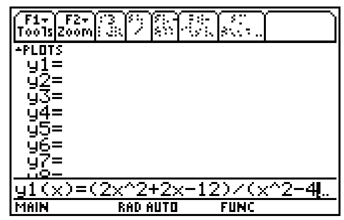
Enter the function

1. To enter this function, press [◆] [Y=] to display the Y= Editor.

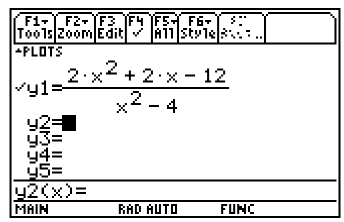


2. Define $y1(x)$. Be sure to use parentheses for the numerator and denominator. Use [^] to designate exponents. The function appears in the entry line.

   [(] **2X**[^] **2** [+] **2X**[−] **12** [)] [÷] [(] **X**[^] **2** [−] **4**[)]

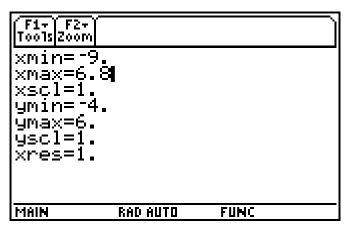


3. Press [ENTER].



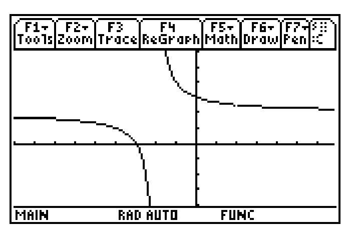
Define the viewing window

To define a viewing window, press [◆] [WINDOW] to display the Window Editor. Use ⊝ or [ENTER] to move through the choices and alter as needed to set the Window variable values as shown.



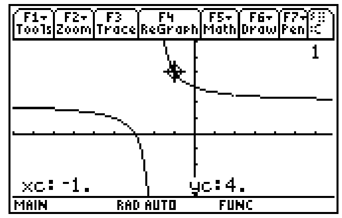
Graph

To plot the graph, press [◆] [GRAPH].

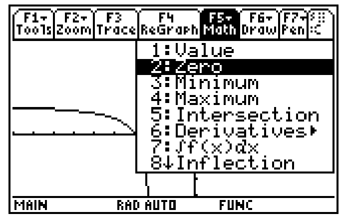


Compute function values

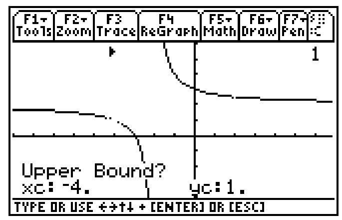
1. To trace the graph, press [F3] **Trace**. Use ◄ or ► to move the cursor along the graph. To move directly to a point, type the x-coordinate. For example, press [(-)] **1** [ENTER].



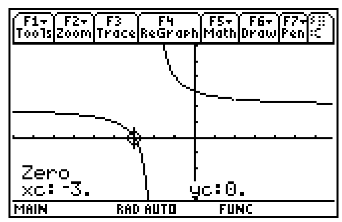
2. To compute an x-intercept, press [F5] **Math** and select **2:Zero**.



3. You have to set bounds for the TI-89 to compute the zero. At the lower bound prompt, press ◄ or ► to move to the left of the x-intercept and press [ENTER]. Or type a value known to be less than the x-intercept and press [ENTER].

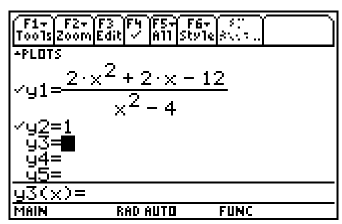


4. Similarly, enter an upper bound and press [ENTER]. The x-intercept of the graph zero of the function will be estimated.

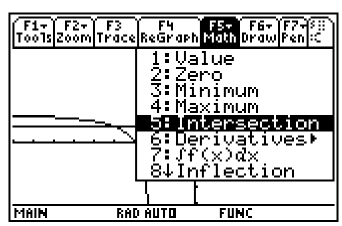


5. Often, you wish to compute an x-coordinate for a given value of y. One easy way to do this is to plot a horizontal line and compute the intersection point(s) of the original function and the horizontal line.

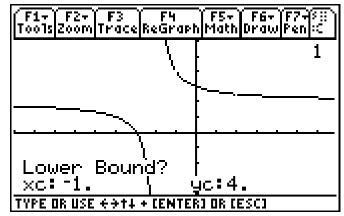
   To compute x when y=1, press [♦] [Y=] and enter a second function y2=1, the horizontal line. Press [ENTER].

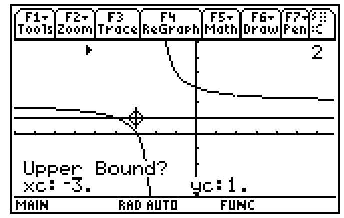


6. Press [♦] [GRAPH] to graph the functions.
7. Press [F5] **Math** and select **5:Intersection**.

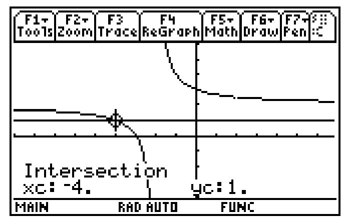


8. Press [ENTER] to select $y1$ as the first and curve. Press [ENTER] again to use $y2$ as the second curve. Press ◀ or ▶, or type values, to set the bounds. This works just like the zero math tool described previously.



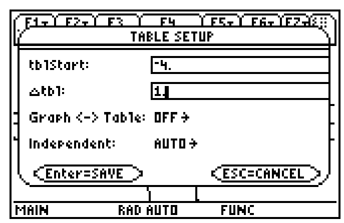


The coordinates of the intersection point are displayed.



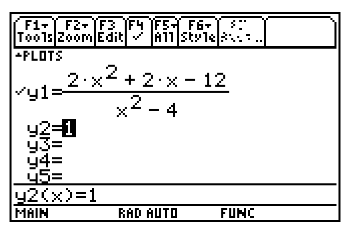
Create a table of values

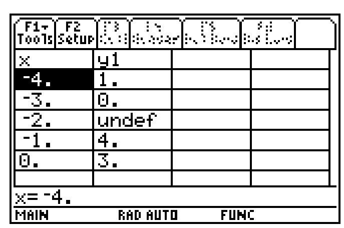
You will often wish to examine the behavior of a function at more than just a single point. The behavior can be examined both graphically and numerically. The numerical information is often best examined by a table of values.



1. To create a table of values, first press ◆ [TblSet]. Enter a starting value (Δ**tbl start** =-4), and an increment (Δ**tbl**=1). Be sure to press [ENTER] to save the new values.

2. Your TI-89 will plot a graph and display values in the table for any selected function. Selected functions can be identified by the check mark at the left of the name in the Y= Editor. In this particular example, the values for $y2$ aren't of interest since they are all 1. You can deselect $y2$ so that the values do not appear in the table. To deselect the function $y2$, press ◆ [Y=] to return to the Y= Editor, place the cursor on $y2$ and press [F4] ✓. The check mark is removed, indicating a deselected function. It will not be graphed or display values in the table.





3. To see the table, press ◆ [TABLE].

Example 2: Investigating limits

In this example, you will continue to investigate this function, particularly continuity, asymptotic behavior, and limits.

Given $f(x)=\dfrac{2x^2+2x-12}{x^2-4}$ (the same function as in Example 1) investigate the following limits:

$$\lim_{x\to 2} f(x)$$

$$\lim_{x\to -2+} f(x)$$

$$\lim_{x\to -2-} f(x)$$

$$\lim_{x\to \infty} f(x)$$

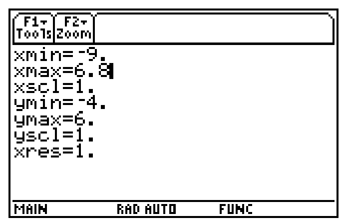
$$\lim_{x\to -\infty} f(x)$$

Solution

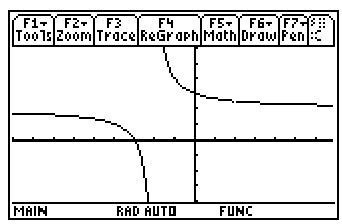
Investigate each limit graphically, numerically by a table of values, and with the **limit(** function.

Investigating graphically

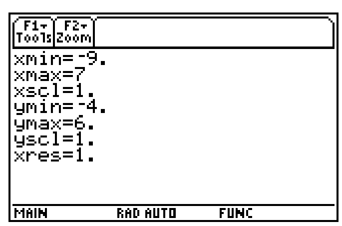
1. Press ◆ [WINDOW] and set the same Window variable values as in Example 1 (at the right).



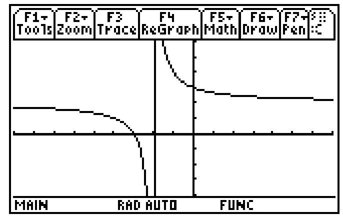
2. Press ◆ [GRAPH] to graph the function. You see evidence of a discontinuity at x=2. This is due to the "decimal" window where each pixel is .1 unit and **xres**=1 so that there is a pixel corresponding to x=2.



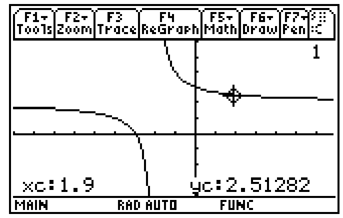
3. Return to the Window Editor, change **xmax** to 7, and graph the function.



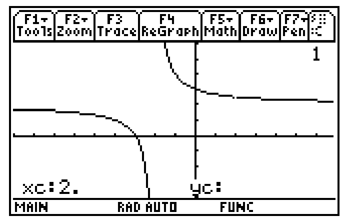
Notice that this minor change in the window settings affects the display. In this new window, there is no evidence of the discontinuity. It also makes a “false connection,” resembling a vertical asymptote.



4. To investigate the limit as $x \to 2$, change **xmax** back to 6.8 and graph. Press [F3] **Trace** and trace near x=2.

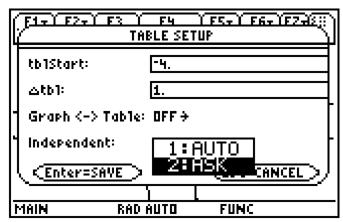


At x=2, no y-coordinate is displayed. Notice also that the display suggests a vertical asymptote near $x = {}^{-}2$ and a horizontal asymptote near y=2.

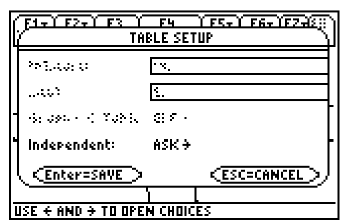


Investigating numerically

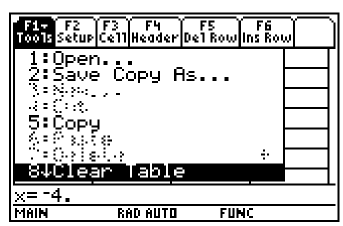
1. To investigate a limit numerically, press [◆] [TblSet].
   a. Use ⊖ to move the cursor to **Independent**.
   b. Press ⊕.
   c. Select **2:Ask** to change the selection.



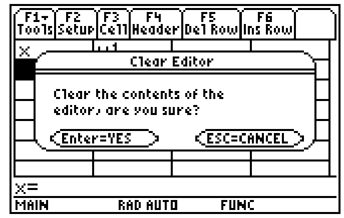
Be sure to press [ENTER] to save the change.



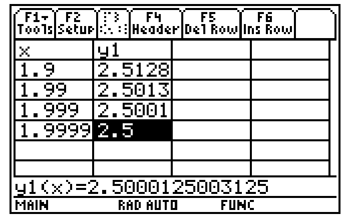
2. Now press [◆] [TABLE]. If necessary, press [F1] **Tools** and select **8:Clear Table**.



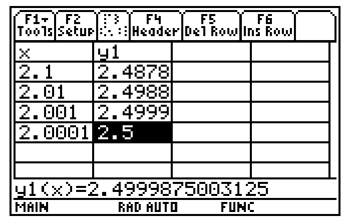
Press [ENTER] at the prompt to clear the table.



3. You can now enter values near 2 to investigate the limit. Press ⊝ to move to the next cell if you want the previous results to remain.



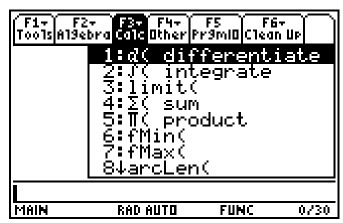
4. Try values larger than 2 to investigate the right-hand limit.



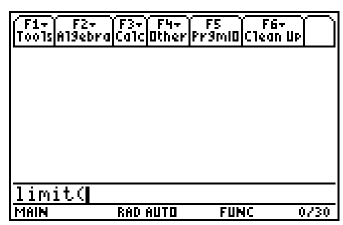
Using the limit command

The limit is the fundamental idea in a calculus course. With the TI-89, you can compute a limit as x approaches a particular value by entering the function, variable, and value.

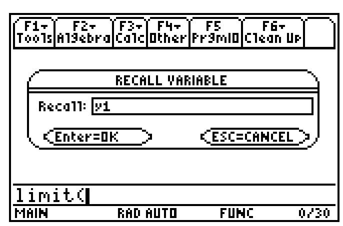
1. Press [HOME] to return to the Home screen. Press [F3] **Calc** for the calculus menu.



2. Select **3:limit(** for the **limit** command. It will be pasted to the entry line on the Home screen.

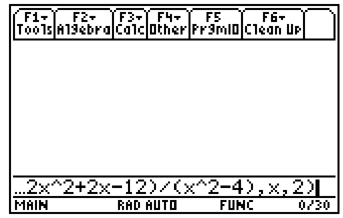


You want the function $y1(x)$ as the first argument of this command. You can type the function again. However, since you have already entered this function as $y1$, it is much easier to press [2nd] [RCL], type **y1** in the dialog box, and press [ENTER] twice.

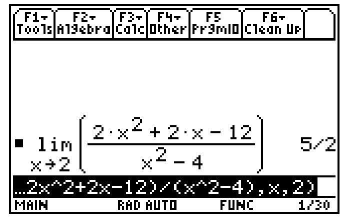


Then type the rest of the command.

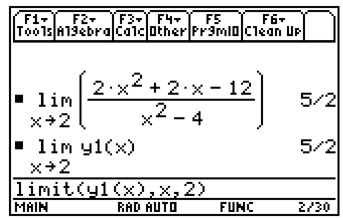
[,] **X** [,] **2** [)]



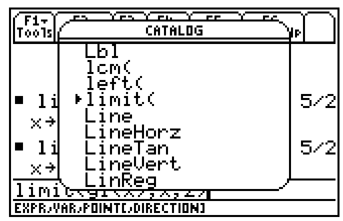
3. To compute the result, press [ENTER].



Alternately, you can use the function name, such as *y*1, in the **limit(** command.

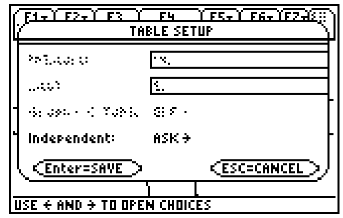


4. You also can find the **limit(** command in the CATALOG. Press [CATALOG]. The calculator assumes alpha mode, so just press **L**. Use ↓ to point to the **limit(** command. Notice the help on the bottom line of the screen. It describes the parameters for the command. Press [ENTER] and proceed as described above.

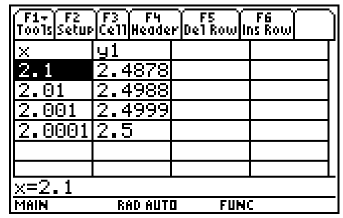


Investigating other table values

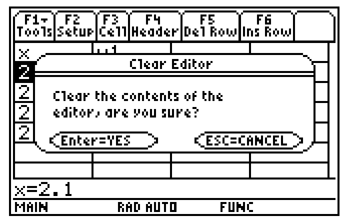
1. To check that the table is still in the ASK setting, press [◆] [TblSet].



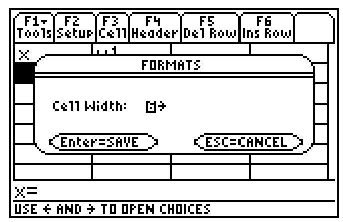
2. Press [◆] [TABLE] to return to the table .



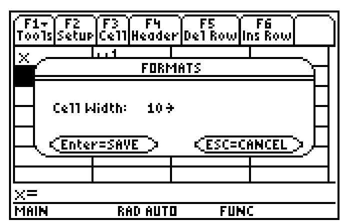
3. To clear the table, press [F1] **Tools** and select **8:Clear Table**. Then press [ENTER].



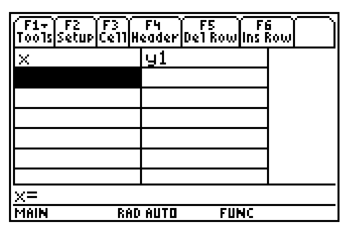
4. To change the format, press [◆] [I]. (You can also get to this screen by pressing [F1] **9:Format**.)



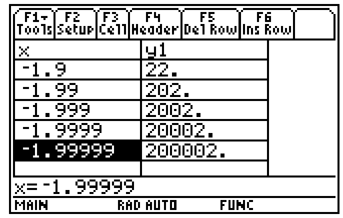
5. Change the cell width by using ⊙, then ⊙ to highlight **10**, and press [ENTER]. Press [ENTER] again to save the change.



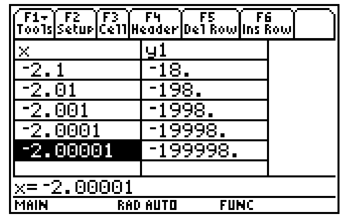
The columns should now be wider so that more digits can be displayed.



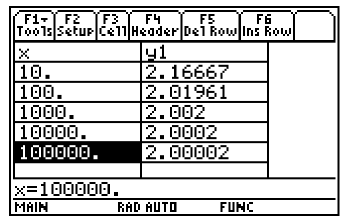
6. Type values near $^-2$ to explore the limit. Press [ENTER] to see the result, but remember to press ⊙ if you want to move to a new cell.



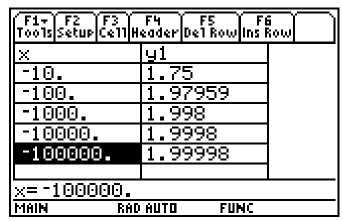
7. Also check values on the other side of $^-2$.



8. Next, check values as $x \to \infty$.



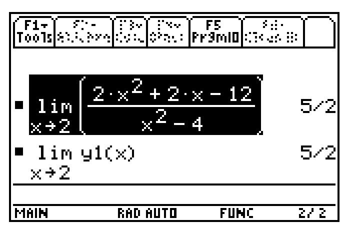
9. Finally, investigate as $x \to -\infty$.



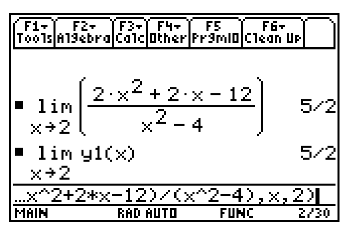
Entering more limit commands

It is also important to investigate limits when the results are infinite, or as *x* approaches positive or negative infinity.

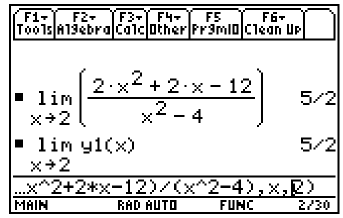
1. To return to the Home screen, press [HOME]. Use ⊙ to highlight the previous **limit(** command. (If this command is not on your Home screen, just type the entire command as shown at the right.)



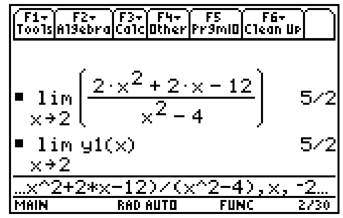
2. Press [ENTER] to copy the command to the entry line.



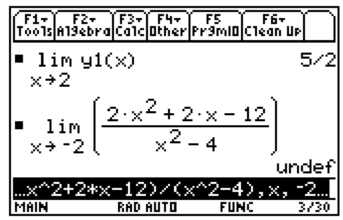
3. An insert cursor (vertical bar) will appear at the end of the line. Press ⓵ twice to place the cursor in front of the 2.



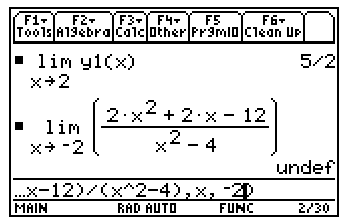
4. Press [(-)] to change the value to ⁻2.



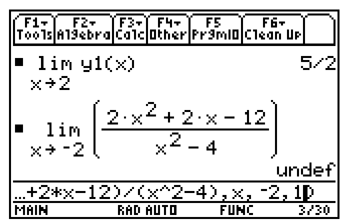
5. Press [ENTER] to evaluate.



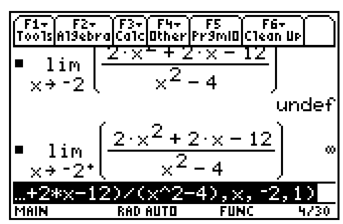
6. To investigate a one-sided limit, press ⓶ to move to the end of the entry line. Then press ⓵ to move the cursor between the 2 and the).



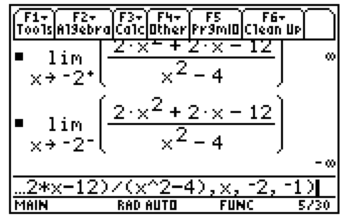
7. To evaluate a one-sided limit, use a fourth argument in the command. Type [,]**1**. (The **1** indicates a right-hand limit; any positive number will do.) Press [ENTER].



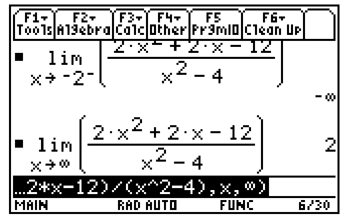
You can see that the result of infinity agrees with the asymptote you saw in the graph and in the numerical behavior of the table.



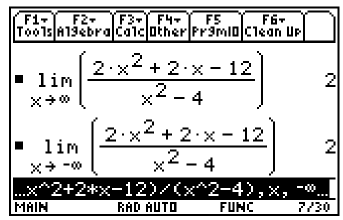
8. Investigate the left-hand limit in a similar manner. Note that entering ⁻1 created a left-hand limit, even though it is to the right of ⁻2. If the fourth argument is positive, the calculator computes a right-hand limit. If the fourth argument is negative, the calculator computes a left-hand limit, regardless of the value *x* approaches. The result of negative infinity again agrees with the graph, vertical asymptote, and the table.



9. Press ▶ to move the edit cursor to the end of the entry line. Press [←] to remove the ⁻1 and the ⁻2 and press [◆] [∞] [)] to insert infinity. Press [ENTER] to see result. This result of 2 reinforces the horizontal asymptote and the numerical values in the table.



10. Finally, investigate the limit as $x \to -\infty$.



Exercises

Given $g(x) = \dfrac{2 + x - x^2}{x^2 - 1}$

1. Graph $g(x)$.
2. What is $g(3)$?
3. What is the y-intercept?
4. What is the x-intercept?
5. What is x if $g(x)=3$?
6. Create a table of values for $x \in \{-3,-2,-1,0,1,2,3\}$.
7. What is $\lim\limits_{x \to -1} g(x)$?
8. What is $\lim\limits_{x \to 1} g(x)$? $\lim\limits_{x \to 1+} g(x)$? $\lim\limits_{x \to 1-} g(x)$?
9. What is $\lim\limits_{x \to \infty} g(x)$? $\lim\limits_{x \to -\infty} g(x)$?