Cylindrical Soda Can

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Abstract: This activity is an application of differentiation. Students use calculus to find the ideal dimensions of a common item - a soda can. They set up the problem and express the function in terms of one variable. They then use the symbolic capacity of their calculator and calculus to determine the dimensions of the can which has the least cost.

NCTM Principles and Standards:
Algebra standards
a) Understand patterns, relations, and functions
b) analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
c) use symbolic algebra to represent and explain mathematical relationships;
d) judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.
e) draw reasonable conclusions about a situation being modeled.
f) interpret representations of functions of two variables

Geometry standards:
a) Analyze characteristics and properties of two- and three-dimensional geometric shapes and mathematical about geometric relationships
b) draw and construct representations of two- three-dimensional geometric objects using a variety of tools;
c) visualize three-dimensional objects and spaces from different perspectives and analyze their cross sections;

Measurement standards:
a) Understand measurable attributes of objects and the units, systems, and of measurement
b) understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders;

Problem Solving Standard build new mathematical knowledge through problem solving; solve problems that arise in mathematics and in other contexts; apply and adapt a variety of appropriate strategies to solve problems; monitor and reflect on the process of mathematical problem solving.

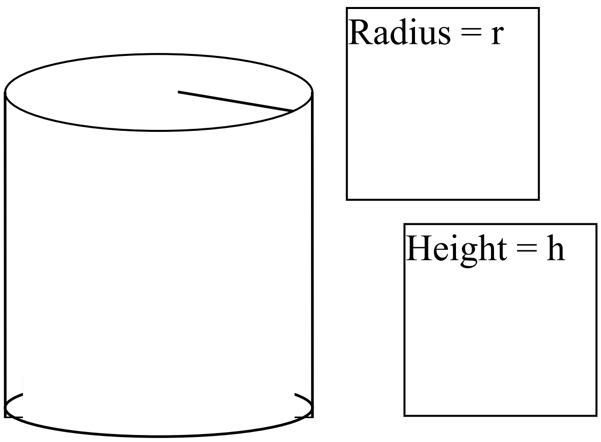
Reasoning and Proof Standard
a) recognize reasoning and proof as fundamental aspects of mathematics;
b) make and investigate mathematical conjectures;
c) develop and evaluate mathematical arguments and proofs;
d) select and use various types of reasoning and methods of proof.

Representation Standard : use representations to model and interpret physical, social, and phenomena.

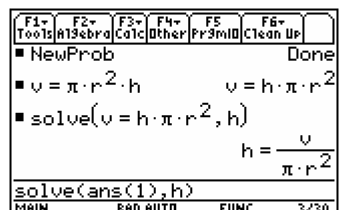
Key topic: Applications of Derivative - determining the minimum of a function

Degree of Difficulty: moderate
Needed Materials: TI-89 calculator

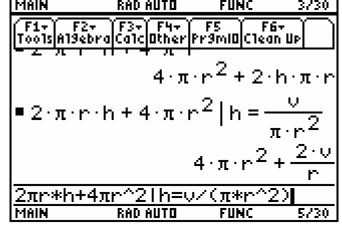
Situation: A cylindrical soda can containing 12 oz. of liquid is to be made for the least cost. The cost per square centimeter of the top is three times the cost for the sides and the bottom of the can. What should the dimensions of the can be?



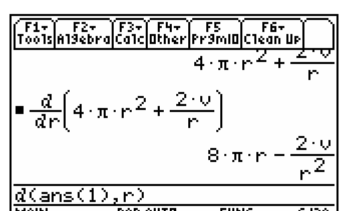
Find an expression for the volume in terms of r: If we let r be the radius and h be the height, the volume of the can is $V=\pi r^2h$. We can solve this expression for h in terms of v and r:



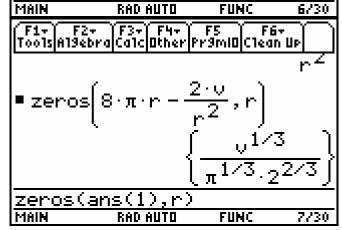
Find an expression for the cost of the can in terms of r: The cost of the can is going to be proportional to sum of the lateral surface area of the cylinder: $2\pi rh$ plus the area of the bottom of the can: πr^2 plus three times the area of the top: $3\pi r^2$ So we can let $2\pi rh+ \pi r^2 + 3\pi r^2 = 2\pi rh + 4\pi r^2$ represent the cost function. We substitute our expression for h to finish this step.



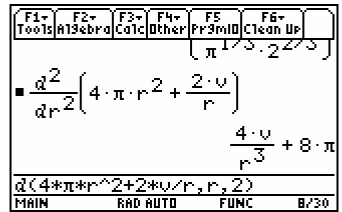
Find the first derivative of the cost function: Since v is a constant in our problem, its derivative with respect to r is zero.



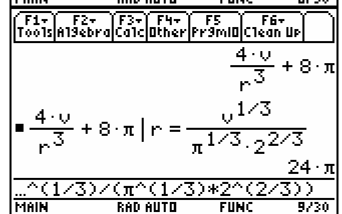
Find the zeros of the derivative of the cost function: This is a critical value for our function.



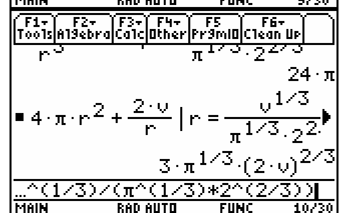
Find the second derivative of the cost function:



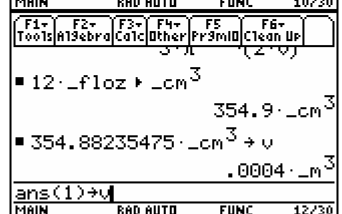
Find the value of the second derivative at the critical value: Since this value is positive, then we have a found a relative minimum.



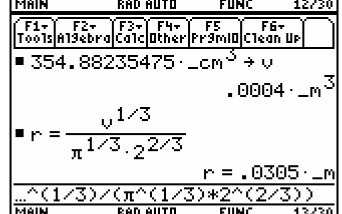
Find an expression for the volume at the critical value.



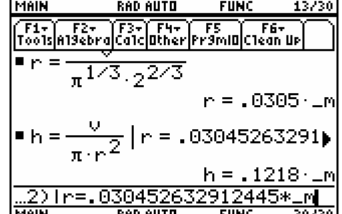
Use the unit menu to convert 12 fluid ounces to cm^3



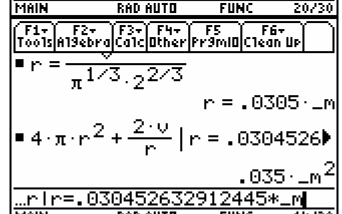
Find the radius of the can



Find the height of the can. What is the ratio of the height of the can to its diameter? How close is this to the ratio from an actual soda can? How would you account for any difference?



Finally, find the value of the cost function.



If we assume that the material for the can costs \$1.35/m^2, what is the cost of the can?