Math Objectives

For the graphs of $f(x) = \log_a x$ where $a > 0$ and $a \neq 1$, students will:

- Infer why the conditions $a > 0$ and $a \neq 1$ are necessary for the function to be logarithmic.
- Determine that for $a > 1$ the function is increasing and for $0 < a < 1$ the function is decreasing.
- Determine the *x*-intercept, *y*-intercept, domain, range, and asymptotes.
- Determine that for *a* >1 the function approaches ∞ as *x* approaches ∞ and that for 0 < *a* < 1 the function approaches –∞ as *x* approaches ∞.

Students will construct viable arguments & critique the reasoning of others (CCSS Mathematical Practice).

Vocabulary

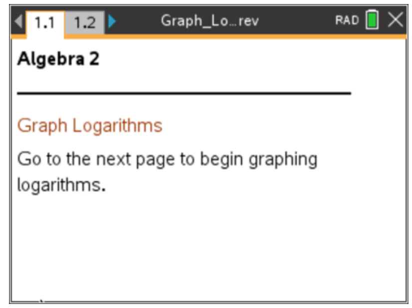
- logarithm function
- end behavior
- intercepts
- domain and range
- asymptotes
- increasing and decreasing functions
- extraneous solution

About the Lesson

- The time varies for this activity depending on whether students create the TI-Nspire document file or use the pre-constructed file.
- Students will investigate the graphs of the family of logarithm functions $f(x) = \log_a x$, by changing the *a*-value over the interval $0 \leq a \leq 4$.
- As a result, students will:
  - Infer why the conditions $a > 0$ and $a \neq 1$ are necessary.
  - Determine how the value of *a* affects the increasing or decreasing behavior of the function.
  - Determine the *x*-intercept, domain, range, and asymptotes.
  - Describe the end behavior.

TI-Nspire™ Navigator™ System

- Use Live Presenter to demonstrate how to use sliders.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Use a minimized slider

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the entry line by pressing ctrl G.

Lesson Materials:

Student Activity

Graph_Logarithms_Student.pdf

Graph_Logarithms_Student.doc

Optional Materials:

Graph_Logarithms _Create.doc

Graph_Logarithms _Create.pdf

TI-Nspire document

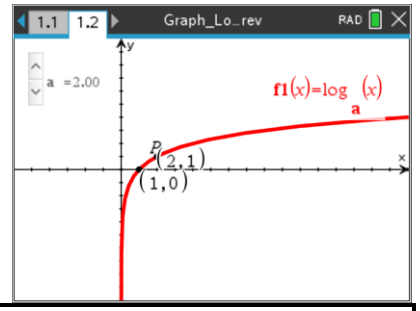
Graph_Logarithms.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

- Use Screen Capture to examine patterns that emerge.
- Use Quick Poll to assess students' understanding throughout the activity.

Discussion Points and Possible Answers

Move to page 1.2.



1. Explore several different **a**-values by clicking ∆ or ∇.

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 1 at the end of this lesson.

a. Set **a** = 1. Describe the graph.

<u>Answer:</u> It is nonexistent.

b. By definition, for the logarithmic function $\mathbf{f}(x) = \log_{\mathbf{a}}(x)$, **a** cannot equal 1. What mathematical reason can you give for this restriction?

<u>Answer:</u> When you have $\log_1 x = y$ this is equivalent to $1^y = x$, and 1 to any power is 1. Therefore, the possible solution is the vertical line x = 1; since this is not a function, x = 1 is an extraneous solution.

c. Set **a** = 0. Describe the graph.

<u>Answer:</u> It is nonexistent.

d. By definition, for the logarithmic function $\mathbf{f}(x) = \log_{\mathbf{a}}(x)$, **a** cannot equal 0. What mathematical reason can you give for this restriction?

<u>Answer:</u> When you have $\log_0 x = y$ this is equivalent to $0^y = x$, and 0 to any power is 0. Therefore, the possible solution is the vertical line x = 0; since this is not a function, x = 0 is an extraneous solution.

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2. Explore several different **a**-values by clicking ∆ or ∇.

   a. For what **a**-values is the function increasing? Why?

   Answer: When **a** > 1, $\mathbf{f}(x) = \log_{\mathbf{a}}(x)$ is increasing. This is because if $y = \log_{\mathbf{a}} x$ then $\mathbf{a}^y = x$. When **a** > 1, as the power increases the resulting value increases. For example, $\log_2 8 < \log_2 16$ because $8 = 2^3$, $16 = 2^4$, and $3 < 4$.

   b. For what **a**-values is the function decreasing? Why?

Answer: When 0 < **a** < 1, $\mathbf{f}(x) = \log_{\mathbf{a}}(x)$ is decreasing. This is because if $y = \log_{\mathbf{a}} x$ then $x = \mathbf{a}^y$. When 0 < **a** < 1, as the power decreases the resulting value decreases. For example, $\log_{0.5} 2 > \log_{0.5} 4$ because $2 = (0.5)^{-1}$, $4 = (0.5)^{-2}$, and $-1 > -2$.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 2 at the end of this lesson.

3. Explore several different *a*-values by clicking ∆ or ∇.

   a. For each **a**-value, identify the *x*-intercept of the function. Interpret your results.

   Answer: The *x*-intercept is always (1, 0) because $\log_{\mathbf{a}} 1 = 0$ for all **a**, which can be written in exponential form $\mathbf{a}^0 = 1$.

   b. When **a** > 0, why is there no *y*-intercept?

   Answer: For there to be a *y*-intercept, then *x* would be 0. This implies that $\log_{\mathbf{a}} 0 = y$ or $\mathbf{a}^y = 0$, which is impossible.

   c. For each **a**-value, what part of point *P* remains the same? Interpret your results.

   Answer: For all **a**-values, the point *P* has the coordinates (**a**, 1) since $\log_{\mathbf{a}} \mathbf{a} = 1$ can be written in exponential form $\mathbf{a}^1 = \mathbf{a}$.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 3 at the end of this lesson.

4. Explore several different *a*-values by clicking ∆ or ∇, such that **a** > 1.

   a. What does **f**(*x*) approach as *x* approaches ∞? Explain.

Answer: ∞ ; As you input larger and larger positive values for *x*, **f**(*x*) gets larger and larger.

b. What does **f**(*x*) approach as *x* approaches 0? Explain.

Answer: $-\infty$; As you input positive *x*-values that approach 0, **f**(*x*) gets smaller and smaller.

c. What is the equation of the vertical asymptote?

Answer: $x = 0$

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 4 at the end of this lesson.

5. Explore several different **a**-values by clicking ∆ or ∇, such that 0 < **a** < 1.

a. What does **f**(*x*) approach as *x* approaches ∞ ? Explain.

Answer: $-\infty$; As you input larger and larger positive values for *x*, **f**(*x*) get smaller and smaller.

b. What does **f**(*x*) approach as *x* approaches 0? Explain.

Answer: ∞ ; As you input positive *x*-values that approach 0, **f**(*x*) gets larger and larger.

c. What is the equation of the vertical asymptote?

Answer: $x = 0$

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 5 at the end of this lesson.

6. Find the domain and range for the family of logarithmic functions $\mathbf{f}(x) = \log_{\mathbf{a}} x$ where **a** > 0 and **a** ≠ 1.

Answer: The domain is $(0, \infty)$, and the range is $(-\infty, \infty)$.

TI-Nspire Navigator Opportunity: *Quick Poll* and *Live Presenter*
See Note 6 at the end of this lesson.

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7. Gail believes $\mathbf{f}(x) = \log_{\mathbf{a}} x$ will eventually intersect the *y*-axis. Is she correct? Why or why not?

Answer: Gail is incorrect. The function will never cross the *y*-axis. It will only approach the *y*-axis.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 7 at the end of this lesson.

8. Judy believes $\mathbf{f}(x) = \log_{\mathbf{a}} x$ has a horizontal asymptote. Is she correct? Why or why not?

Answer: Judy is incorrect. When $x > 1$, the function decreases very slowly when 0 < **a** < 1 and increases very slowly when **a** > 1 .

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 8 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand that for the graph of $f(x) = \log_a x$:

- The conditions $a > 0$ and $a \neq 1$ are necessary.
- When *a* > 1, the function is increasing and when $0 < a < 1$, the function is decreasing.
- The *x*-intercept is always (1, 0) and there is never a *y*-intercept.
- When *a* > 1, the function approaches ∞ as *x* approaches ∞, and the function approaches $-\infty$ as *x* approaches 0.
- When 0 < *a* < 1, the function approaches $-\infty$ as *x* approaches ∞, and the function approaches ∞ as *x* approaches 0.
- The domain is $(0, \infty)$ and the range is $(-\infty, \infty)$.
- The function has a vertical asymptote of $x = 0$.

TI-Nspire Navigator

Note 1

Question 1, *Live Presenter:* You may want to demonstrate how to change the *a*-values by clicking the arrows using *Live Presenter*.

Note 2

Question 2a and 2b, *Screen Capture:* Take a *Screen Capture* of page 1.2 where students are on different *a*-values. As a class, discuss the various cases that occur.

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Note 3

Question 3a and 3b, *Screen Capture:* Take a *Screen Capture* of page 1.2 where students are on different *a*-values. As a class, discuss the various cases that occur.

Note 4

Question 4a–4c, *Screen Capture:* Take a *Screen Capture* of page 1.2 where students are on different *a*-values. As a class, discuss the various cases that occur.

Note 5

Question 5a–5c, *Screen Capture:* Take a *Screen Capture* of page 1.2 where students are on different *a*-values. As a class, discuss the various cases that occur.

Note 6

Question 6, *Quick Poll (Open Response):* Send two *Open Response Quick Polls*, asking students to submit their domain and range. If students struggle to identify the domain and range, consider taking a *Screen Capture* and discussing how, for all of the graphs, the possible *x*- and *y*-values are similar.

Note 7

Question 7, *Quick Poll (Open Response):* Send an *Open Response Quick Poll*, asking students to submit their answer to question 7.For students having difficulty, use Live Presenter and change the *a*-value to 0.9 and the *y*-max to 200. Repeatedly zoom in on the "*y*-intercept."

Note 8

Question 8, *Quick Poll (Open Response):* Send an *Open Response Quick Poll*, asking students to submit their answer to question 8.

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