Linear Approximation

ID: 9470

Time required
45 minutes

Activity Overview

In this activity, students will graph the functions, construct the tangent line at a point, find an estimate from a linear approximation. They will then determine whether the estimate is an overestimate or an underestimate, and find an interval for a desired accuracy.

Topic: Application of Derivatives

- *Calculate the equation of the tangent line to a graph at any given point.*
- *Construct a tangent line to the graph of a differentiable function at x = a to approximate its value near x = a.*
- *Construct a tangent line to the graph of a differentiable function at x = 0 to approximate its value near x = 0.*

Teacher Preparation and Notes

- *This investigation offers an opportunity for students to develop an understanding of how a tangent line to a curve can be used to approximate the values of a function near the point of tangency. Linear approximations are often used in scientific applications, including formulas used in physics where* $\sin\theta$ *is replaced with its linear approximation,* θ.
- *Before starting this activity, students should go to the home screen and select* ***F6:Clean Up > 2:NewProb****, then press* [ENTER]*. This will clear any stored variables, turn off any functions and plots, and clear the drawing and home screens.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "9470" in the keyword search box.***

Associated Materials

- *LinearApproximation_Student.doc*

Suggested Related Activities

To download any activity listed, go to <u>*education.ti.com/exchange*</u> *and enter the number in the keyword search box.*

- *Slope of a Tangent Line (TI-Nspire technology) — 9018*
- *Secant and Tangent Lines (TI-89 Titanium) — 11140*

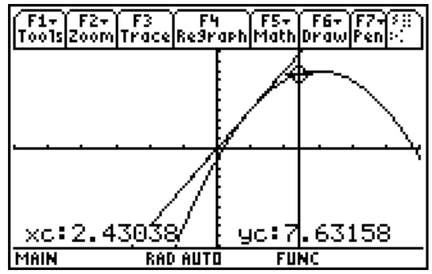
Introduction

From top to bottom, the points where the horizontal lines cross the *y*-axis are *L*(*x*), *f*(*x*), *f*(*a*).

L(*x*) will be the linear approximation.

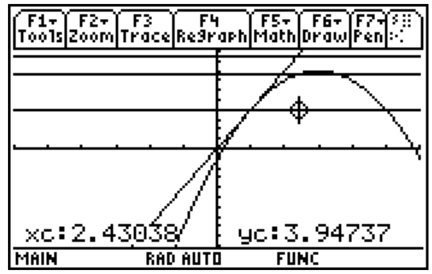
L(*x*)–*f*(*x*) = error of this estimate.

Since *L*(*x*) > *f*(*x*), this estimate is an overestimate.



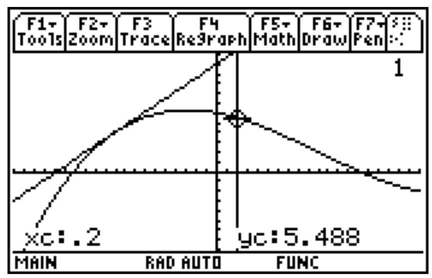
To get the horizontal line, students can use **graph > F7:Pencil > 5:Horizontal**

To get the vertical line, they can use **graph > F7:Pencil > 6:Vertical.**



Investigating linear approximation

Students are to graph the function of $f1(x) = x^3 - 3x^2 - 2x + 6$ and the tangent at *a* = –1. The point p is where the vertical line crosses the function. The point q is where the vertical line crosses the tangent line.



For the x-value 0.2:

L(0.2) = 12.4 is the linear approximation.

The error is the length *pq* = 6.912 (12.4–5.488).

The true value, *f*(0.2) = 5.488.

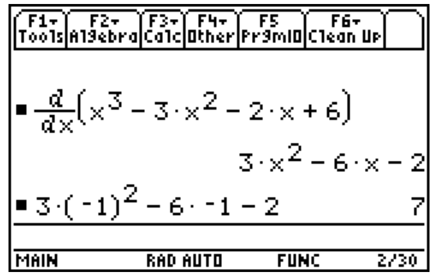
It is an overestimation.

	linear approx. of f1(q)	**real value of f1(q)**	**error**	**underestimation/ overestimation**
x = –0.2	9.6	6.272	3.328	overestimation
x = –0.5	7.5	6.125	1.375	overestimation
x = –0.6	6.8	5.904	0.896	overestimation
x = –1.2	2.6	2.352	0.248	overestimation

As you get close to the point of tangency, the graph of the function and the graph of the tangent line appear to be the same.

They are called local linearization because the graph acts like a straight line at the point of tangency and that line is the tangent line.

Students are to find the derivative of f1(x) = x^3 - 3x^2 - 2x + 6 and evaluate it at x = –1. They should use the slope and the point (–1, 4) to get the equation of the line.



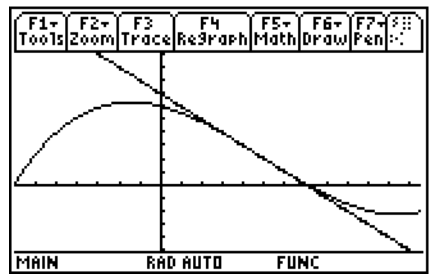
$$y - 4 = 7(x + 1) \rightarrow y = 7x + 11$$
$$L(x) = 7x + 11$$

L(–1.03) = 3.79. This is the linear approximation

Error = 3.79 – 3.78457=0.005427

Underestimates versus overestimates

Students are to graph f1(x) = x^3 - 3x^2 - 2x + 6 and place a tangent line a = 1.



If p is to the left of a = 1, then the tangent line will be above the function and the linear approximation is an overestimate.

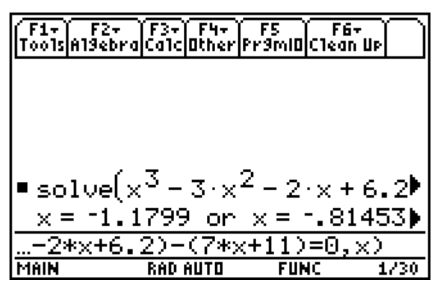
If p is to the right of a = 1, then the tangent line will be below the function and the linear approximation is an underestimate.

The point a = 1 is a point of inflection. Students can look at the second derivative and set it equal to zero to confirm this.

Students should see that in general the linear approximation will overestimate if the curve is concave down and it will underestimate when the curve is concave up.

Finding intervals of accuracy

Students are posed with the question: *How close to –1 must x be for the linear approximation to be within 0.2 units of the true value of* ***f1(x)****?*

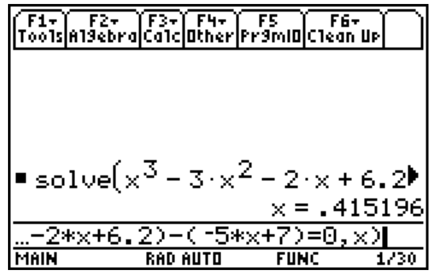


When students look at the graph, they want the tangent line to be within the bound of **f1(*x*) + 0.2** and **f1(*x*) – 0.2.**

Since the linear approximation overestimates in this region, they want to compare L(x) and **f1(*x*) + 0.2.**

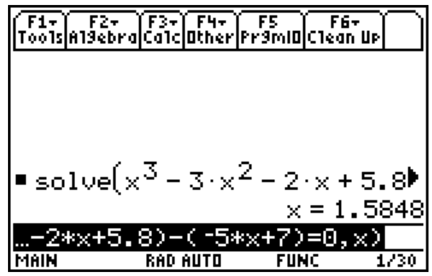
Students can zoom in graphically or solve algebraically. The interval is (–1.1799, –0.81453) using the **solve** command.

Since the tangent line overestimates to the left of *x* = 1 and underestimates to the right of *x* = 1 students will have to do this in two parts.



Since the linear approximation overestimates in the region to the left of *x* = 1, students need to compare *L*(*x*) and **f1(x) + 0.2** there.

They can zoom in graphically or solve algebraically. The interval is (0.415196, 1) using the **solve** command.



Since the linear approximation underestimates in the region to the right of x = 1, students want to compare *L*(*x*) and **f1(x) – 0.2** there.

They can zoom in graphically or solve algebraically. The interval is (1, 1.5848) using the **solve** command.