Math Objectives

- Students will connect the area under a derivative curve to the graph of the antiderivative.

Activity Types

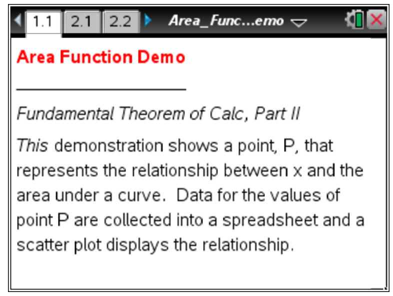
- Teacher Demonstration
- Student Exploration

About the Lesson

- This demonstration shows a point *P* that represents the relationship between *x* and the area under a curve. Data for the values of point *P* are collected into a spreadsheet and a scatter plot displays the relationship.

Directions

- This demonstration shows a point *P* that represents the relationship between *x* and the area under a derivative curve. Data for the values of point *P* are collected into a spreadsheet and a scatter plot displays the relationship.
- Grab and move point *P* across the screen. Move to page 2.3 to examine the graph of the antiderivative.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point
- Edit a function

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.

Lesson Files:

Student Activity

- Area_Function_Demo_Student.pdf
- Area_Function_Demo_Student.doc

TI-Nspire document

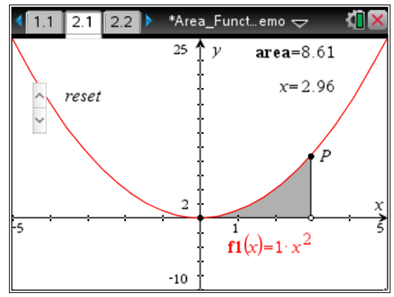
- Area_Function_Demo.tns

Visit www.mathnspired.com for lesson updates.

Discussion Points and Possible Answers

Move to page 2.1.

Teacher Tip: Students explore various functions by changing **f**(x) on the screen. The x-value, representing the right end value of an interval starting at zero, and the area under the derivative curve are shown. As students move the open circle on the x-axis, the area and point P move.
The example **f**(x) = x^2 is shown at the right.



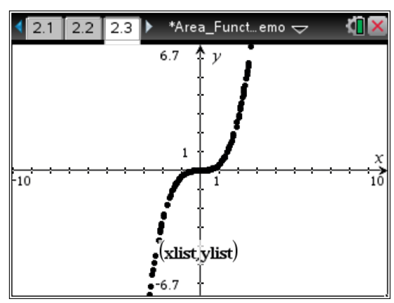
Tech Tip: Page 2.2 automatically collects the x-value and the area under the curve. The first two columns will fill as the student moves the empty circle along the x-axis.

1. Can you predict what function point P is tracing?

   Answer: Point P is tracing a cubic function.

Move to page 2.3.

Tech Tip: This graph will display a scatter plot of the points collected on page 2.2. Students can enter a function they think the scatter plot represents in the entry line. When students have entered the correct function, they have found the antiderivative function for the function from page 2.1.



2. What is the antiderivative function of **f**(x) = x^2?

   Sample answer: The antiderivative function of **f**(x) = x^2 is $\frac{1}{3}x^3$ or $\frac{1}{3}x^3 + C$.

Tech Tip: To clear the collected data, follow the steps in the student worksheet.

Explorations 1 and 2 of the student worksheet ask students to explore variations of x^2 and x^3 to find their antiderivative functions.

$\mathbf{f}(x) = ax^2$, where *a* equals 2, 3, 4, etc., until students see a pattern.
$\mathbf{f}(x) = ax^3$, where *a* equals 2, 3, 4, etc., until students see a pattern.

$\mathbf{f}(x) = ax^2 + b$; keep *a* constant and change *b.*
$\mathbf{f}(x) = ax^3 + b$; keep *a* constant and change *b.*

Teacher Tip: You can return to this lesson file as the course continues to have students explore non-polynomial functions such as $sin(x)$, $\ln(x)$, e^x, and others.

Exploration 1: Now that you have found the antiderivative function for $\mathbf{f}(x) = x^2$, explore some other variations of this function and see if you can find a pattern in their antiderivatives.

3. Record the antiderivative functions and any patterns you saw here:
   a. $\mathbf{f}(x) = ax^2$, where *a* equals 2, 3, 4, etc., until you see a pattern.

   <u>Answer:</u> If $\mathbf{f}(x) = 2x^2$, then the antiderivative function of $\mathbf{f}(x)$ is $\frac{2}{3}x^3$.
   If $\mathbf{f}(x) = 3x^2$, then the antiderivative function of $\mathbf{f}(x)$ is x^3.
   If $\mathbf{f}(x) = 4x^2$, then the antiderivative function of $\mathbf{f}(x)$ is $\frac{4}{3}x^3$.

   b. $\mathbf{f}(x) = ax^2 + b$; keep *a* constant and change *b*

   <u>Answer:</u> If $\mathbf{f}(x) = 3x^2 + 1$, then the antiderivative function of $\mathbf{f}(x)$ is $x^3 + x$.
   If $\mathbf{f}(x) = 3x^2 + 2$, then the antiderivative function of $\mathbf{f}(x)$ is $x^3 + 2x$.
   If $\mathbf{f}(x) = 3x^2 + 3$, then the antiderivative function of $\mathbf{f}(x)$ is $x^3 + 3x$.

Exploration 2: Begin by finding the antiderivative function for $\mathbf{f}(x) = x^3$.
What is the antiderivative function of $\mathbf{f}(x) = x^3$?

<u>Answer:</u> The antiderivative function of $\mathbf{f}(x) = x^3$ is $\frac{1}{4}x^4$ or $\frac{1}{4}x^4 + C$.

Now explore some other variations of this function and see if you can find a pattern in their derivatives.

4. Record the derivative functions and any patterns you saw here:
   a. **f**(x) = ax^3, where a equals 2, 3, 4, etc., until you see a pattern.

Answer: If **f**(x) = $2x^3$, then the antiderivative function of **f**(x) is $\frac{1}{2}x^4$.

If **f**(x) = $3x^3$, then the antiderivative function of **f**(x) is $\frac{3}{4}x^4$.

If **f**(x) = $4x^3$, then the antiderivative function of **f**(x) is x^4.

   b. **f**(x) = $ax^3 + b$; keep a constant and change b.

Answer: If **f**(x) = $3x^3+1$, then the antiderivative function of **f**(x) is $\frac{3}{4}x^4+x+C$.

If **f**(x) = $3x^3+2$, then the antiderivative function of **f**(x) is $\frac{3}{4}x^4+2x+C$.

If **f**(x) = $3x^3+3$, then the antiderivative function of **f**(x) is $\frac{3}{4}x^4+3x+C$

Teacher Tip: Students may include + C with each answer if the concept has been discusses prior to the lesson. If not, then the concept can be explored following this lesson.