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Triangles: Law of sines and cosines
Heron's Formula

NCTM standard: Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

Analyze properties and determine attributes of two- and three-dimensional objects.
Introduction: This application shows how to derive the area of a triangle given the three sides.

1) Turn the calculator on.
2) Press APPS. Press 1:FLASHAPPS:Sketchpad, and then press ENTER.
3) Make a triangle. Press 2nd F3 for the Toolbox, and go to the point tool (second down). Press ENTER. Make three points by moving the cursor to three different non-collinear locations and pressing ENTER after each time. Press ESC, then highlight each point by pressing ENTER on them. Press F3 and then press C:Polygon interior. Press ESC.

4) Find the area. Press ENTER on the interior of the triangle. Press F5 6:Area. Record the value displayed.
5) Find the perimeter. Press ESC, then highlight the interior of the triangle. Press F5 3:Perimeter. Record the value, and then calculate half of the perimeter. Record that value as well.
6) Measure the length of each side, but first, hide the measurements. Press F2 1:Hide. Highlight the area measurement by pressing F2 1:Hide. Press ESC. Measure the left side of the triangle, call it side 1.
Highlight the left and top vertices, press F5 2:Distance. Record the value. Press ESC. Repeat for the right side of the triangle, and record this value as side 2 . Then repeat for the measurement process for the bottom of the triangle, and record the value as side 3 .
7) If $1 / 2$ perimeter is $s$, and the three sides are $a, b$, and $c$, calculate $s-a, s-b, s-c$, and record the value for each.
8) Take the three totals from \#7, multiply them together, then multiply that value by s. Compare that to the area of the triangle. Do you see any connection? If so, what?
9) Take the square root of the total in 8 . Compare that to the area. What do you observe?
10) You have just used Heron's formula to calculate the area of a triangle. If $A$ is used for area, then use A, s, a, b, and c to write Heron's formula based on the calculations above.

|  | Area | Perimeter | Side 1 <br> (a) | Side 2 <br> (b) | Side 3 <br> (c) | $(1 / 2)$ perimeter <br> (s) | s-a | s-b | s-c |
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