

Difference Between Two Proportions

ID: 10082

Time required

40 minutes

Activity Overview

Students use confidence intervals to estimate the difference of two population proportions. First they find the intervals by calculating the critical value and the margin of error. Then, they use the **2-prop z Interval** command. Students find confidence intervals for differences in proportions in real-life situations and use them to make judgments about claims. Last, they determine required sample size, n , when given a confidence interval and margin of error.

Topic: Sampling Distributions

- Use the fact that the sampling distribution of the difference $p_1 - p_2$ (where p_1 and p_2 are the proportions of some attribute in two different populations) is approximately a normal distribution with mean $p_1 - p_2$ and standard deviation $\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$ to calculate a confidence interval for $p_1 - p_2$.

Teacher Preparation and Notes

- This activity is designed to be used with students who are studying Statistics and Probability.
- Students should be familiar with estimating a confidence interval for a single population proportion and determining the sample size that is required for a given confidence interval and margin of error.
- Using a confidence interval to make a decision, as done in Problem 2, is a precursor to hypothesis testing.
- To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "10082" in the keyword search box.

Associated Materials

- Difference_Two_Proportions_Student.doc
- Difference_Two_Proportions.tns
- Difference_Two_Proportions_Soln.tns

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- Testing Claims About Proportions (TI-Nspire technology) — 10131

Problem 1 – Estimating a Confidence Interval

To begin the activity, explain to students that they can estimate the true difference between two population proportions and that subscripts will be used to differentiate between the two populations.

Then introduce the formulas for the margin of error and confidence interval.

A **confidence interval** for the difference between two population proportions is

$$(\hat{p}_1 - \hat{p}_2) - E < p_1 - p_2 < (\hat{p}_1 - \hat{p}_2) + E,$$

where $E = z \frac{\alpha}{2} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$.

The scenario on page 1.4 describes, from those customers randomly selected, the numbers of men and women that used coupons at a grocery store. Students are to use the math boxes on the *Notes* application on page 1.5 to find each sample proportion, the difference between the sample proportions and the margin of error.

Find \hat{p}_1 , \hat{p}_2 , $\hat{p}_1 - \hat{p}_2$, and E at the 95% level.

$$p_1 := \frac{74}{172} \rightarrow 0.43 \quad p_2 := \frac{12}{45} \rightarrow 0.27 \quad p_1 - p_2 \rightarrow 0.16$$

$$z_{\alpha 95} := \text{invNorm}(0.975, 0, 1) \rightarrow 1.96$$

$$E := z_{\alpha 95} \cdot \sqrt{\frac{p_1 \cdot (1-p_1)}{172} + \frac{p_2 \cdot (1-p_2)}{45}} \rightarrow 0.15$$

To use the math boxes for p_1 and p_2 , the students are to enter the correct values in each proportion and press **enter**. To use the math box that will calculate $p_1 - p_2$, have the students click the math box, arrow over to the end of the math box and press **enter**.

To use the math box that will calculate the margin of error, E, have the students click the math box, arrow over to the end of the math box and press **enter**. The margin of error is about 15%.

Note: The required z-score for the margin of error formula is already calculated on the notes page and is stored as variable **zα95**.

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 1 at the end of this lesson.

On page 1.6, students are to construct the confidence interval by subtracting from and adding to the difference of the sample proportions. This has already been set-up using the math boxes. As before, to have the math box calculate the desired value, have the students click the math box, arrow over to the end of the math box and press **enter**.

Use E and $(\hat{p}_1 - \hat{p}_2)$ to find the 95% confidence interval.

$$(\hat{p}_1 - \hat{p}_2) + z_{\alpha 95} \cdot \sqrt{\frac{p_1 \cdot (1-p_1)}{172} + \frac{p_2 \cdot (1-p_2)}{45}} \rightarrow 0.31$$

$$(\hat{p}_1 - \hat{p}_2) - z_{\alpha 95} \cdot \sqrt{\frac{p_1 \cdot (1-p_1)}{172} + \frac{p_2 \cdot (1-p_2)}{45}} \rightarrow 0.01$$

Have students state their result in a sentence: *We are 95% certain that the difference in the proportion of men and women who use coupons at that store is between 1% and 31%.*

Now students will repeat the process on page 1.7 to construct the 90% confidence interval. The 90% confidence interval is from 3.9% to 28.9%.

Find E at the 90% level to find to 90% confidence interval.

$$z_{\alpha 90} = \text{invNorm}(0.95, 0, 1) \rightarrow 1.64$$

$$E1 = z_{\alpha 90} \cdot \sqrt{\frac{p_1 \cdot (1-p_1)}{172} + \frac{p_2 \cdot (1-p_2)}{45}} \rightarrow 0.12$$

$$(\hat{p}_1 - \hat{p}_2) + E1 \rightarrow 0.289$$

$$(\hat{p}_1 - \hat{p}_2) - E1 \rightarrow 0.039$$

In a sentence: *We are 90% certain that the difference in the proportion of men and women who use coupons at that store is between 3.9% and 28.9%.*

TI-Nspire Navigator Opportunity: Quick Poll and Class Capture
See Note 2 at the end of this lesson.

Let students check their work on page 1.8 by pressing **MENU > Statistics > Confidence Intervals > 2-Prop z Interval** and entering the required information.

The first screenshot shows the "2-Prop z Interval" dialog box with the following values: Successes, x1: 74; n1: 172; Successes, x2: 12; n2: 45; C Level: 0.95. The second screenshot shows the command: `zInterval_2Prop 74,172,12,45,0.95: stat.resu` and the resulting list: `"Title" "2-Prop z Interval"`, `"CLower" 0.014675`, `"CUpper" 0.312457`, `"pDiff" 0.163566`, `"ME" 0.148891`, `"p1" 0.430233`. The third screenshot shows the list of results: `"CUpper" 0.312457`, `"pDiff" 0.163566`, `"ME" 0.148891`, `"p1" 0.430233`, `"p2" 0.266667`, `"n1" 172`, `"n2" 45`.

Problem 2 – Practice Problems

Ask students what $p_1 - p_2$ would be if the two proportions were equal or close to equal (0 or close to 0).

Once students have read the scenario on page 2.1, they are to find the confidence interval on the next page using the **2-prop z Interval** command or step by step as done in Problem 1.

Ask what the value of $p_1 - p_2$ is (about 7.5%). The value for $p_1 - p_2$ is *pDiff* in the generated list of values. Ask what the margin of error is (ME is about 14.8%.) Discuss why the lower bound is negative.

Find the 95% **confidence interval** for the difference between the proportions.

"CLower"	-0.073437
"CUpper"	0.222869
"pDiff"	0.074716
"ME"	0.148153
"p1"	0.776471
"p2"	0.701754

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TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 3 at the end of this lesson.

Students should discuss what they think about the principal's claim and the reasoning behind their statements. There is no reason to believe the principal's claim is incorrect.

What do you think of the principal's claim?

Appears correct. 0%, or no difference, is in the interval.

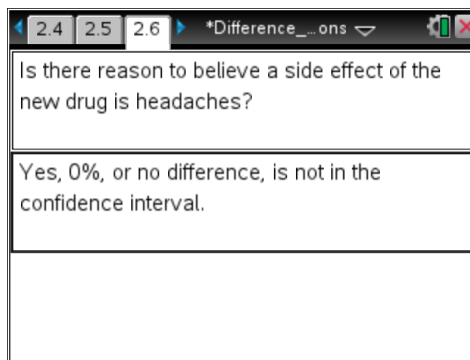
Students are to repeat this process for the scenario on page 2.4. Remind them that in an experiment where patients are given either the drug or a placebo, the patients do not know to which group they belong.

Find the 95% **confidence interval** for the difference between the proportions.

"CLower"	0.018945
"CUpper"	0.205113
"pDiff"	0.112029
"ME"	0.093084
"p1"	0.37931

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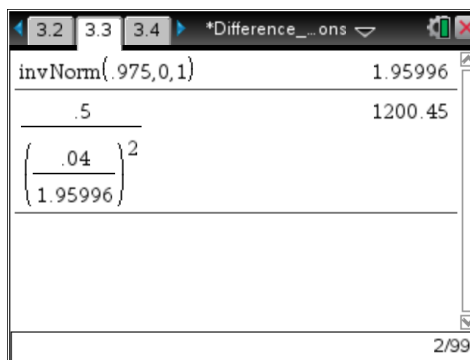
This time, 0%, or no difference, is not in the confidence interval, so there is reason to believe that the true proportions are different. Since the first proportion was larger, and this was the group that received the drug, it would appear that the drug may cause headaches.



Problem 3 – Sample Size

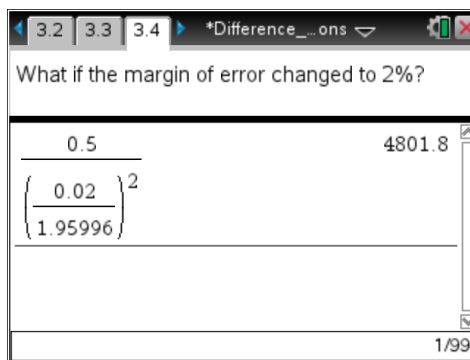
Introduce the formula for determining the required sample size for a given confidence level and margin of error.

On page 3.3, students are to find the sample size needed (1201 men and 1201 women would need to be surveyed).



Ask students to predict how many more people would need to be surveyed if the margin of error were reduced to 2%. Then have them advance to page 3.4 to find the actual sample size

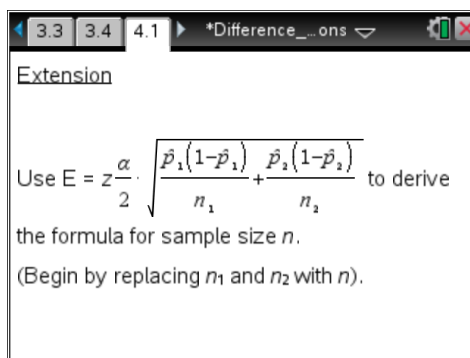
The number that would need to be surveyed is 4802 men and 4802 women. This is about four times as many of each.



Problem 4 – Extension

Have students use algebra to derive the formula used in Problem 3 from the formula for the margin of error by replacing both n_1 and n_2 with n .

If needed, remind or tell students that when an estimate of a sample proportion is unknown, 0.5 is used.



TI-Nspire™ Navigator™ Opportunities**Note 1****Question 1, *Live Presenter***

Use Live Presenter to demonstrate how to use the math boxes

Note 2**Question 1, *Quick Poll* and *Class Capture***

Send students a Quick Poll asking for their calculated confidence interval. For those incorrect intervals, use Class Capture and have the class find the error.

Note 3**Question 3, *Live Presenter***

Use Live Presenter to facilitate the discussion of the following questions: What is the margin of error? (ME is about 14.8%.) Why is the lower bound negative?