

One-sided Limits and Continuity with Piece-Wise Defined Functions

by – Matt Bohon

Activity overview

Piece-wise defined functions are used extensively in PreCalculus and Calculus. This concept will be used to introduce one-sided limits and continuity. Students will engage in a discovery activity using the TI-Nspire. This activity is meant as an introduction to the limit finding capabilities of the TI-Nspire.

Concepts

- *Piece-wise defined functions*
 - *Limits*
 - *Continuity*
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Teacher preparation

Students should have a firm grasp of graphing many types of functions. Also general knowledge of the TI-nspire is required. Graphing and/or defining piece-wise defined functions is a definite prerequisite.

Classroom management tips


This activity will be primarily student driven. Students could work in groups to complete the attached activity sheet. Be sure students understand these concepts numerically, graphically, and algebraically. The example demonstrated in the step-by-step directions would be a great first example to be led by the teacher using either a TI-Nspire or the TI-Nspire computer software.

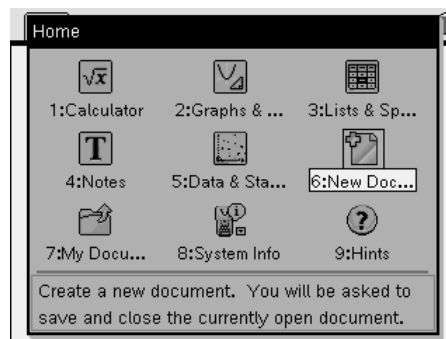
TI-Nspire Applications

Graphs & Geometry, Calculator

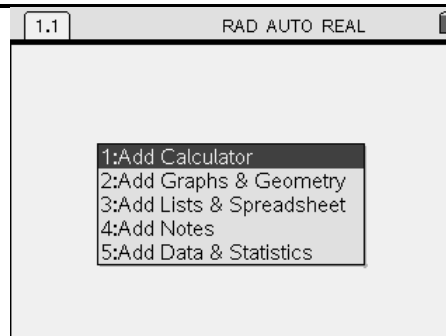
Step-by-step directions

These instructions are for the first exercise. Similar steps can be used to solve the remaining problems.

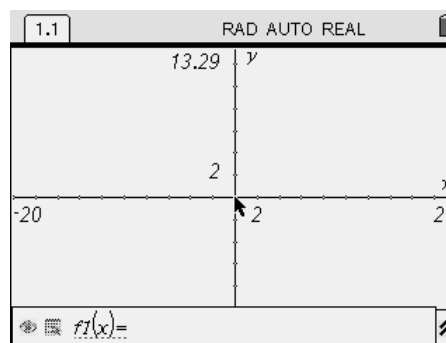
Press , taking you to the home screen.



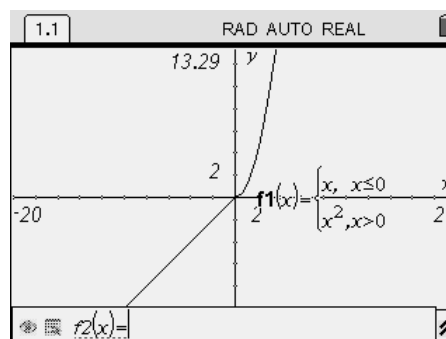
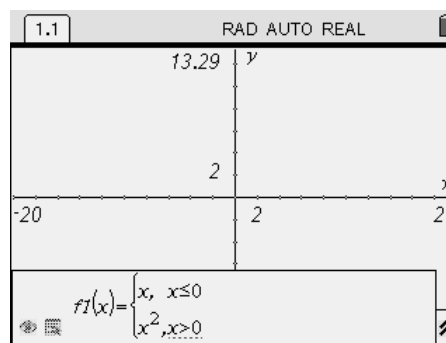
Press **6** for a New Document. You may be Asked if you want to save this document. Answer 'NO' to move on.



Type **2** to add 'Graphs and Geometry'.

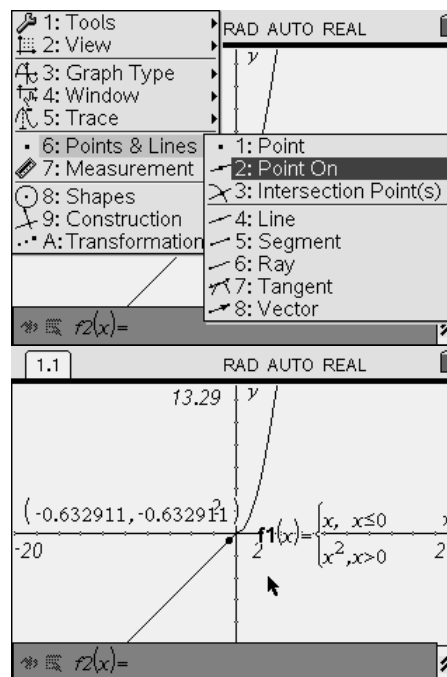


Enter the function definition from Exercise 1 on the student worksheet. Then press enter to graph it.



Inspect this graph at $x=0$. Does the graph appear to be continuous there?

Using **(menu)**, 6: Points & Lines, 2: Point On, place a point on the graph

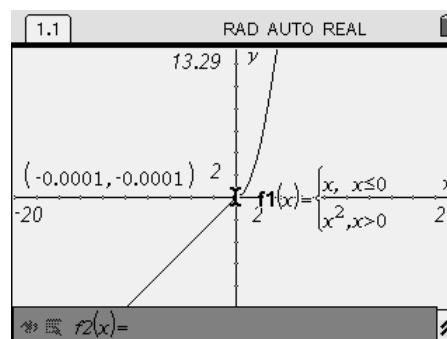
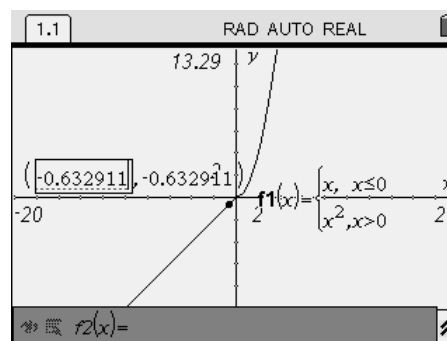


Use the NavPad to point at the x-coordinate of the newly placed point. Hit Enter twice to select this coordinate. Enter a value of $x = -.0001$, just to the left of $x=0$. Notice the y-value.

The same method can be used to check a value just to the right of $x=0$.

Do the y-values in both these cases agree?

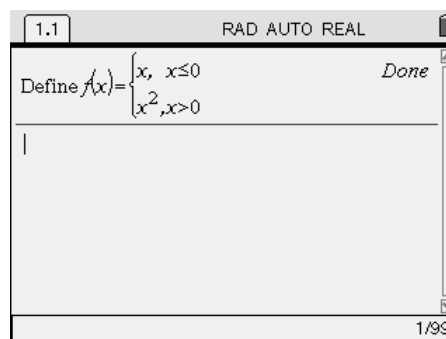
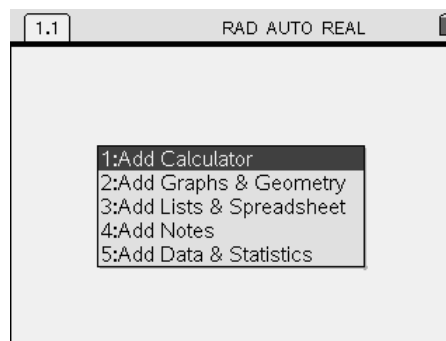
Note: The point can also be grabbed and dragged using the grab-hand method on the nav-pad.



The same problem can be worked without using the Graphs and Geometry Application. Start a new problem with a

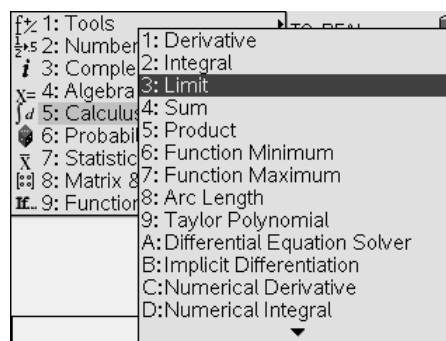
Calculator Application. Type **menu**, 1: Tools, 1: Define, to define the same function.

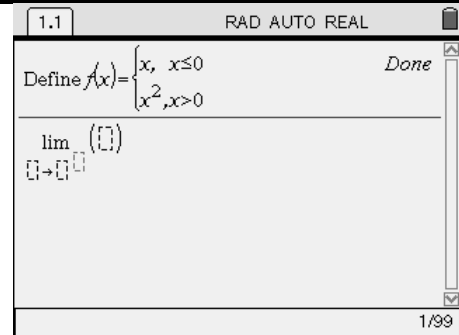
Note: This would work exactly the same if Calculator App were added to our graph. The variable $f(x)$ would still be defined and all the following activities would still work.



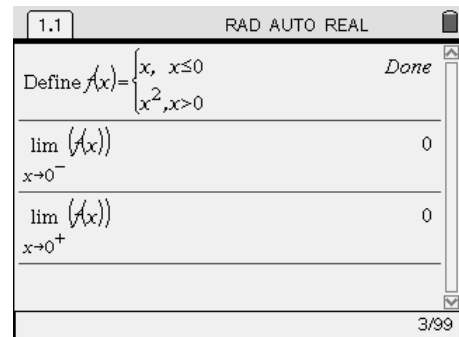
We can now use the built in Limit command to check our previous graphical and numerical answers.

menu, 5: Calculus, 3: Limit is an easy way to get to this command.

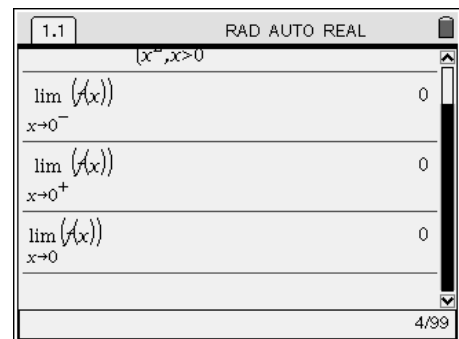




The left and right hand limits are easily calculated in the shown usual manner.



If no left or right hand limit is entered, the limit at the point will be calculated.



Notice that all three of the calculated limits equal zero. Does this support our other work?

Is this enough to prove continuity? We lack one other value, $f(0)$, which is easily calculated in the calculator application. Notice $f(x)$ will be bold-faced, when it is entered in, to indicate that is an already defined variable.

1.1		RAD AUTO REAL	
$\lim_{x \rightarrow 0^-} f(x)$	0		
$\lim_{x \rightarrow 0^+} f(x)$	0		
$\lim_{x \rightarrow 0} f(x)$	0		
$f(0)$	0		
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Assessment and evaluation

- Successful completion of the attached worksheet.
- Teachers may limit questions to more easily understood equations.

Activity extensions

- Finding limits using much more complex functions would be useful for advanced students.
- In Calculus, the concepts of derivatives and integrals of piece-wise defined functions would follow this activity.