Math Objectives

- Given two functions, students will be able to evaluate the composition of the two functions numerically.
- Given two functions, students will be able to write the symbolic representation of the composition.
- Students will find the composition of two functions using proper notation.
- Students will construct viable arguments and critique the reasoning of others. (CCSS Mathematical Practice)
- Students will look for and make use of structure. (CCSS Mathematical Practice)

Vocabulary

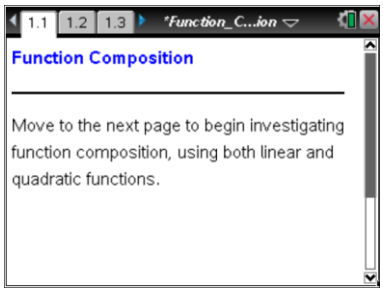
- composition
- functions

About the Lesson

- This lesson involves students exploring the composition of a linear and a quadratic function.
- As a result, given two functions, students will:
  - Write and evaluate the symbolic representation of a composition of two functions.
  - Find the composition of the two functions.

TI-Nspire™ Navigator™ System

- Transfer a document.
- Use **Live Presenter** to demonstrate.
- Use **Screen Capture** to monitor students' progress.
- Use **Quick Poll** to assess students' understanding.
- Use Teacher Software to review student documents.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire™ document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire™ handheld is set to Medium.
- You can hide the entry line by pressing ctrl G.

Lesson Materials:

Student Activity

Function_Composition_Student.doc

Function_Composition_Student.pdf

TI-Nspire document

Function_Composition.tns

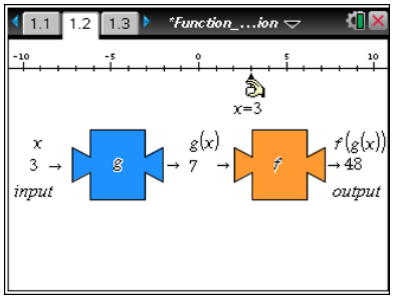
Visit www.mathnspired.com for lesson updates and tech tip videos. (optional)

Discussion Points and Possible Answers

Tech Tip: All the pages after the title screen are designed to easily allow students to change the value of *x* by moving the arrow along the number line. Instruct students to move the cursor to the open point at the bottom of the arrow until they get the open hand (✋) and press ctrl ✋ or press the ✋ to close the hand. The point should then slowly blink. Then the point can be moved by pressing the directional arrows of the touchpad.

TI-Nspire™ Navigator™ Opportunity: File Transfer
See Note 1 at the end of this lesson.

Move to page 1.2.



1. Grab and move the point to change the value of *x*. Record your observations in the tables provided below.

x	**g**(*x*)		**g**(*x*)	**f**(**g**(*x*))
0	4		4	15
1	5		5	24
2	6		6	35
3	7		7	48

Answer: The table above includes possible answers students might collect.

2. Identify the patterns in the tables.

   a. What is a possible formula for **g**(*x*)?

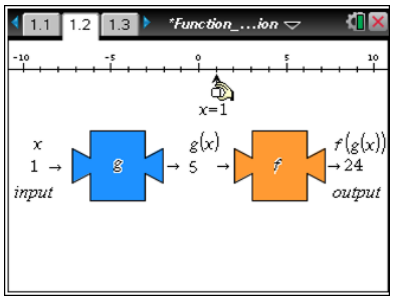
   Answer: **g**(*x*) = *x* + 4

   b. What is a possible formula for **f**(*x*)?

   Answer: **f**(*x*) = $x^2 - 1$

TI-Nspire™ Navigator™ Opportunity: Screen Capture or Live Presenter
See Note 2 at the end of this lesson.

Teacher Tip: How can you determine whether the pattern is linear? If the differences in the *y*-values are constant, it must be a linear pattern. Ask students to explain how they could find the *g* function. Can they explain the process they used to find *f* and *g*? Differences can be used to find *f*, but second differences may be needed to find *g*.

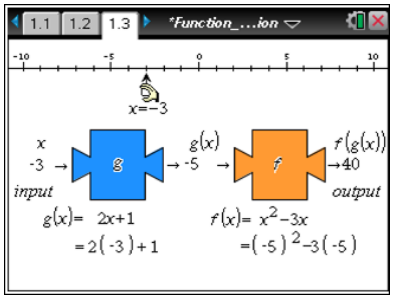


3. A function machine can be thought of as a substitution machine. The function $(f \circ g)(x)$ (also notated as **f**(**g**(*x*)) and read as "*f* composed with *g* of *x*") is shown as a double substitution machine. First, *x* is substituted into the *g* function. What happens to the result of this substitution?

Answer: The result, **g**(*x*), is substituted into the *f* function.

Teacher Tip: Page 1.2 does not use the function notation $(f \circ g)(x)$ because this page is designed to emphasize the double substitution concept. More formal notation will be introduced on the next page.

Move to page 1.3.

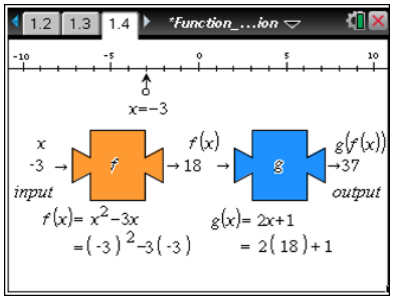


4. On page 1.3, there are new functions for *f* and *g*. If you grab and move the open point, the handheld will allow only integer values from –9 to 9 to be substituted into the function composition. What is the value of **f**(**g**(–10))?

Answer: **f**(**g**(–10)) = **f**(–19) = 418

Move to page 1.4.

5. This page shows the *g* function composed with the *f* function, notated as **g**(**f**(*x*). Which function is *x* substituted into?



Answer: First, *x* is substituted into the *f* function. The result, **f**(*x*), is then substituted into the *g* function.

6. Given **f**(*x*) = x^2 – 3*x* and **g**(*x*) = 2*x* + 1:

   a. What is the value of **g**(**f**(–2))?

   Answer: **g**(**f**(–2)) = **g**(10) = 21

   b. What is the value of **f**(**g**(–2))?

   Answer: **f**(**g**(–2)) = **f**(–3) = 18

> **Teacher Tip:** Students may confuse function composition with multiplication. Because the notation looks similar, emphasize this distinction to students.

7. Function compositions are not just represented as values in a table. When two functions are composed, the resulting function can be written in terms of *x*. This can save time if there are many values to substitute into the function composition.

   a. The Commutative Property of Multiplication says that $a \cdot b = b \cdot a$. Does **g**(**f**(*x*)) = **f**(**g**(*x*))? Why or why not?

   Answer: Function composition is not commutative, so they are not equal.

   b. What is the resulting formula for **g**(**f**(*x*))? Use the resulting formula to find **g**(**f**(–2)). Does it match your answer from question 6a?

   Answer: **g**(**f**(*x*)) = **g**(x^2 – 3*x*) = 2x^2 – 6*x* + 1
   g(**f**(–2)) = 2(–2)^2 – 6(–2) + 1 = 21; Yes, it matches.

c. What is the resulting formula for **f**(**g**(*x*))? Use the resulting formula to find **f**(**g**(–2)). Does it match your answer from question 6b?

Answer: **f**(**g**(*x*)) = **f**(2*x* + 1) = $4x^2 - 2x - 2$
f(**g**(–2)) = $4(-2)^2 - 2(-2) - 2 = 18$; Yes, it matches.

TI-Nspire™ Navigator™ Opportunity: Quick Poll
See Note 3 at the end of this lesson.

Teacher Tip: Be careful on question 7b. Many students will forget to use the Distributive Property properly when simplifying $(2x + 1)^2 - 3x$.

Teacher Tip: Extend learning on pages 1.3 and 1.4 of the file. The *f* and *g* functions can be changed. For **g**(*x*) = *mx* + *b*, *m* and *b* can be changed to integers from –9 to 9. For **f**(*x*) = $x^2 + a$, *a* can be changed to an integer from –9 to 9. This allows the students to work more example problems if needed.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- How to write and evaluate the symbolic representation of a composition of two functions.
- How to write the resulting formula for the composition of two functions.

TI-Nspire™ Navigator™

Note 1

Before the activity: Use *File Transfer* to efficiently send the TI-Nspire™ document to the students. Using TI-Nspire™ Navigator™ will allow students to receive the document without having to leave their seats or use extra cables.

Note 2

Entire Document: If students experience difficulty with operation of a file or a question, use **Screen Capture** or **Live Presenter** with TI-Nspire™ Navigator™. You can also use these features to facilitate student discussion.

Note 3

Questions 7b and 7c, *Quick Poll:* A *Quick Poll* can be given at the conclusion of the lesson. You can either save the results or show the results after each question.

The following are some sample questions you can use:

1. Given that $\mathbf{f}(x) = (x + 1)^2$ and $\mathbf{g}(x) = -x + 3$, find the value of $\mathbf{f}(\mathbf{g}(-4))$.
   a. –6
   b. 0
   c. 12
   d. 64

2. Given that $\mathbf{f}(x) = (x + 1)^2$ and $\mathbf{g}(x) = -x + 3$, find the value of $\mathbf{g}(\mathbf{f}(-4))$.
   a. –6
   b. 0
   c. 12
   d. 64

3. Given that $\mathbf{f}(x) = (x + 1)^2$ and $\mathbf{g}(x) = -x + 3$, find the value of $\mathbf{f}(\mathbf{g}(x))$.
   a. $-x^2 - 2x + 2$
   b. $x^2 - 8x + 16$
   c. $x^2 + 2x - 2$
   d. $x^2 + 8x - 16$