Kicking for Goals

Student Activity

7 8 9 10 **11** **12**

TI-Nspire™ Investigation Student 75 min

Introduction

When it comes to kicking for goals, most players would rather be 'straight' in front of the goals and as close as possible. Opposition players try to defend the *region* around the goals. Where is this region? The region can be considered as a balance between distance and *shooting angle*. As a player runs towards the goals he or she may choose to pass the ball to a team member off to one side if they are closer to the goals. There is a compromise between proximity and shooting angle. This investigation focuses on optimising the shooting angle.

In extreme cases such as a corner penalty in soccer, or a mark near the behind post in Australian Rules Football (AFL), the shooting angle is 0°; commentators often refer to this as: "the player cannot see daylight between the posts". This investigation starts with the mathematically simpler soccer scenario where a player is not directly in front of the goals, no consideration is given to the variation in accuracy due to range. Part two of the investigation considers the more complicated scenario where a player moves along the curved boundary line to improve the shooting angle, a tactic used to an extreme by one AFL player!

Warm Up

Every athlete knows the importance of warming up prior to activity. The following '*exercises'* constitute an essential warm up before commencing this activity.

Question: 1.

a) Show that: $1+\tan^2(x)=\sec^2(x)$

b) Given $x=\tan(y)$ show that: $\dfrac{d\tan^{-1}(x)}{dx}=\dfrac{1}{1+x^2}$

c) Given $\dfrac{d\ Tan^{-1}(x)}{dx}=\dfrac{1}{1+x^2}$ use the chain rule to show that $\dfrac{d\ Tan^{-1}\left(\frac{a}{x}\right)}{dx}=\dfrac{-a}{a^2+x^2}$

d) Graph the function: $y=Tan^{-1}\dfrac{10}{x}-Tan^{-1}\dfrac{5}{x}$ with *y* measured in degrees. Identify key features of the graph including any intercept(s), turning point(s), points of inflection and asymptotes.

Shooting for Goal - Soccer

Sam is a mathematically inspired soccer coach, she is trying to teach the team the finer points of shooting for goal. The highest percentage shots are those with the maximum *shooting angle*. The *shooting angle* has the player at the vertex and the opening defined by the goal posts.

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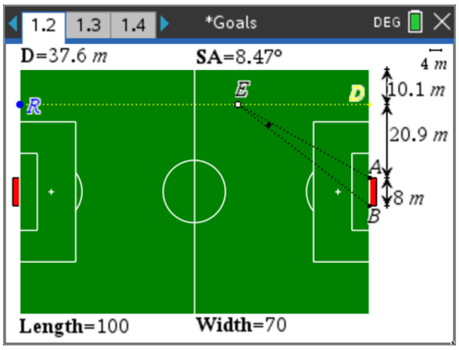


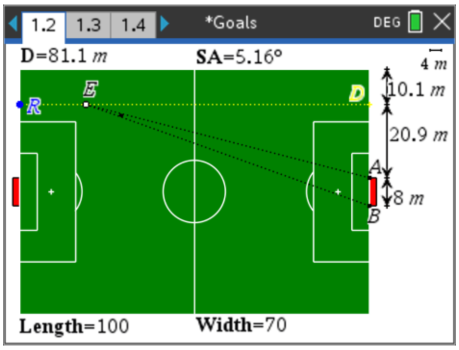
Open the TI-Nspire file: Goals

Navigate to page 1.2. This page contains an interactive layout of a soccer field. For the following questions, the length of the field is 100m, the width is 70m and the goals 8m wide.

Ellie is positioned at point E. She is planning on shooting for goals somewhere along the line RD which is 10m from the boundary. The goal posts are at A and B, so the shooting angle is ∠AEB.

Question: 2.

Let distance ED = x, determine an expression for the shooting angle ∠AEB in terms of x.



Move Ellie (point E) along the line RD. As Ellie moves parallel to the boundary watch the range of shooting angles produced. (SA)

The distance (D) and shooting angle (SA) are automatically captured. Navigate to page 1.3 to see the data.

Navigate to page 1.4 to see a graph of the data.

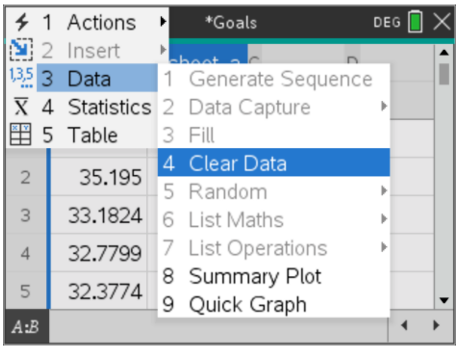
Check your answer to Question 2 by graphing your equation.

Question: 3.

Sam instructs Ellie to 'take a shot' as long as the shooting angle is greater than 9°, determine the range of positions along RD where Ellie can take her shot.

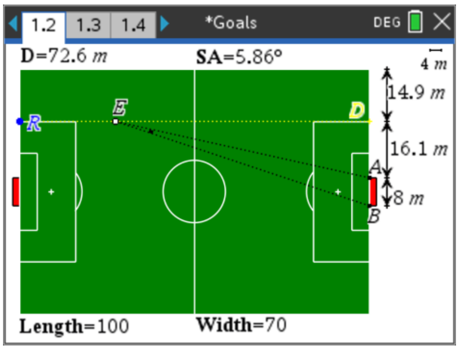
Question: 4.

Use calculus to determine the maximum shooting angle and corresponding location.



The line RD is moved to a distance 15m from the boundary. Grab point R and move it so the line is approximately 15m from the boundary.

The original data must be cleared. Navigate to Page 1.3 and select the two columns, then use the menu to '**clear**' the data.



Drag E along RD (≈15m from boundary), determine a new equation and check your equation against the data. You will need to adjust the window settings on Page 1.4 to see the entire graph.

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Question: 5.

Use calculus to determine the new maximum shooting angle and corresponding location.

Question: 6.

Repeat this process with the line RD located 20m and then 25m from the boundary.

Question: 7.

Determine the general location for Ellie's best shooting angle based on her distance from the boundary. Example: If Ellie is 28m from the boundary, the rule should return the distance x (ED) for the optimum shooting angle, but not the angle itself.

AFL Kicking Angle

In an AFL match between Collingwood and Essendon in the early 1990s, Paul Salmon (Essendon) marked the ball near the point post. As per rule 20.5.1 (below), the umpire lined up the centre of the goals with the point post where the mark was taken. Paul walked back along this line with his opponent standing 'on the mark'. Paul couldn't walk back very far before bumping into the boundary fence, so he proceeded to walk around the boundary, distancing himself from the player on the mark in preparation for his natural run up. Paul just keep walking, almost all the way to the 50m arc. Paul managed to improve his shooting angle. One of Paul's attributes was that he was an excellent long and straight kick of the ball. The question is: "*Did Paul shoot from the optimum location*?"

20.5.1 Line of The Mark

Where a Player from the Attacking Team is Kicking for a Goal after being awarded a Mark or a Free Kick, the Kick shall be taken along a direct line from The Mark to the centre of the Attacking Team's Goal Line, except in the following cases:

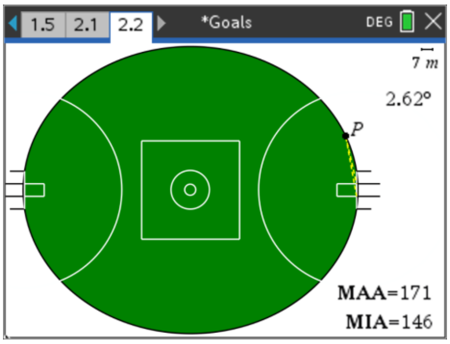
(a) Where the Mark or Free Kick is awarded within or on a line of the Goal Square, the Kick shall be taken from directly in front of the Goal Line from a spot horizontally across from where the Mark or Free Kick was awarded;

(b) Where the Kick will occur after the siren, the Player shall be entitled to approach The Mark from any direction, as long as the location of the Kick does not improve the angle to the goal posts.

MCG – Kicking Angle

Each AFL ground is slightly different. The MCG 'oval' is almost perfectly elliptical with a major axis 171m and a minor axis 146m. A scaled version of the oval is located on Page 2.2. Paul's position is marked by point P. You can move point P along the boundary line and watch the angle change.

You can change the scale in the top right corner to 5 to get a closer view of Essendon's goals and Paul's position.



Point P has been placed on an ellipse. If you move point P inside the goal line point P will end up 'behind' the goals which of course is not permitted in the game and geometrically results in much larger angles!

Question: 8.

Let the centre of the ground represent the origin (Cartesian Plane). Determine the equation for the 'oval', given it is well approximated by an ellipse.

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Question: 9.

The goal posts are 6.4m apart, so too the point posts to the goal posts. Given the point posts (two outer posts) are on the boundary line (ellipse), determine the coordinates of each post.

Question: 10.

Determine an expression for the goal kicking angle for any point along the playable boundary.
Note: The ground is symmetrical so the expression only needs to work along one side of the ground.

Question: 11.

Determine the location for the optimum shooting angle.

Question: 12.

Given Paul kicked the ball from approximately 40m away from the goal line, determine if he managed to find the best location (within the rules) to take his kick. (Also assume that Paul is a very long and accurate kick of the ball).

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