FTC Changed History

ID: 12148

Time Required
15–20 minutes

Activity Overview

In this activity, students will graphically investigate the Fundamental Theorem of Calculus and properties of definite integrals. Self-check questions engage students and deepen understanding of the topic. Multiple choice exam-like questions are also included.

Topic: Fundamental Theorem of Calculus

- *Graphical explanation of the Fundamental Theorem of Calculus*
- *Properties of Definite Integrals*

Teacher Preparation and Notes

- *Teachers may want to use the Sum Rectangles activity (12100) to help students see that the limit as n approaches infinity is* $\int_a^b f(x)dx = F(b) - F(a)$.
- *Students will write their responses directly into the TI-Nspire handheld and/or on the accompanying handout. On self-check questions, after answering the question students can press* (menu) *and select* ***Check Answer*** *(or* (ctrl) *+▲).*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "12148" in the quick search box.***

Associated Materials

- *FTCChangedHistory_Student.doc*
- *FTCChangedHistory.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

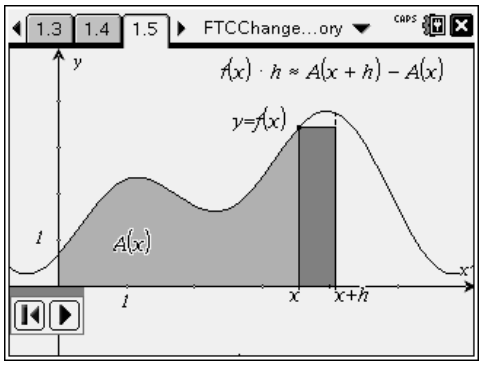
- *Sum Rectangles (TI-Nspire CAS technology) — 12100*
- *Introduction to the Fundamental Theorem (TI-Nspire CAS technology) — 9779*
- *Fundamental Theorem Follow Up (TI-Nspire CAS technology) — 9995*
- *Exploring the Fundamental Theorem of Calculus (TI-Nspire CAS technology) — 9205*

Part 1 – Fundamental Theorem of Calculus

The Fundamental Theorem of Calculus (FTC) is important in that it links differential and integral calculus. Students are motivated in this study by considering applications of the FTC.

Feel free to expand on the list of majors on page 1.3 and encourage science, technology, engineering, and mathematics by reading a more expanded list: economics, systems engineering, nuclear engineering, mechanical engineering, electrical engineering, industrial engineering, civil engineering, chemical engineering, biomedical engineering, aerospace engineering, medicine, biological science, chemistry, physics, mathematics, and more.

The animation on page 1.5 and the explanation on pages 1.4 and 1.6 provide a graphical discovery of the relationship between the area and the derivative. Help students make the connection between $A(x)$, the area function, and what is generally referred to as $F(x)$ in the FTC. If $x + h = b$ and $x = a$, then the conclusion of FTC follows for small h.



Use the Riemann sums to show the limit as n approaches infinity = $\int_a^b f(x)dx = F(b) - F(a)$.

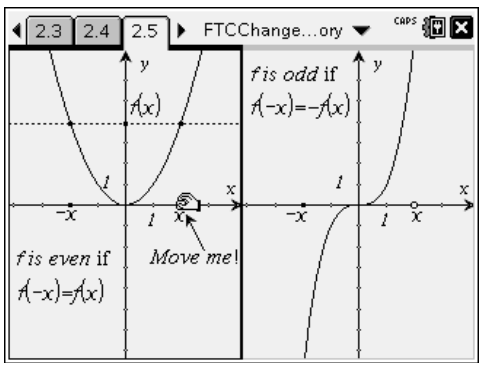
Student Solutions

1. The FTC is of fundamental importance because it links the two branches of calculus: differentiation and integration. The application of FTC changed civilization.
2. The derivative of the **area** function. (Note: This is equal to the graph $f(x)$.)
3. $$\int_1^2 \left(x^2 + 2\right)dx = \left(\frac{1}{3}\right)(2^3) + 2(2) - \left(\frac{1}{3}\right)(1^3) - 2(1)$$
$$= \frac{8}{3} + 4 - \left(\frac{1}{3} + 2\right)$$
$$= \frac{7}{3} + 2$$
$$= \frac{13}{3} \approx 4.333$$

Part 2 – Properties of Definite Integrals

This section enables students to dynamically explore properties of definite integrals.

An alternative for page 2.5, the odd and even function definition, would be to delete the definition and have the students discover the definition. Ask them: "How do $f(x)$ and $f(–x)$ compare for the even function **f2**$(x) = x^2$? How do they compare for the odd function **f3**$(x) = x^3$?"



Student Solutions

4. $\int_a^b f(x)dx > 0$ *if* $f(x) > 0$, *and* $\int_a^b f(x)dx < 0$ *if* $f(x) < 0$

5. Odd functions include $f(x) = x$, x^3, $\sin(x)$, $\tan(x)$, $\sec(x)\tan(x)$
Even functions include $f(x) = 1$, $\cos(x)$, x^2, $\sin^2(x)$, $\sec(x)$. ($x^{1/2}$ and $x^{3.2}$ are example of neither)
For odd, $\int_{-a}^{a} f(x)dx = 0$. For even, $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$

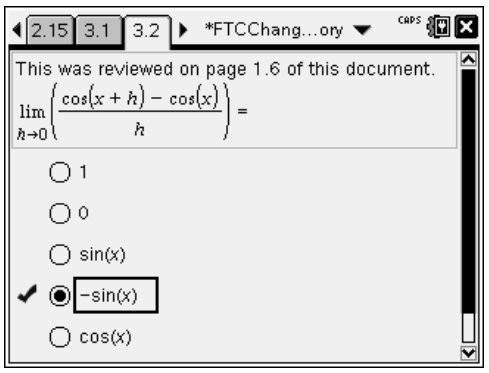
6. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

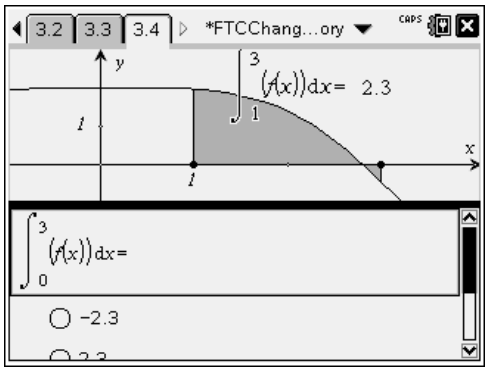
7. The sum doesn't change no matter what the value of c is changed to.

8. Use integral of sum property to distribute the integral over addition. Integral over symmetry limits of integration for the odd functions is zero. This leaves only $2\int_0^2 5dx = 2(10) = 20$.

Extension/Homework – Exam-type questions

Students will practice the FTC by solving the multiple-choice problems.





Student Solutions

9. $\frac{d}{dx}(\cos(x)) = -\sin(x)$

10. $\int_1^3 (f(x)+2)\,dx = \int_1^3 f(x)dx + \int_1^3 2\,dx$
$= 5 + 2x\,|_1^3 = 5 + 2(3) - 2(1) = 9$

11. $\int_0^3 f(x)dx = \int_0^1 f(x)dx + \int_1^3 f(x)dx$
$= 1(2) + 2.3 = 4.3$
The area of from 0 to 1 has a base of 1 and height of 2.