

Activity 10

**Analyzing Variance (ANOVA)**

Topic 49 uses **STAT** <TESTS> **F:ANOVA** with lists of data gathered from a completely randomized design. Program **A1ANOVA** (see *Appendix B*) is introduced with its capability of using summary statistics for input. Program **A1ANOVA** can also use raw data stored in a matrix as opposed to a list. This technique is required for those who will analyze randomized block designs and two-factor factorial experiments, which are presented in Topics 50 and 51.

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**Topic 49—Completely Randomized Designs (One-Way ANOVA)**

To compare the distances traveled by three different brands of golf balls when struck by a driver, we use a completely randomized design. A robotic golfer, using a driver, is set up to hit a random sample of 24 balls (8 of each brand) in a random sequence. The distance is recorded for each hit, and the results are shown in the table below, organized by brand.

	L1	L2	L3
<b>Brand</b>	Brand A	Brand B	Brand C
<b>Distance</b>	264.3	262.9	241.9
	258.6	259.9	238.6
	266.4	264.7	244.9
	256.5	254	236.2
	182.7	191.2	167.3
	181	189	165.9
	177.6	185.5	162.4
	187.3	192.1	172.5
<b>Mean</b>	221.8	224.9	203.7
<b>StdDev</b>	42.58	38.08	39.4
<b>n</b>	8	8	8

## Activity 10, Analyzing Variance (ANOVA) (cont.)

Test the null hypothesis  $H_0: \mu_A = \mu_B = \mu_C$ .

- With the data stored in lists  $L_1$ ,  $L_2$ , and  $L_3$ , press **[STAT]** **<TESTS>** **F:ANOVA(L1 [ ] L2 [ ] L3)**, as shown in screen 1.
- Press **[ENTER]** to display the next two screens (2 and 3).

With the p-value of **0.531**, the data shows no significant difference between the mean distance traveled by the three brands of balls. We do not reject the null hypothesis.

### Bonferroni Multiple Comparison Procedure

Since we do not reject the null hypothesis, there is no significant difference between any of the means. A multiple comparison procedure is not needed or appropriate.

Topic 50 gives an example of the multiple comparison procedure that relates back to this topic, so you will know how to proceed if the null hypothesis above is rejected.

### Program A1ANOVA

Program **A1ANOVA** is available from Texas Instruments over the internet ([www.ti.com](http://www.ti.com)) or on disk (**1-800-TI-CARES**) and can be transferred to your TI-83 using TI-GRAPH LINK™. (The program listing is in Appendix B.)

- Press **[PRGM]**, highlight program **A1ANOVA**, and then press **[ENTER]** to paste the name, as shown in screen 4.
- Press **[ENTER]** for the menu on screen 5.
- Press **1:ONE-WAY ANOVA** for screen 6, which informs you of two options for input of the data: a matrix or summary statistics. The procedures for using these options follow.

### Using Summary Statistics

- Press **[ENTER]**, and the menu (screen 7) presents the options mentioned in screen 6.
- Select **2: $\bar{x}1, Sx1, n1, \bar{x}2$**  for screen 8.
- When prompted with **HOW MANY LEVELS?** (screen 8) type **3**, and then press **[ENTER]**. (There are three levels or brands.)

(1) ANOVA(L1,L2,L3

(2) One-way ANOVA  
F=.6531481238  
P=.5306573758  
Factor  
df=2  
SS=2096.76083  
↓ MS=1048.38042

(3) One-way ANOVA  
↑ MS=1048.38042  
Error  
df=21  
SS=33707.4975  
MS=1605.11893  
Sxp=40.0639355

**Note:**  $Sxp = \sqrt{(MSE)} = \sqrt{(1605.11893)} = 40.0639355$ . (See screen 3.)

(4) PrgmA1ANOVA

(5) ANOVA USES (0-4) OF  
1: ONE-WAY ANOVA  
2: RAN BLOCK DESI  
3: 2WAY FACTORIAL  
4: QUIT

(6) 1. USE A N\*2 MAT:  
[D], OBSERVATIONS  
IN COL 1, FACTOR  
LEVEL IN COL 2-  
INTEGERS 1,2..N.  
2. ENTER MEAN, SD  
AND SAMP SIZES.

(7) DATA INPUT MODE  
1: DATA MAT [D]  
2:  $\bar{x}1, Sx1, n1, \bar{x}2$ ..  
3: QUIT

(8) HOW MANY LEVELS  
?3  
LEVEL  
MEAN=?221.8  
S.D.=?42.58  
SIZE=?8

- Enter the means, standard deviations, and sample sizes for each level, as shown in screens 8 and 9. After you press **ENTER** the final time, the **ANOVA** table appears (screen 10).

The results are basically the same as before. The differences occur because the means and standard deviations were rounded with the summary statistics.

The mean squares, MS, are not given in the table but are easily calculated with  $MS = SS/DF$  or, for **Factor**:  $2097.76/2 = 1048.9$  and for **Error**:  $33708.52/21 = 1605.2$ .

- Press **ENTER** again, and 95 percent confidence intervals (screen 11) for each mean are calculated based on the pooled standard deviation **SP** or **Sxp**.

Note all of these intervals overlap, indicating there is no significant difference between the means.

### Using Matrix [D]

- Enter the 24 data values in a 24x2 matrix [D] (see screen 12) with all the distance data in column 1 and the level or brand data in column 2 (eight 1s, eight 2s, and eight 3s).

This is explained in the informational screen (screen 6) that appears after you select **ONE-WAY ANOVA**. (See "Storing Data in a Matrix" in Topic 48. You may enter data by column by pressing **▾** after each value instead of **ENTER** as shown in Topic 48.)

- Select **1:DATA MAT [D]**, as shown in screen 13.

The **ANOVA** table appears. (See screen 14.)

Press **ENTER** again to get the sample size, means, and standard deviations for the three levels. (See screen 15. Use **▾** to view the standard deviation.)

- Press **ENTER** for the confidence intervals, which are the same as above. (See screen 11.)

If you wonder how a robot swinging a driver could get such wide variations in distances, several possible explanations exist:

- The wind was shifting and gusting.
- The balls were inconsistently made.
- The robot hit with a wide variability of forces.

Other reasons are given in the next two topics.

```
MEAN=?224.9
S.D.=?38.08
SIZE=?8
LEVEL
MEAN=?203.7
S.D.=?39.4
SIZE=?8
```

```
DF SS
FAC 2 2097.76
ERR 21 33708.519
6
F=.65
P=.531
SP=40.06454293
```

```
0.95 C.I.S -LEVEL 1 AT TOP
_____
_____
_____
```

```
MATRIX[D] 24x2
[ 264.3 1 ]
[ 258.6 1 ]
[ 266.4 1 ]
[ 256.5 1 ]
[ 182.7 1 ]
[ 181 1 ]
[ 177.6 1 ]
[ 187.3 1 ]
[ 262.9 1 ]
[ 259.9 1 ]
[ 264.7 1 ]
[ 254 1 ]
[ 191.2 1 ]
[ 189 1 ]
[ 185.5 1 ]
[ 182.7 1 ]
[ 241.9 1 ]
[ 238.6 1 ]
[ 244.9 1 ]
[ 236.3 1 ]
[ 187.3 1 ]
[ 185.9 1 ]
[ 182.4 1 ]
[ 172.5 1 ]
[ 24, z=3 ]
```

```
DATA INPUT MODE
1:DATA MAT [D]
2:X1,Sx1,n1,X2..
3:QUIT
```

```
DF SS
FAC 2 2096.7608
ERR 21 33707.497
5
F=.65
P=.531
SP=40.06393551
```

```
[LEV N MEAN SD]
[1 8 221.8 ...]
[2 8 224.9125 ...]
[3 8 203.7125 ...]
```

Activity 10, Analyzing Variance (ANOVA) (cont.)

Topic 50—Randomized Block Design (Program A1ANOVA)

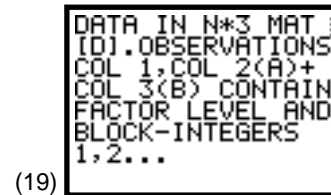
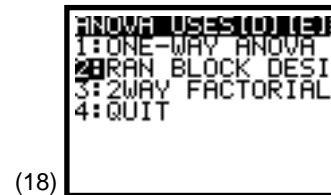
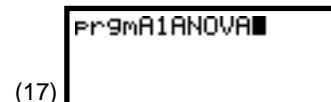
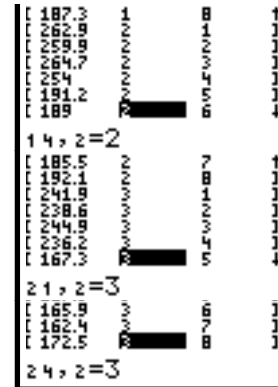
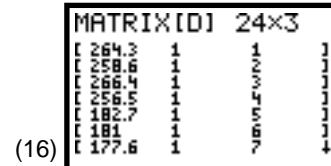
Suppose eight golfers are randomly selected and each golfer hits three balls, one of each brand, in a random sequence. The distance is measured and recorded, as shown in the table below and in matrix [D] on the TI-83. (See screen 16.)

Golfer (Block)	Brand A	Brand B	Brand C
1	264.3	262.9	241.9
2	258.6	259.9	238.6
3	266.4	264.7	244.9
4	256.5	254	236.2
5	182.7	191.2	167.3
6	181	189	165.9
7	177.6	185.5	162.4
8	187.3	192.1	172.5
Mean	221.8	224.9	203.7
StdDev	42.58	38.08	39.4
n	8	8	8

Test the null hypothesis  $H_0: \mu_A = \mu_B = \mu_C$ .

- Press **PRGM**, highlight program **A1ANOVA**, and then press **ENTER** so the name is pasted, as shown in screen 17.
- Press **ENTER** for the menu in screen 18.
- Press **2:RAN BLOCK DESIGN** for screen 19.

This screen informs you how to input the data into matrix [D]. You need a 24x3 matrix with the 24 distances in column 1, the factor levels (brands) in column 2 (eight 1s, eight 2s, and eight 3s), and the block integer (golfer) in column 3 (1 to 8 three times). See the matrix example in screen 16.



4. Press **ENTER**, and then “continue” for the **ANOVA** table shown in screen 20.

The very large F-value of **168.24** and a p-value of 0 to 3 decimal places (**0.000**) leads us to reject the null hypothesis and conclude that the mean distances are not all the same for the three brands of balls.

(20)

	DF	SS
A	2	2096.76083
B	7	33620.2583
ER	14	87.2391667
F(A)=168.24		
P=0.000		
F(B)=770.76		

5. Press **ENTER** to see screen 21.

Screen 21 shows  $S = \sqrt{(MSE)} = \sqrt{(SS/DF)} = \sqrt{(87.2392/14)} = \mathbf{2.49627}$ .

(21)

B	P=0.000
S=2.496271029	

### Bonferroni Multiple Comparison Procedure

We will use the Bonferroni Procedure to see which means differ. The table below shows the means ranked in order. (Note that the **ONE-WAY ANOVA** option of program **A1ANOVA** could be run with the current matrix [D] and used to find the means for each brand of ball.)

<b>Brand:</b>	C	A	B
<b>Mean:</b>	203.7	221.8	224.9

### Number of Pairwise Comparisons: $C = k(k - 1)/2$

There are three **nCr 2** ways of picking pairs from three means or  $3 * 2/2 = 3$  (with **nCr** under **[MATH] <PRB>**). These are CA, CB, and AB. Note that if there were four means, this would be four **nCr 2** =  $4 * 3/2 = 6$  pairs.

### Comparisonwise Significance Level ÷ 2: $\alpha / (2C)$

In doing multiple *t*-tests of  $H_0: \mu_1 = \mu_2$  with the alternate  $H_a: \mu_1 \neq \mu_2$ , and holding the overall experimental significance level to  $\alpha = 0.05$ , you will need to use a comparisonwise significance level =  $0.05/3$ . Because you are doing a two-tail test, you must divide this by 2 for  $0.05/6 = 0.00833$  in each tail.

### Critical *t*-value

With **0.00833** in the right tail of a *t*-distribution with 14 degrees of freedom (**Error** degrees of freedom), use the **MATH** equation solver to solve **tcdf( X, E99, 14) = 0.00833** for **x**, as explained on the last page of Topic 34. **X** is the critical value and equals **2.718**, as verified with **[2nd] [DISTR] 5:tcdf( 2 . 718 , E99 , 14 [ENTER]**, for an area of **0.00833**, as shown in screen 22.

(22)

tcdf(2.718, E99, 14)
.0083293348

## Activity 10, Analyzing Variance (ANOVA) (cont.)

### Comparisons

Calculate  $t = (\bar{x}_2 - \bar{x}_1) \div (\sqrt{MSE} * \sqrt{(1/n_1 + 1/n_2)})$  for each comparison.

For **AC**:

$$t = (221.8 - 203.7) \div (2.49627 * \sqrt{(1/8 + 1/8)}) = 14.502 > 2.718$$

For **BC**:

$$t = (224.9 - 203.7) \div (2.49627 * \sqrt{(1/8 + 1/8)}) = 16.985 > 2.718$$

For **AB**:

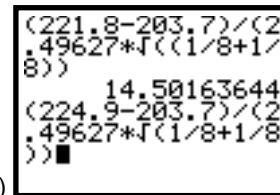
$$t = (224.9 - 221.8) \div (2.49627 * \sqrt{(1/8 + 1/8)}) = 2.484 < 2.718$$

The  $\boxed{2nd}$  [ENTRY] feature is helpful for doing the previous calculations on the home screen, as shown in screen 23.

Notice that Brand C has a smaller mean distance than either Brand A or Brand B, but brands A and B do not have significantly different means. We show this with a line over or under A and B, but not C:

$$C \overline{A} \overline{B} \text{ or } C \underline{A} \underline{B}$$

(23)



```
(221.8-203.7)/(2.49627*sqrt((1/8+1/8)))
14.50163644
(224.9-203.7)/(2.49627*sqrt((1/8+1/8)))
```

### Bonferroni for Completely Randomized Designs (Topic 49)

In Topic 49, we did not do the multiple comparisons procedure with the example because there was no significant difference between any of the means. If we could have rejected the null hypothesis, then we would have done the Bonferroni Procedure as it was done above. The means were the same but  $S = \sqrt{MSE} = 40.0639$  and the **Error** degree of freedom was 21 (which leads to a critical t-value of 2.601). The largest difference is between Brand B and Brand C for the following  $t$  statistic:

$$t = (224.9 - 203.7) \div (40.0639 * \sqrt{(1/8 + 1/8)}) = 1.0583 < 2.601$$

There is, therefore, no significant difference.

Note that for the completely randomized designs, the sample sizes do not have to be of the same size ( $n_1 \neq n_2$ ).

Although the data is the same, there is a large difference in the MSE in Topics 49 and 50. Because of the different power in which different golfers could hit the ball, we were able to block out much of the variability in Topic 50. All golfers were not able to hit Brand C as far as the other two brands. There was no such blocking possible in Topic 49, where the variability was due to other unknown causes. Although the means were the same in the two cases, there was no significant difference in Topic 49. If the null hypothesis were true, it is quite possible that another sample of 24 balls would have Brand C with the greater mean distance, but not significantly different from the other two brands.

## Topic 51—Two-Factor Designs With Equal Replicates

Suppose we test three brands of golf balls and two different clubs (driver and five-iron) in a completely randomized design. Each of the six ball-club combinations is randomly and independently assigned to four experimental units, each of which consists of a specific position in the sequence of hits by a golf robot. The distance response is recorded for each of the 24 hits, and the results are shown in the table below and in the matrix [D] shown in screen 24.

Factor B	Factor A		
(Club)	Brand A	Brand B	Brand C
Driver	264.3	262.9	241.9
	258.6	259.9	238.6
	266.4	264.7	244.9
	256.5	254	236.2
Five-iron	182.7	191.2	167.3
	181	189	165.9
	177.6	185.5	162.4
	187.3	192.1	172.5

The data is stored in matrix [D] of order 24x3 with the distance data in column 1, Factor-A level in column 2 (1, 2 or 3 for Brand A, B, or C) and Factor-B level in column 3 (1 for driver or 2 for five-iron). For example, the last value in the table above and in the last row of matrix [D] is 172.5, which is the distance a Brand C (3) ball is hit with a five-iron (2).

1. Press **[PRGM]**, highlight program **A1ANOVA**, and then press **[ENTER]** so the name is pasted, as shown in screen 25.
2. Press **[ENTER]** for the menu on screen 26.
3. Press **3:2WAY FACTORIAL** for screen 27, which informs you how to input the data into matrix [D] as was done above.
4. Press **[ENTER]** and then 'continue' for the **ANOVA** table at the top of the next page.

(24)

```

MATRIX[D] 24x3
[ 264.3  1  1 ]
[ 258.6  1  1 ]
[ 266.4  1  1 ]
[ 256.5  1  1 ]
[ 182.7  1  1 ]
[ 181    1  2 ]
[ 177.6  1  2 ]
[ 187.3  2  2 ]
[ 185.5  2  2 ]
[ 192.1  2  2 ]
[ 191.2  2  2 ]
[ 189    2  2 ]
[ 185.5  2  3 ]
[ 192.1  2  3 ]
[ 241.9  2  3 ]
[ 238.6  2  3 ]
[ 244.9  2  3 ]
[ 236.2  2  3 ]
[ 167.3  2  3 ]
[ 165.9  2  3 ]
[ 162.4  2  3 ]
[ 172.5  2  3 ]
24, z=3
  
```

(25) **PRGM A1ANOVA**

(26) **ANOVA: USES (0) (1)**  
**1: ONE-WAY ANOVA**  
**2: RAN BLOCK DESI**  
**3: 2WAY FACTORIAL**  
**4: QUIT**

(27) **EQUAL REPLICATES**  
**DATA IN N\*3 MAT**  
**[D], 1ST COL-DATA**  
**2ND COL-A LEVELS**  
**3RD COL-B LEVELS**  
**LEVELS-INTEGERS**  
**STARTING WITH 1.**

**Activity 10, Analyzing Variance (ANOVA) (cont.)**

We see from the results that there is no significant interaction with  $F(AB) = 2.21$  and a p-value =  $0.139$ . This is also clear from the **xyLine** plots (screen 30) of the mean for each of the six ball-club combinations recorded in the table below.

The plots are obtained as follows.

1. Store **1, 2, 3** in **L1**, the three driver means in **L2**, and the three five-iron means in **L3**.
2. Set up **Plot1** (for **L1** and **L2**) and **Plot2** (for **L1** and **L3**) as **xyLine** plots (see in Topic 1).

There is a significant difference between the B factors (p-value =  $0.000$ ). It is clear that the driver drives the ball further on average than the five-iron, just as we would expect.

Because there is also a significant difference between the different balls (Factor A), Brand C seems least effective for distance. Multiple comparisons could also be done similar to those in Topic 50.

Factor B (Club)	Factor A (L1)		
	Brand A (1)	Brand B (2)	Brand C (3)
Driver (L2)	261.45	260.38	240.4
Five-iron (L3)	182.15	189.45	167.03

