



## Activity Overview

In this activity, students first compare the  $t$  distribution with a mean of 0 for  $n = 4$  to the standard normal distribution. They increase the value of  $n$  to larger values until the  $t$  distribution is nearly the same as the normal distribution. Students then use the **invT** command to find critical values for a  $t$  distribution and again note that as  $n$  increases, the critical values become closer to those used for a normal distribution. They use the critical values and margins of error to construct confidence intervals, noting that the margin of error becomes smaller as  $n$  increases. Finally, they use the **Tinterval** command to find the confidence intervals.

## Topic: Continuous Distributions and their Properties

- Graph the probability density function on the  $t$  distribution of  $n$  degrees of freedom
- Determine the probability that it takes a value in a given interval.
- Calculate a confidence interval for the  $t$  distribution of  $n$  degrees of freedom.

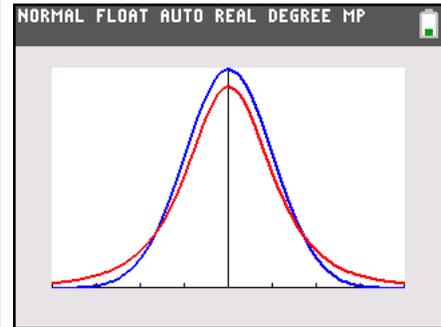
## Teacher Preparation and Notes

- This activity is designed to be used for students studying Statistics and Probability. Students should already be familiar with the normal distribution and its characteristics, as well as finding and interpreting confidence intervals for normal distributions.
- **To download the student worksheet, go to [education.ti.com/exchange](http://education.ti.com/exchange) and enter “9617” in the keyword search box.**

## Suggested Related Activities

To download any activity listed, go to [education.ti.com/exchange](http://education.ti.com/exchange) and enter the number in the keyword search box.

- Special Cases of Normally Distributed Data (TI-89 Titanium) — 4363
- One and Two Sample  $t$ -Distributions (TI-Navigator) — 1967
- F Distribution (TI-84 Plus family) — 9780
- Chi-Square Distributions (TI-84 Plus family) — 9737



This activity utilizes MathPrint™ functionality and includes screen captures taken from the TI-84 Plus C Silver Edition. It is also appropriate for use with the TI-83 Plus, TI-84 Plus, and TI-84 Plus Silver Edition but slight variances may be found within the directions.

### Compatible Devices:

- TI-84 Plus Family
- TI-84 Plus C Silver Edition

### Lesson Files:

- tDistributions\_Student.pdf
- tDistributions\_Student.doc

Click [HERE](#) for Graphing Calculator Tutorials.



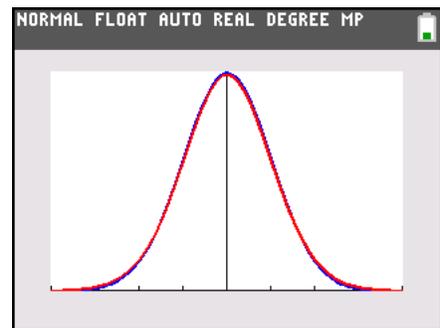
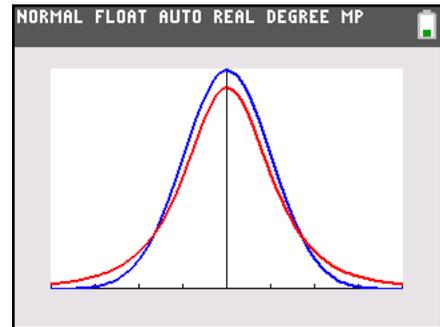
### Problem 1 – Characteristics of the t Distribution

Discuss with students that the  $t$  distribution is used when  $n$  is small (less than 30) and the population standard deviation is unknown. For a sample size of  $n$ , the number of degrees of freedom is  $n - 1$ .

Students will first compare the graph the standard normal distribution (thin line) and the  $t$  distribution graph with d.f. of 3 (thick line). They should notice that both have the same overall shape but have small differences.

Students will then increase the d.f. in **Y2** and view the graph after each change. They should see that the graph more closely resembles the normal distribution with each increase in  $n$ .

Following Question 2, explain that when the sample size is greater than or equal to 30, the normal distribution can be used instead of the  $t$  distribution.  $t$  distributions have more area, and therefore, more probability in the tails, which will affect confidence intervals as will be seen later in the activity.

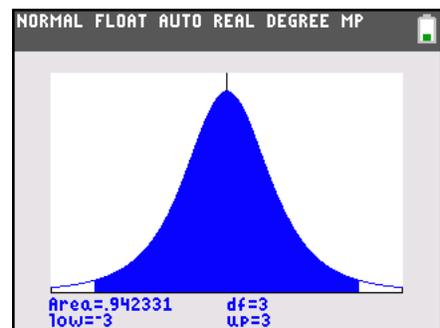
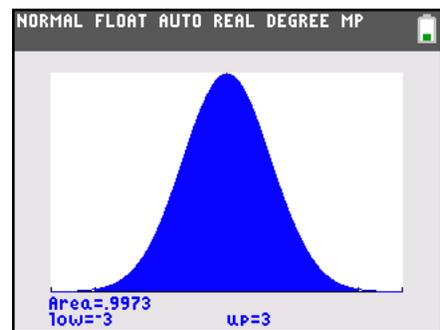


### Problem 2 – Comparing Areas

After clearing all of the entries from the  $\boxed{Y=}$  screen, students will use the **ShadeNorm** command to find the value of the area under the normal curve within three standard deviations of the mean. Students should understand that the total area under the curve is 1, so the area they are finding should be less than but close to 1.

When students have cleared the drawing, they can use the **Shade\_t** command to draw the  $t$  distribution and find the area within three standard deviations of the mean for d.f. of 3.

Then they should change the degrees of freedom to 8, 15, and 25 to see the areas approach that for the normal distribution.





### Problem 3 – Critical Values for a t Distribution

Students will now find the five critical values listed on the worksheet, each at the 95% level. They must remember that for a 95% confidence interval, 5% is left in *both* tails, so 2.5% is left in *each* tail. For the positive critical value (2.5% to the right), they are looking for the value where 97.5% of the area is to the left.

Students will use the **invT** command to find a critical value for a *t* distribution and use the **invNorm** command to find the critical value for the normal distribution. Remind them that d.f. is  $n - 1$ .

Students can also check their answers against a *t* distribution chart. Students should notice that as  $n$  increases, the critical values approach the *z* critical value of about 1.96.

Then students should predict how the critical values in Question 9 will compare and give their reasoning for their predictions.

Students should be able to make statements such as, *For  $n = 4$ , the critical values for an 80% confidence interval will be greater than for a 50% confidence interval because the distance (number of standard deviations) away from the mean increases as the confidence increases. For each percent, the critical value will be greater for  $n = 4$  than for  $n = 28$ , because smaller samples have more possible error.*

After discussing what should happen generally, Students will find the actual values on the Home screen.

HISTORY	
invT(.975,2)	4.302652729
invT(.975,7)	2.364624235
invT(.975,14)	2.144786681
invT(.975,24)	2.063898542
invNorm(.975,0,1)	1.959963986

NORMAL FLOAT AUTO REAL DEGREE MP	
invT(.75,3)	.7648923284
invT(.9,3)	1.637744314
invT(.995,3)	5.84090931

NORMAL FLOAT AUTO REAL DEGREE MP	
invT(.75,27)	.6836849788
invT(.9,27)	1.313702913
invT(.995,27)	2.770682946

### Problem 4 – Constructing a Confidence Interval

When introducing the formula for the margin of error in Problem 4, have students note the similarities between it and the formula used for a normal distribution.

For a *t* distribution, the margin of error for estimating the

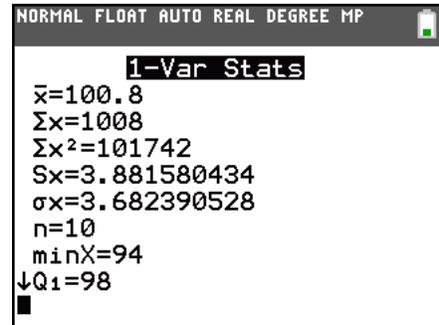
population mean is given by  $E = t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$ .

Students are to enter the weights given on the worksheet into list L1.

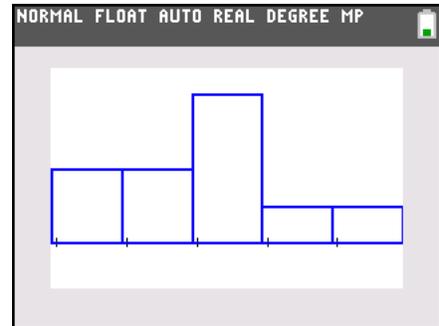
NORMAL FLOAT AUTO REAL DEGREE MP					
L1	L2	L3	L4	L5	1
98	-----	-----	-----	-----	
107					
101					
102					
94					
103					
105					
97					
99					
102					
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L1(11)=					



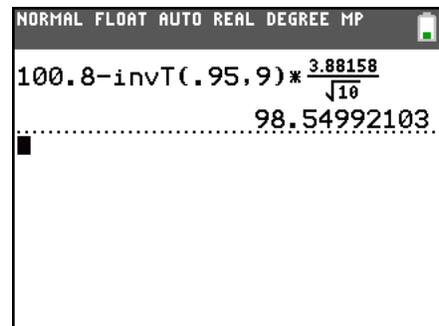
They will then use the **1-Var Stats** command to find the mean and standard deviation of the weights.



To verify that the distribution is roughly normal, students can create a histogram of the weights. Instructions are given on the worksheet for students to set up **Plot1** and graph the histogram.



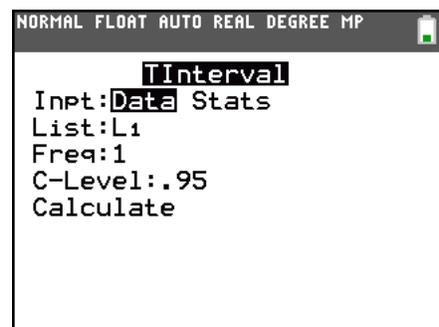
Students will construct 90% and 95% confidence intervals for the population mean by first finding the critical value, then the margin of error, and then subtracting and adding the error from and to the mean.



Students should use the confidence intervals in sentences, such as, *Based on these 10 data values, we are 95% confident that the true mean weight is between 98.0233 grams and 103.577 grams.*

They will repeat this process after adding the 10 more weights from the worksheet.

The calculations of the confidence intervals can be checked using the **TInterval** command. To move through each category, students need to press **ENTER**. Due to rounding in the students' calculations, this command may produce slightly different answers.





## Solutions

1. Both are bell-shaped with a mean of 0, but the  $t$  distribution has more area in the tails. The standard deviation in the  $t$  distribution must be greater than one.
2. As  $n$  increases, the distribution more closely follows the normal distribution. Smaller samples have more variability than larger samples.
3. 0.9973
4. 0.942331
5. 8 d.f.: 0.982928, 15 d.f.: 0.991027, 25 d.f.: 0.993962
6. The area between these points is greater for the standard normal distribution than for the  $t$  distributions, but as  $n$  increases, the areas under the  $t$  distributions also increase, approaching the area under the normal distribution. This is because the  $t$  distribution approaches the normal distribution as  $n$  increases.
7. 2.365, 2.145, 2.064, 1.96
8. As  $n$  increases, the critical values approach the  $z$  critical value of about 1.96.
9. Answers will vary. Students should be able to make statements such as, *For  $n = 4$ , the critical values for an 80% confidence interval will be greater than for a 50% confidence interval because the distance (number of standard deviations) away from the mean increases as the confidence increases. For each percent, the critical value will be greater for  $n = 4$  than for  $n = 28$ , because smaller samples have more possible error.*
10. From left to right, top to bottom: 0.765, 0.684, 1.638, 1.314, 5.841, 2.771
11. mean = 100.8, standard deviation  $\approx$  3.88158
12. 90%: 1.83311, 2.25008, (98.5499, 103.05)  
95%: 2.26216, 2.77672, (98.0233, 103.577)
13. mean = 100.05, standard deviation  $\approx$  4.87178
14. 90%: 1.72913, 1.88365, (98.1663, 101.934)  
95%: 2.09302, 2.28006, (97.7699, 102.33)