Proof of Identity

ID: 9847

Time required
60 minutes

Activity Overview

Students use graphs to verify the reciprocal identities. They then use the handheld's manual graph manipulation feature to discover the negative angle, cofunction, and Pythagorean trigonometric identities. Geometric proofs of these identities are given as well.

Topic: Trigonometric Identities

- *Verify trigonometric identities by graphing.*
- *Use the Pythagorean Theorem to prove the trigonometric identities* $\sin^2\theta + \cos^2\theta = 1$ *and* $\sec^2\theta = 1 + \tan^2\theta$.

Teacher Preparation and Notes

- *This activity is appropriate for an Algebra 2 or Precalculus classroom.*
- *Students should have experience graphing and translating trigonometric functions.*
- *This activity is intended to be* ***teacher-led*** *with students in* ***small groups****. You should seat your students in pairs so they can work cooperatively on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "9847" in the keyword search box.***

Associated Materials

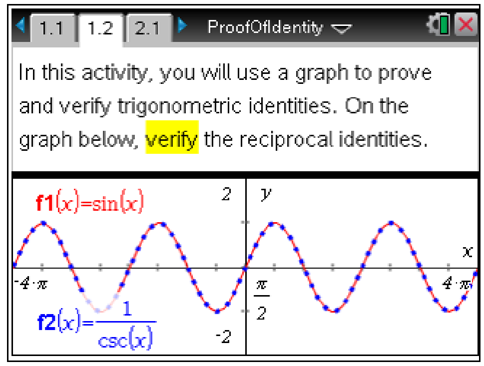
- *ProofOfIdentity_Student.doc*
- *ProofOfIdentity.tns*
- *ProofOfIdentity_Soln.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Discovering Basic Pythagorean Trig Identities (TI-Nspire technology) — 13868*
- *Trigonometric Identities (TI-84 Plus family) — 9848*
- *Verifying Trigonometric Identities (TI-Nspire technology) — 12942*

Problem 1 – Reciprocal Identities



Begin the activity by discussing the term *identity*. An *identity* is a statement about two expressions that are the same, or *identical*. Trigonometric identities are used to simplify trigonometric expressions and solve trigonometric equations.

Students may be surprised to learn that they already know several trigonometric identities just by knowing the definitions of the trigonometric functions.

TI-Nspire Navigator Opportunity: *Screen Capture*

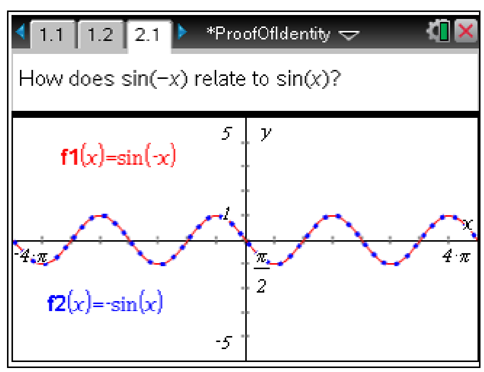
See Note 1 at the end of this lesson.

Ask: *If two expressions are the same, what will their graphs look like?* Page 1.2 shows the graphs of two identical functions: **f1**(x) = 1 and **f2**(x) = 1. Throughout this activity, the graph of the first function is shown as a thin, solid line and the graph of the second is shown as a thick, dashed line.

Demonstrate verifying a trigonometric identity by graphing. Click on the equation for **f1**(x) and change it to csc(x). Then click on the equation for **f2**(x) and change it to $\frac{1}{\sin(x)}$. The graphs are the same because $\csc(x) = \frac{1}{\sin(x)}$.

Explain that is equation is one of the *reciprocal identities* because it shows that one trigonometric function is equal to the reciprocal of another. Have students work independently on page 1.2 to verify all six reciprocal identities and record them on their worksheets.

Problem 2 – Negative Angle Identities

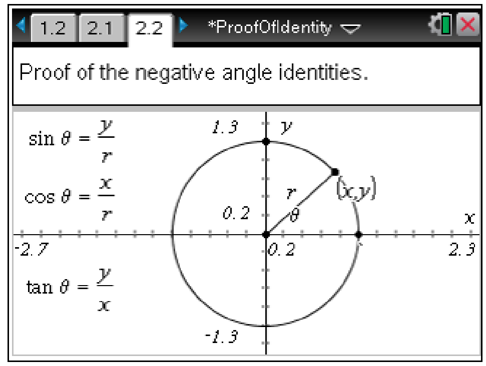


In Problem 2, students progress from graphing to *verify* trigonometric identities to graphing to *discover* them. We are now using the idea that two expressions that are equal have the same graph in the other direction: *Two expressions that have the same graph are equal*.

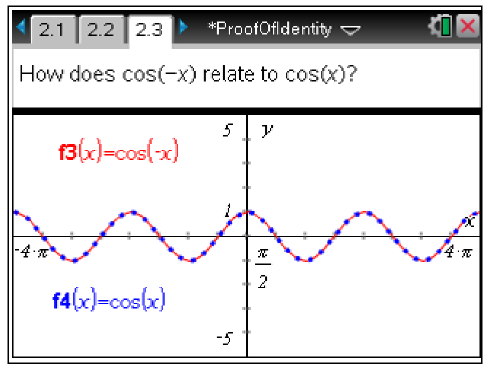
Students are to graph **f1**(x) = sin(–x) and **f2**(x) = sin(x) on page 2.1. Ask: *How are these graphs related?*

Demonstrate grabbing and dragging the graph of **f2** until it lines up with **f1**. You can either reposition the graph without affecting the shape of the curve (cursor looks like ✥) or you can adjust the *reshaping* shape of the curve (cursor looks like ⤡). In this case, you want to flip the graph by the curve, not *repositioning* it. The equation updates as you drag the graph. Reading the equation for **f2** (some rounding may be required) and setting it equal to **f1** gives sin(–*x*) = –sin(*x*).

Students can prove the negative angle identities geometrically on page 2.2 by following the directions on their worksheets. (If students are not familiar with the unit circle definition of the trigonometric functions, you may choose to skip this step.)



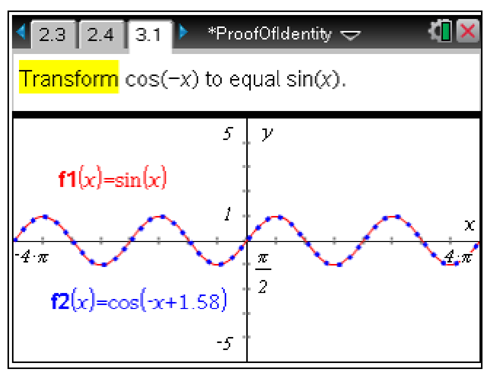
Have students work independently on pages 2.3 and 2.4 to verify the remaining negative angle identities.



Problem 3 – Cofunction Identities

By now, students have seen enough trigonometric graphs to realize that the sine and cosine functions are more similar to each other than they are to the tangent function. This intuition is formalized by the cofunction identities.

Students are to graph **f1**(*x*) = sin(*x*) and **f2**(*x*) = cos(–x) on page 3.1.

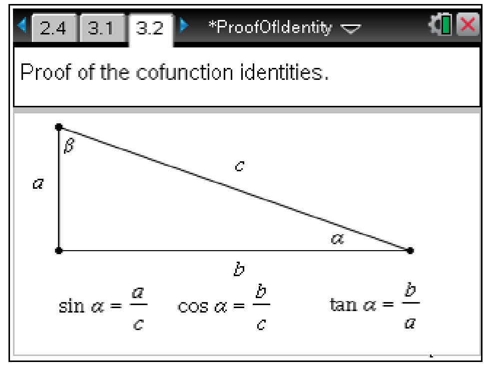


They should then grab and drag the graph of **f2** until it lines up with **f1**. In this case they want to reposition the curve, not reshape it. Reading the equation for **f2** and setting it equal to **f1** gives $\sin(x) = \cos\left(\frac{\pi}{2} - x\right)$ or a similar identity, depending on which way the graph was dragged. Show students that are there many possible ways to make the curves align.

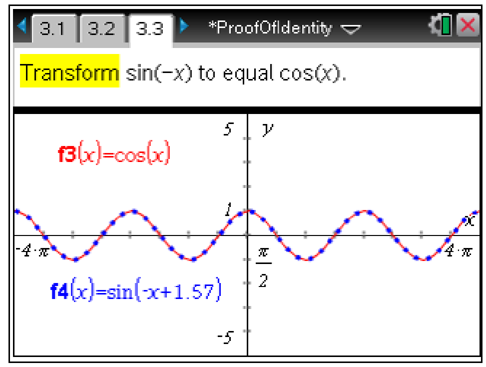
TI-Nspire Navigator Opportunity: *Quick Poll* and Screen *Capture*

See Note 2 at the end of this lesson.

Students can prove the cofunction identities geometrically on page 3.2 by following the directions on their worksheets.



Have students work independently on pages 3.3 and 3.4 to verify the remaining cofunction identities.

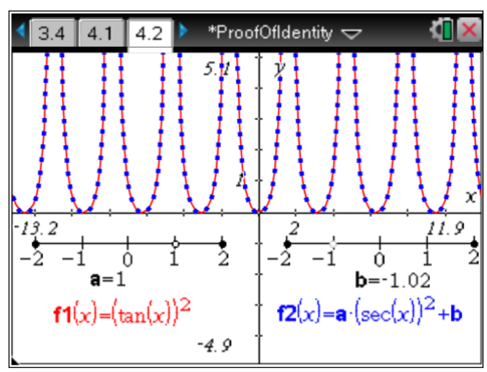


TI-Nspire Navigator Opportunity: *Quick Poll* and Screen *Capture*

See Note 3 at the end of this lesson.

Problem 4 – Pythagorean Identities

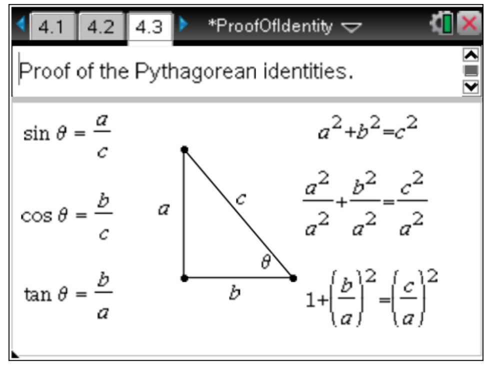
The final set of trigonometric identities that students explore in this activity relates the squares of the different trigonometric functions. Page 4.2 shows the graphs of **f1**(x) = $\tan^2(x)$ and **f2**(x) = $\sec^2(x)$. The graphs of these functions are more complex and cannot be dragged into place like those in the other problems of this activity.



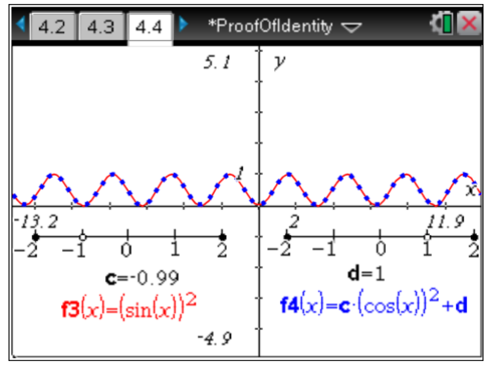
Instead, two *parameters* are introduced into one of the function equations. The parameters can then be adjusted with the sliders on the screen. Demonstrate by clicking on the equation for **f2** and changing it to read **f2**(x) = $a \cdot (\sec(x))^2 + b$. Move the open circle on the sliders to adjust the values of a and b and hence the graph.

When the graphs align, substitute the values of a and b into the equation for **f2** and set it equal to **f1**, yielding the identity $\tan^2(x) = \sec^2(x) - 1$.

Students can prove the Pythagorean identities using the diagram on page 4.3 by following the directions on their worksheets.



Have students work independently on pages 4.4 and 4.5 to verify the remaining Pythagorean identities.



TI-Nspire Navigator Opportunities

Note 1
Problem 1, *Screen Capture*

Use *Screen Capture* to monitor student progress through the problems on pages 1.1 to 2., offering help where needed.

Note 2
Problem 3, Quick Poll and *Screen Capture*

Send a *Quick Poll* for student responses to the transformation to page 3.1. Use this opportunity to discuss equivalent transformations if anyone used π notation in their transformation. Use *Screen Capture* to illustrate these equivalent transformations.

Note 3
Problem 3, *Quick Poll*

Send a *Quick Poll* for student responses to the transformation to page 3.3. Use this opportunity to discuss equivalent transformations if anyone used π notation in their transformation. Use *Screen Capture* to illustrate these equivalent transformations.