NUMB3RS Activity: Walkabout
Episode: "Mind Games"

Topic: Random Walks & π **Grade Level:** 8 - 12
Objective: Learn about random walks and use them to compute an approximation to π.
Materials: TI-83/84 Plus calculator or other means of generating random sequences of 0s and 1s.
Time: 30 minutes

Introduction

In "Mind Games," Charlie uses the Fokker-Planck equation to predict the movement of illegal immigrants. The equation is used to predict the likely movement of something under the influence of external forces. It was originally developed to study Brownian motion, the apparently random movement of a particle as it bounces off other particles. Brownian motion is an example of a random walk. In this activity, we study a simple random walk in which there are only two directions of movement: up and down.

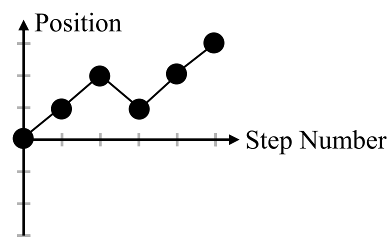
In a random walk with N steps and equal probability at each step of moving either up or down, the expected value of the final distance from the starting point approaches $\sqrt{\frac{2N}{\pi}}$ as N increases. This surprising appearance of π is one of the topics discussed in the Extensions page.

Discuss with Students

A random walk describes the movement of a person, where at each step, the direction is chosen at random with a certain probability assigned to each direction. In this activity, the choice of movement is limited to up or down, with a 50% chance for each direction. We record each walk with "U" for up and "D" for down. There are several questions we can ask about the walks that have a given number of steps, say ten steps. What are the possible final positions? What is the probability of winding up in each of these final positions? What is the expected value (long-term average) for the final distance from the horizontal axis?

Each student will create three 10-step random walks. The class will compute the average of all of their distances from the starting point. As the number of steps N gets larger, the expected value approaches $\sqrt{\frac{2N}{\pi}}$. The students will determine how close their average and the class' average are to that value.

The graph to the right is a function representation of the 5-step walk UUDUU. This results with the walker at position 3 after the 5th step.



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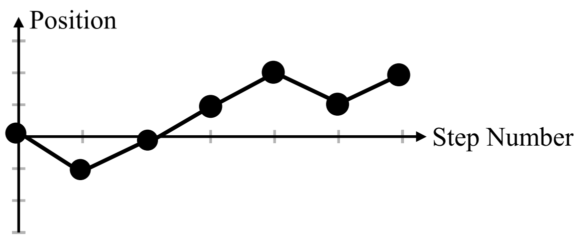
Student Page Answers:

1.

	Number of Possible Walks								
Number of Steps	**D4**	**D3**	**D2**	**D1**	**0**	**U1**	**U2**	**U3**	**U4**
0					1				
1				1	0	1			
2			1	0	2	0	1		
3		1	0	3	0	3	0	1	
4	1	0	4	0	6	0	4	0	1

2.



The walk ends at position 2.

3. *$2^6 = 64$* **4.** *1* **5.** *$\frac{1}{64}$* **6.** *Not possible; after an even number of steps, a person would always be an even number of positions from 0.* **7.** *6* **8.** *$\frac{6}{64}$* **9.** *Answers will vary. The graph records three walks.* **10.** *π can be approximated by calculating 2N divided by the square of the average distance.* **11.** *Yes*

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Name: ________________________________ Date: _______________

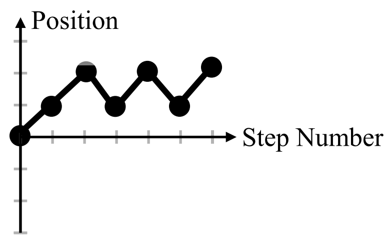
NUMB3RS Activity: Walkabout

In “Mind Games,” Charlie uses the Fokker-Planck equation to predict the movement of illegal immigrants. This equation enables us to predict the likely movement of someone for whom each step might be in any of several different directions, with a certain probability assigned to each direction. The path this person follows is called a **random walk**. In this activity, each step of the random walk is either up (U) or down (D), and the probability of moving in either direction is 50%.

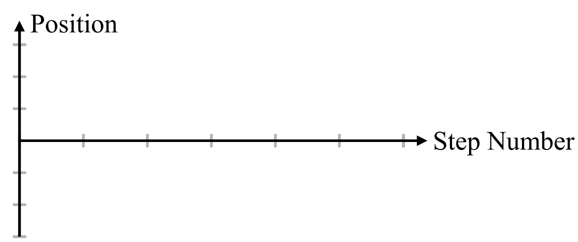
1. Complete the table below, which shows the number of possible ways for a person to end a random walk (that ranges from 0–4 steps) in a certain position. For example, D1 means down 1 unit, U1 means up 1 unit, D2 means down 2 units, etc. The first two rows are completed showing the number of different ways to make a 0-step and a 1-step walk. There is only 1 possible 0-step walk, which ends at position 0. For a 1-step walk, there is 1 way for a person to end down 1 unit (D1) and 1 way for a person to end up 1 unit (U1).

	Number of Possible Walks								
Number of Steps	**D4**	**D3**	**D2**	**D1**	**0**	**U1**	**U2**	**U3**	**U4**
0					1				
1				1	0	1			
2									
3									
4									

A random walk can also be recorded by listing the steps in order. For example, UUDUDU goes up 2 steps, down 1 step, up 1 step, down 1 step, and up 1 step. The walk ends 2 units above the horizontal axis (position 2). A graphical representation of this path is shown at the right.



2. Show the graph representing the walk DUUUDU. What is the final position of this walk?



3. How many different 6-step walks are there?______________________________

4. How many of the 6-step walks end at position 6? _____________________________

5. What is the probability that the 6-step walk ends at position 6? __________________

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6. Is it possible for a 6-step walk to end at position 5? Give an example or explain why it cannot be done.

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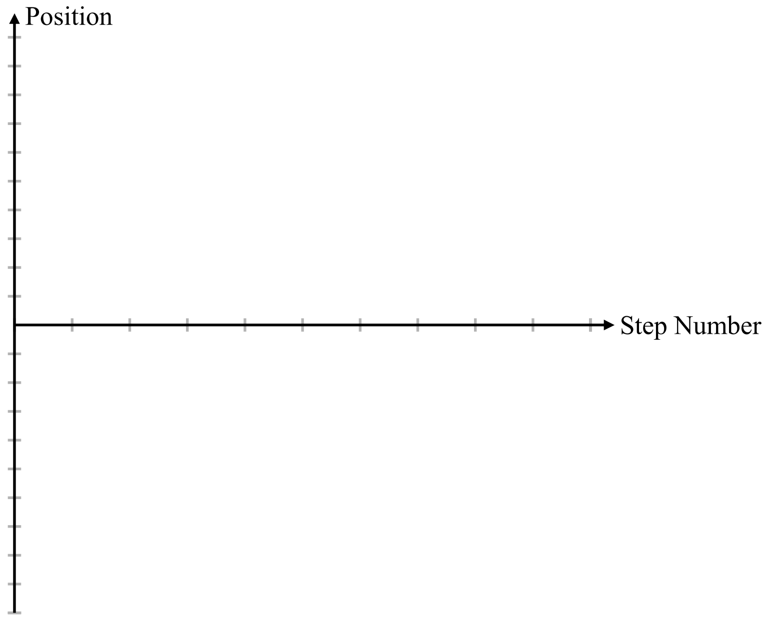
7. How many of the 6-step walks end at position 4? ________________________________

8. What is the probability that a 6-step walk ends at position 4? ____________________

9. On the graph below, show three 10-step random walks. Either flip a coin or use your calculator to simulate coin tossing: 0s move up, 1s move down. To generate a list of 10 random numbers from 0 to 1 on your calculator, enter the command **randInt(1, 0, 10)**. (To find **randInt(** press $\boxed{\text{MATH}}$ $\boxed{\blacktriangleleft}$ $\boxed{5}$.)



Compute the average distance from the ending position of the walk to the horizontal axis for the three random walk simulations you completed (remember that distance is always non-negative). The expected value of the distance for the 10-step path is 2.46. The expected value means that in the long run, the average distance the end of the walk moved away from zero is 2.46 units. Compare your value to the expected value of the distance for a 10-step walk.

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10. As the number of steps N increases, the expected value approaches $\sqrt{\frac{2N}{\pi}}$. Explain how you could use this average distance to approximate the value of π.

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11. Generate a 15-step walk and compute the distance from the ending position of the walk to the horizontal axis. Use the values from the entire class to find the class average distance for a 15-step walk. Is this value closer to $\sqrt{\frac{2N}{\pi}}$ than the 10-step walk? _________

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The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extension: Connection to Binomial Coefficients

Introduction

The table in question 1 includes Pascal's triangle. We can see why binomial coefficients appear if we think about all 6-step walks that end 2 steps above the starting level. Such a path must have exactly 4 steps up and 2 steps down (# of steps = $U + D = 6$, final position = $U - D = 2$, so $U = 4$ and $D = 2$). The 2 down steps can be in any of the 6 positions. The number of different ways for this to occur is ${}_6C_2$.

For the Student

Explain why a walk of N steps must end an even distance from 0 if N is even and an odd distance from 0 if N is odd. Show that the number of walks of N steps that end at position p is given by the binomial coefficient ${}_NC_{(p+N)/2}$.

Additional Resources

- Another way of estimating π was discovered by 18th century mathematician Georges-Louis Leclerc, Comte de Buffon. He showed that when a needle is dropped over a series of equally spaced parallel lines, the probability of the needle touching the lines is equal to $\frac{2L}{D\pi}$, where L is the length of the needle and D is the spacing of the parallel lines.

  For more about this method and a computer program to illustrate it, see: **http://www.angelfire.com/wa/hurben/buff.html**

- For a description of random walks, including the derivation of the expected value formula, see: **http://mathworld.wolfram.com/RandomWalk1-Dimensional.html**

- The connection to π comes from Wallis' formula: $\frac{\pi}{2} = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdots$. For more information, see: **http://mathworld.wolfram.com/WallisFormula.html**

- An applet for one-dimensional random walks can be found at: **http://polymer.bu.edu/java/java/1drw/RandWalk1D.html**

Related Topics

- An applet for exploring two-dimensional random walks, can be found at: **http://polymer.bu.edu/java/java/2drw/RandWalk2D.html**

- For further explanation of the binomial theorem, see the activity **Right or Wrong**, which also accompanies the *NUMB3RS* episode "Mind Games." This activity can be downloaded for free from the Web site below.
  http://www.cbs.com/primetime/numb3rs/ti/activities/Act1_RightOrWrong_MindGames_final.pdf

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