All Systems Go!

ID: 10086

Time required
45 minutes

Activity Overview

Students will learn about inverse matrices in the context of encoding/decoding secret messages and then use inverse matrices to solve systems of equations.

Topic: Linear Algebra: Vectors & Matrices

- *Compute the product of any pair of matrices.*
- *Compute the inverse of a square non-singular matrix.*
- *Solve a linear system of n equations in n variables by computing the inverse of the coefficient matrix.*

Teacher Preparation and Notes

- *This investigation illustrates to students that (some) matrix multiplications can be "undone" by multiplying by an inverse matrix. Such inverse matrices can also be used to solve systems of equations.*
- *Prior to beginning this activity, review with students how to organize information into matrices. Students should also be familiar with multiplying matrices in TI-Nspire technology.*
- *This activity does not teach students the process used to set up and/or multiply matrices; rather it explains how to find the inverse matrix of an invertible 2 × 2 matrix and provides several examples of the types of problems that can be solved using inverse matrices. Students should already know how to multiply matrices by hand.*
- ***To download the student and solution TI-Nspire document (.tns files) and student worksheet, go to education.ti.com/exchange and enter "10086" in the quick search box.***

Associated Materials

- *PrecalcWeek30_AllSystemsGo_worksheet_TINspire.doc*
- *PrecalcWeek30_AllSystemsGo.tns*
- *PrecalcWeek30_AllSystemsGo_Soln.tns*

Suggested Related Activities

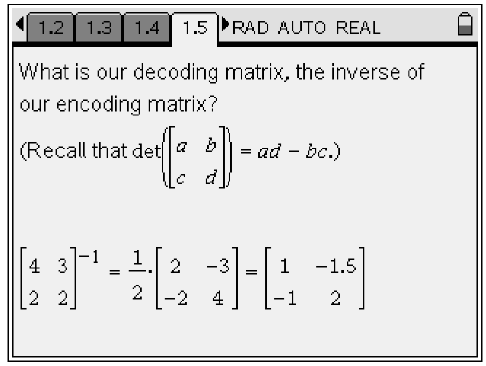
To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *How Many Solutions? (TI-Nspire technology) — 9284*
- *Solving Systems of Linear Equations with … Augmented Matrices (TI-Nspire technology) — 8838*
- *Solving Systems of Linear Equations from Four Perspectives (TI-Nspire technology) — 9210*

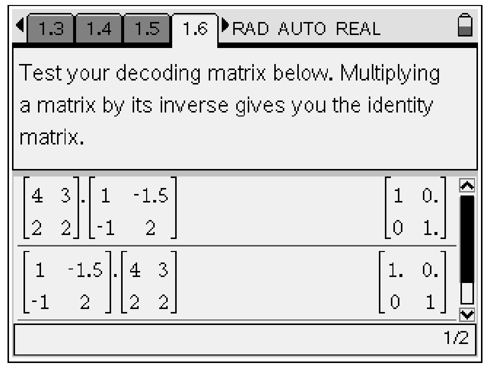
Problem 1 – A secret message

This problem introduces students to the concept of an inverse matrix by having them explore using matrices to encode and decode secret messages.

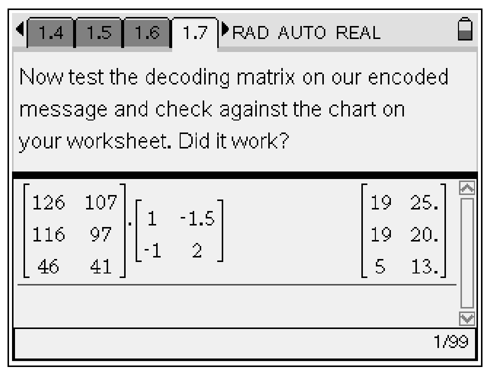
On page 1.4, students are given the definition of the inverse of a 2 × 2 matrix, and they are asked to find the inverse of the encoding matrix on page 1.5.



Multiplying the encoding matrix by the decoding matrix should result in the identity matrix, since they are inverses. Be sure that students understand that this is true *regardless* of the order in which the inverse matrices are multiplied.

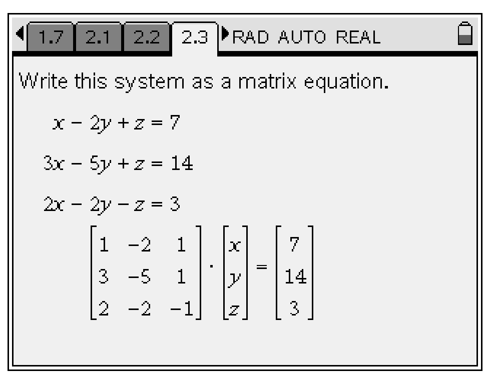


Finally, students are asked to use the decoding matrix on the encoded message to check that they are able to retrieve the original message.

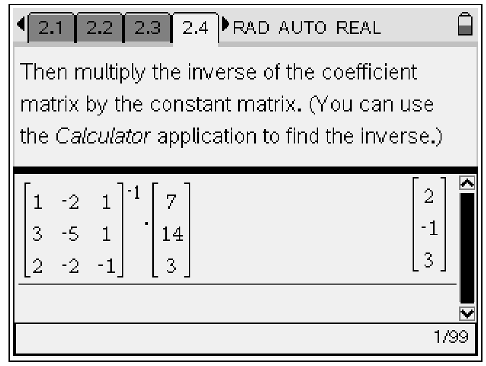


Problem 2 – Solving systems of equations

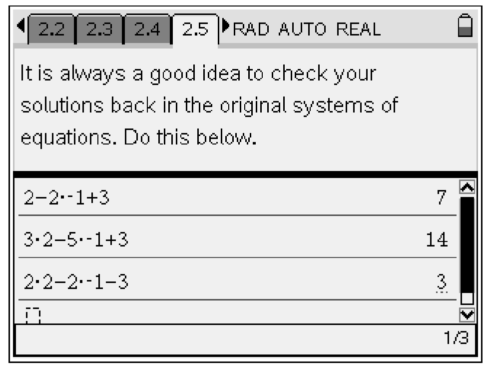
This problem helps students use inverse matrices to solve a system of equations. When students are translating the system to a matrix equation, be sure that they include all appropriate negative signs in the coefficient and/or constant matrices. You might also want to provide students with several examples of systems with “missing terms” that will translate into 0’s in a coefficient matrix.



On page 2.4, students will use the *Calculator* application to multiply the inverse of the coefficient matrix by the constant matrix to obtain the solution.

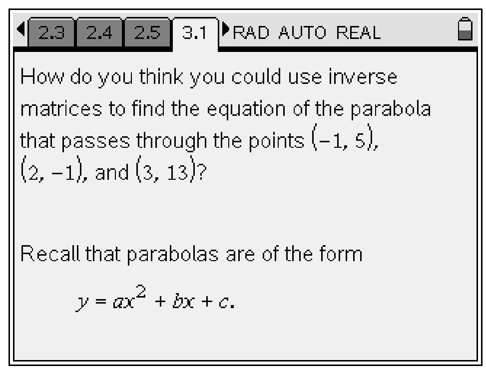


Be sure that students check their solutions on page 2.5. It is here that any errors in translating the system to a matrix equation will be revealed.



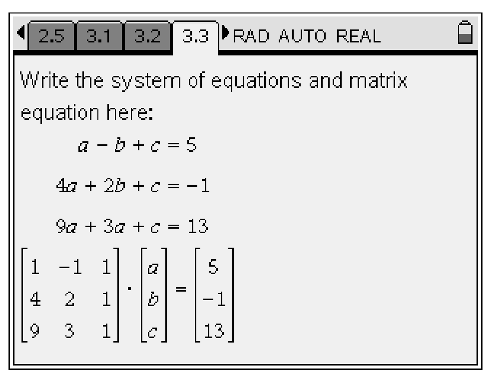
Problem 3 – Finding the equation of a parabola

This final problem asks students to find the equation of the parabola that passes through three specific points. Pages 3.1 and 3.2 provide steps that will guide students through how to use systems and matrices to solve the problem.

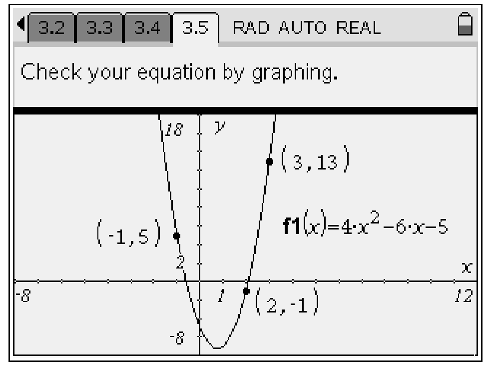


You may wish to perform the first substitution as an example for students. Substituting the first point (–1, 5) into the quadratic equation $y = ax^2 + bx + c$ yields $5 = a(-1)^2 + b(-1) + c$, which simplifies to $a - b + c = 5$.

The other equations follow, and result in the system and matrix equation shown to the right.



Students can confirm the solution (a = 4, b = –6, and c = –5) by substituting the coordinates and verifying the equation holds. They can also confirm the solution by graphing the equation to see if it indeed passes through the three points.



Solutions – Student Worksheet Exercises

1. The inverse of the encoding matrix is $\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix}$, so

$$\begin{bmatrix} 18 & 27 \\ 51 & 81 \\ 37 & 58 \\ 60 & 100 \\ 18 & 27 \\ 85 & 137 \\ 59 & 93 \\ 51 & 79 \end{bmatrix} \cdot \begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 12 & 9 \\ 11 & 5 \\ 0 & 20 \\ 9 & 0 \\ 14 & 19 \\ 16 & 9 \\ 18 & 5 \end{bmatrix}$$

, which translates to **I LIKE TI NSPIRE.**

2. Writing the system as a matrix equation gives $\begin{bmatrix} 7 & -8 & 5 \\ -4 & 5 & -3 \\ 1 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 18 \\ -11 \\ 1 \end{bmatrix}$.

Solving for the variable matrix yields a solution of x = 2, y = –3, and z = –4.

3. Writing as a system of equations:

$$a - b + c = 3$$
$$a + b + c = -3$$
$$4a + 2b + c = 0$$

Writing as a matrix equation:

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix}$$

Solving for the variable matrix, the solution is a = 2, b = –3 and c = –2, which substituted into the equation for a parabola yields $y = 2x^2 - 3x - 2$.