EXPLORATIONS

Activity 8

Cooling Off

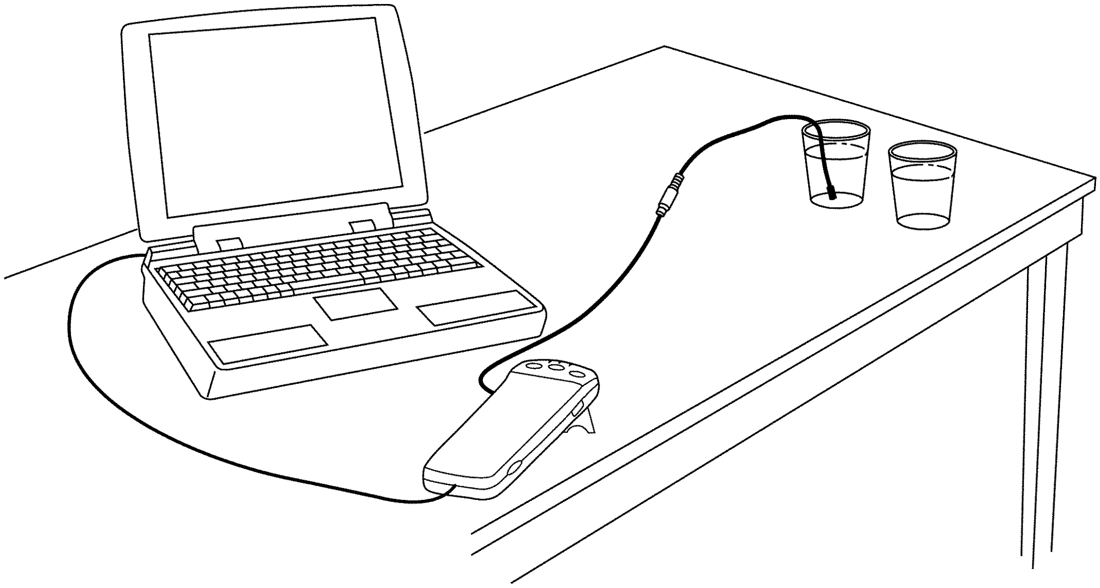
If you pour yourself a hot cup of coffee and place it on the counter, it immediately starts to cool off. The cooling process is quick at first, and then slowly levels off as the beverage approaches room temperature. In this situation, the rate at which the temperature of the drink changes is approximately proportional to the difference between its temperature and the temperature of its surroundings. This rule is commonly called *Newton's Law of Cooling*.

Introduction

In this activity, you will use a CBL™ or CBL 2™ together with TI InterActive!™ software and a temperature sensor to collect data that will simulate the temperature variations that occur as a liquid is cooling. You will then use the mathematical techniques and the statistical features of TI InterActive! to build a mathematical model for the resulting data set.

Equipment Required

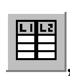
- Computer
- TI InterActive! software
- CBL/CBL 2
- TI temperature probe
- TI-GRAPH LINK™ cable
- Cup of hot water
- Cup of ice water

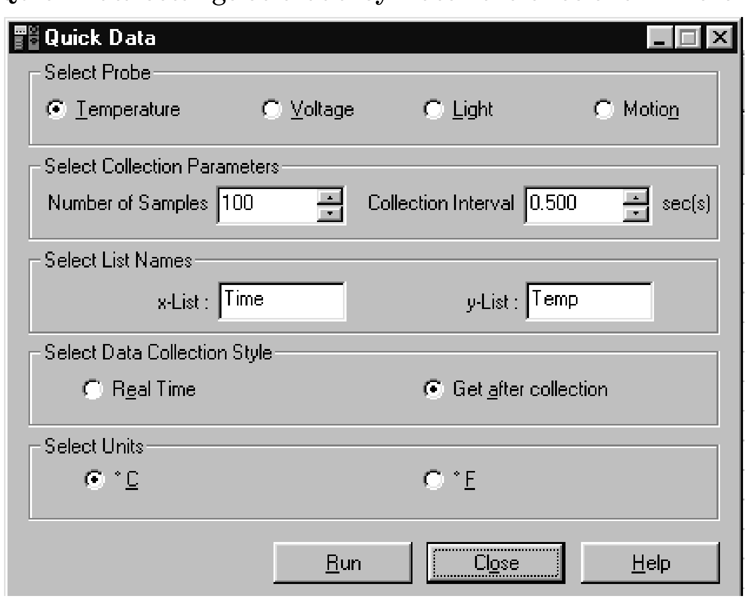


Setup

1. Plug the TI-GRAPH LINK™ cable into your computer.
2. Plug the other end of the TI-GRAPH LINK cable into the CBL™/CBL 2™.
3. Connect the TI temperature probe into the Channel 1 (**CH1**) port on the CBL/CBL 2.
4. If you are using a CBL, turn it on by pressing the red [ON/HALT] button.

Collecting the Data

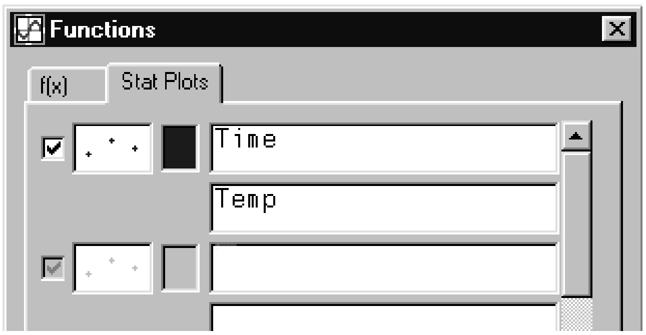
1. Start TI InterActive!™ The software opens to a new, blank document.
2. Title your new document *Cooling Off* and add your name and the date. Click the Save button  to save and name your document.
3. Click the List button , then click the Quick Data button  on the menu bar to open the data collection setup window.
4. Adjust the Quick Data settings so that they match the ones shown here:

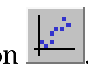


5. Place the temperature probe in the cup of hot water for about a minute. You will simulate the action of a cooling liquid by recording temperature data as the probe cools in a cup of ice water.

6. When you are ready to start collecting data, remove the temperature probe from the hot water, plunge it into the ice water and, *at the same time*, click **Run** in the Quick Data box. It will take about 50 seconds to collect the data. While the data is being collected, stir the temperature probe in the ice water.

7. As soon as the CBL™/CBL 2™ is finished collecting the temperature data, it will be transferred to the computer and a Graph window will open on the screen. The plot setup box shown below will appear.

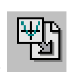


8. Press Enter, then click the Zoom Statistics button . The viewing boundaries adjust automatically to show all the plotted data.

   The plot of Celsius temperatures versus time should be curved and decreasing.

   If you are not satisfied with your results, start again and collect a new set of data.

   If you are satisfied with your data, you can make a sketch of the temperature versus time data that you collected on a blank grid in the Appendix. Label the horizontal and vertical axes on your sketch.

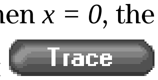
9. Click the Save to Document  button to save the graph to your TI InterActive!™ document.

Analysis and Questions

1. According to Newton's Law of Cooling, the temperature difference between a hot object (in this case, the temperature probe) and its surroundings (in this case, the ice water) decreases exponentially with time. Since the temperature of the ice water is 0 degrees Celsius, we can model this data set with an equation of the form:

   $$y = A\,B^x$$

   where x is time and y is temperature.

   When $0 < B < 1$, y is said to *decay exponentially* with x. The constant B is called the *base*. Notice that when $x = 0$, the y value is equal to A. Double-click the graph to open the Graph window. Click 

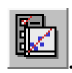
 and note the y-coordinate shown in the Trace Value dialog box that corresponds to $x = 0$. Click the Copy button. Close the Trace Value box, the Functions dialog box, and the Graph window. In a blank spot in your TI InterActive! document, Click on Edit, Paste, to past the Trace Value result in the document.

2. In order to find an exponential model for data you collected, you will need to find an appropriate value for *B*. We will use the guess-and-check method. Double-click the graph to open it, and click the f(x) tab in the upper left corner of the Functions dialog box. For exponential decay, the model demands that $0 < B < 1$, so start with an initial guess of $B = 0.5$. Type **f(x):= A*.5^x** in the uppermost text box of the f(x) tab, using the numerical value of *A* from above. Press Enter to superimpose the graph on the plotted data.

   It is unlikely that your first guess for the value of B produced a model that matched the data closely. Double-click the graph to open it, click in the text box of the f(x) tab again and edit the exponential equation, replacing the old value, B = 0.5, with your new guess for B. Press Enter to update the graph. Repeat the guess-and-check procedure until you find a B-value that models the data well. Press the Copy button. Close the Trace Value dialog box, Functions dialog box, and the Graph window. In a blank spot in your TI InterActive!™ document, click on Edit, Paste, to paste the Trace Value result in the document.

3. TI InterActive! lets you check the values of *A* and *B* you just found by calculating the exponential curve of best fit. Close the Function dialog box, and the Graph window to return to your TI InterActive! document.

   a. Double-click on the list to open the Data Editor.

   b. Click Statistical Regressions .

   c. Click the down arrow  next to Calculation Type, scroll down the list and click on **Exponential Regression**.

   d. In the text box labeled **X List**, type **time**; in the box labeled **Y List**, type **temp**. Click **Calculate** to find the regression equation, $y = a\,b^x$, and its variables.

   e. Click the checkbox next to *a* and *b* to select these values. Click the **Save Results** button. TI InterActive! stores the results in variables, closes the Statistical Regressions tool, and displays the selected results in your document.

   How do the values of *a* and *b* in the exponential regression equation compare with the *A*-value and *B*-value you found by guess-and-check?

4. Double-click on the graph in the TI InterActive! document to refresh the Graph window. In the second text box of the f(x) tab, type **f(x):= regeq(x)** and then press Enter. TI InterActive! graphs the original data set, the equation that was created as the Statistical Regressions result, and your modeling curve. How well does each equation fit the data?

5. Describe how the value of *B* affects the shape of the temperature versus time graph, $y = A B^x$.

6. Why must the value of *B* be less than 1 for this modeling activity? What shape does the graph have if *B* is greater than 1?

7. For the experiment you performed in this activity, does it make a difference whether the temperature readings are taken in Celsius degrees or Fahrenheit degrees? Explain.

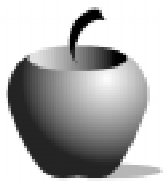
8. Save and print your TI InterActive!™ document.

Extensions

- Repeat the experiment, but this time allow the temperature sensor to cool off in the air rather than in ice water. In this case, the temperatures will level off at a non-zero value. This means that the modeling equation will be shifted: $y = A B^x + C$. Can you find values for *A*, *B*, and *C* so that this equation provides a good fit for the data you collected? What variable (or variables) represents the starting temperature? What is the physical meaning of the constant *C* in the modeling equation? Is it possible to use the TI InterActive! exponential regression feature to model this set of data? Why or why not?
- What would a temperature versus time data set look like if the sensor *started* in ice water and then was submerged in hot water? Sketch your prediction for the resulting heating curve, then perform an experiment to check your prediction. Is it possible to find a mathematical equation that describes this type of phenomenon?

Teacher Notes

Activity 8: Cooling Off



Math Concepts

- CBL/CBL 2
- Exponential Function

Activity Notes

- The water used in this activity should be very hot, but does not need to be boiling. Students should work quickly to collect the data before the water cools off too much.
- You might want to discuss the concept of asymptotic behavior with your students since the temperatures in this experiment approach the surrounding temperatures, in this case 0°C.

Sample Data

***Note**: This data set is based on readings every 2 seconds, not every 0.5 seconds as prescribed in the activity.*

Time (sec)	Temp (F)	Time(sec)	Temp (F)	Time (sec)	Temp (F)
0	41.59	18	11.86	36	5.22
2	36.99	20	10.85	38	3.35
4	30.49	22	10.07	40	2.69
6	25.99	24	10.07	42	2.36
8	24.69	26	7.53	44	2.24
10	19.84	28	8.19	46	1.7
12	19	30	7.53	48	2.58
14	15.26	32	4.89	50	0.47
16	13.89	34	4.78		

Analysis and Questions - Key

1. $A = 41.59$.
2. $B = 0.93$.
3. $a = 43.987$, $b = 0.933$; they match closely.
4. Both equations fit the data well.
5. The value of B changes the steepness of the curve.
6. It must be less than 1 so that the curve shows decay; for B-values greater than 1, the curve is increasing, not decreasing.
7. The data would have the same shape but the level-off value would be non-zero and, as a result, the modeling equation would be $y = AB^x + C$, where C represents 32°F.