Graphing Quadratic Functions

ID: 9186

Time required
60 minutes

Activity Overview

In this activity, students will graph quadratic functions and study how the constants in the equations compare to the coordinates of the vertices and the axes of symmetry in the graphs. The first part of the activity focuses on the vertex form, while the second part focuses on the standard form. An optional extension allows students to see how the two forms are related.

Topic: Quadratic Functions & Equations

- *Determine the vertex, the zeros, and the equation of the axis of symmetry of the graph of the quadratic function* $y = ax^2 + bx + c$.
- *Determine the vertex, the zeros, and the equation of the axis of symmetry of the graph of* $y = a(x – h)^2 + k$

Teacher Preparation and Notes

- *This activity is designed to be used in an Algebra 1 classroom. It can also be used as review in an Algebra 2 classroom.*
- *Problem 1 introduces students to the vertex form of a quadratic equation, while Problem 2 introduces the standard form. You can modify the activity by working through only one of the problems.*
- *If you do not have a full hour to devote to the activity, work through Problem 1 on one day and then Problem 2 on the following day. Each problem includes a game that can be skipped or extended to fit your schedule.*
- *This activity is intended to be mainly* ***teacher-led****, with breaks for individual student work. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "9186" in the keyword search box.***

Associated Materials

- *GraphQuadratic_Student.doc*
- *GraphQuadratic.tns*
- *GraphQuadrati_Soln.tns*

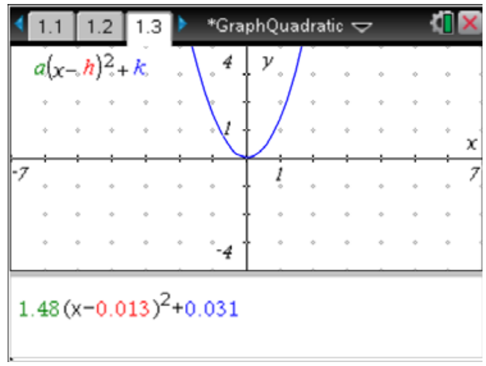
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange *and enter the number in the keyword search box.*

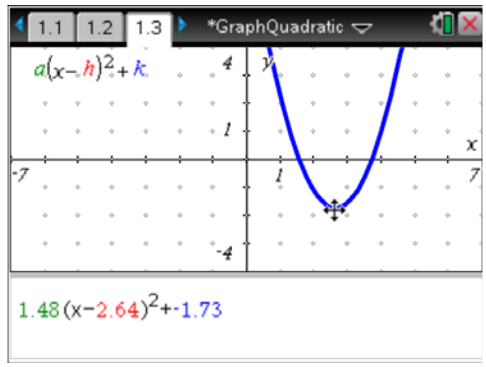
- *Curve Shifters (TI-Nspire Technology) — 13057*
- *An Application of Parabolas (TI-Nspire Technology) — 13364*
- *Quadratic Transformation with Sliders (TI-Nspire Technology) — 13727*

Problem 1 – Vertex form

On page 1.3, Ask students to describe the shape of the graph. Be sure to mention that this curve is called a *parabola*.



Moving the cursor near the vertex of the parabola (so that the cursor looks like this: (✥), have them press ctrl . Now moving the cursor translates (slides) the parabola. Have them observe the changes in the equation. Explain that the equation is written in what is called the ***vertex form*** of a quadratic equation: $y = a(x - h)^2 + k$. For now, students will look at functions for which $a = 1$.

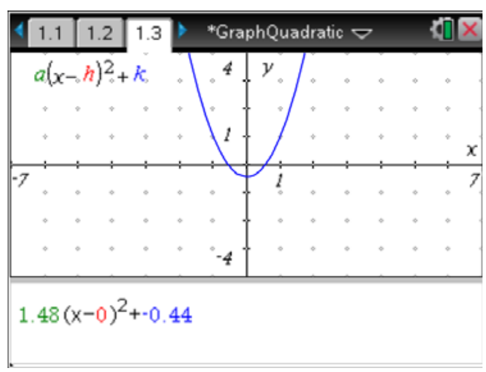


As students move the parabola, have them pay attention to the changing value of *h*. Ask: *When is h positive? negative? When does the absolute value of h get larger? smaller?*

TI-Nspire Navigator Opportunity: *Live Presenter*

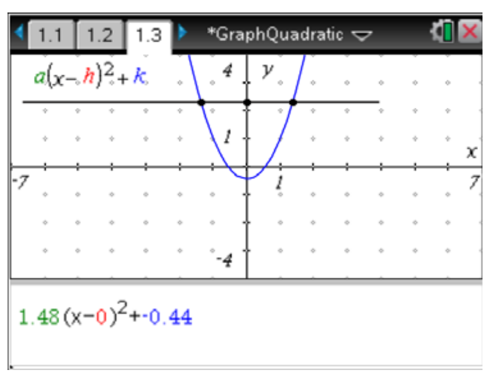
See Note 1 at the end of this lesson.

Ask students what they think will happen to the parabola if *h* was zero. Then have students drag the parabola around until h is zero. All student graphs should have a parabola centered about the *y*-axis.

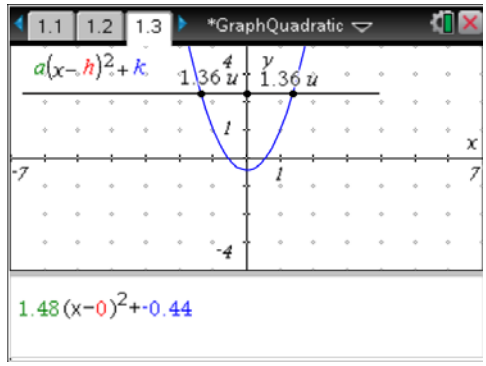


Students are to use the **Parallel** tool from the Construction menu to display a line parallel to the *x*-axis such that it intersects the parabola twice. Next, have them create the intersection points (**MENU > Points & Lines > Intersection Point(s)**) for where the parallel line intersects with the parabola and the *y*-axis.

Be sure students' graphs still represent a quadratic function with $h = 0$.



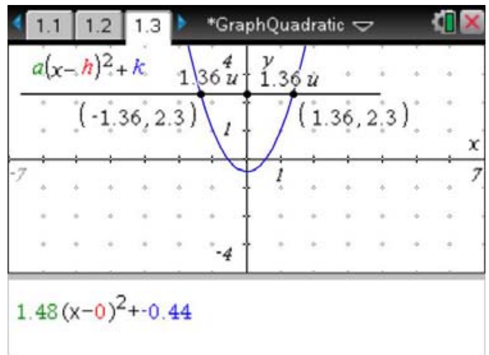
Next, use the **Length** tool from the Measurement menu to measure the distances from each point on the parabola to the point on the *y*-axis.



As they grab and drag the intersection point of the parallel line and the *y*-axis to shift the parallel line up and down, ask students what they notice (The lengths just measured are always equal.)

Caution students to select the intersection point and not the parabola. The parabola should remain unchanged; only the parallel line should be moved.

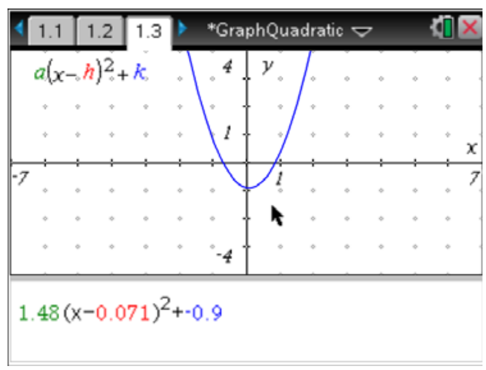
Use the **Coordinates and Equations** tool from the Actions menu to display the coordinates of the points where the line intersects the parabola.



Ask how the distances from the *y*-axis and the coordinates are related. *Why does the absolute value of each x-coordinate equal the measured length? Why are the y-values the same?*

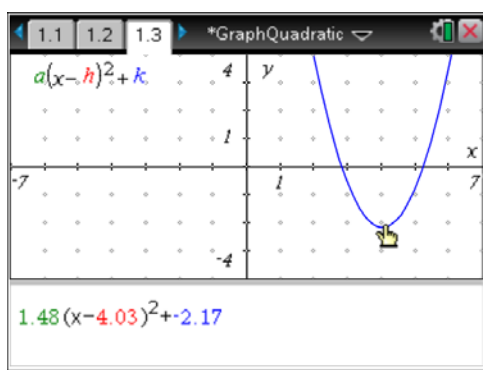
Discuss how the graph is symmetric and how the *y*-axis is the *axis of symmetry* when *h* = 0. Have students write an equation for this axis of symmetry (*x* = 0).

To continue exploring the graph and its equation, students should return the screen as it was just before creating the parallel line. They may do so by pressing ctrl Z (undo) repeatedly, or by clicking once on the line parallel to the *x*-axis and pressing del. This deletes the line, and every subsequent construction that was based off of the line.



Ask students which way they think the graph will move if *h* is changed from 0 to 4. (Students may need to be reminded that the negative sign is part of the general equation.)

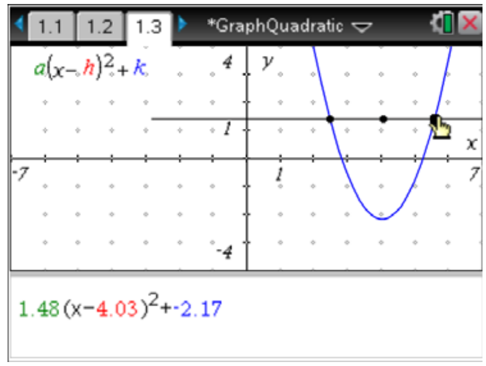
Have them drag the equation by the vertex so that *h* is 4 (it may be difficult to become exactly 4, but have the students get as close as they can.)



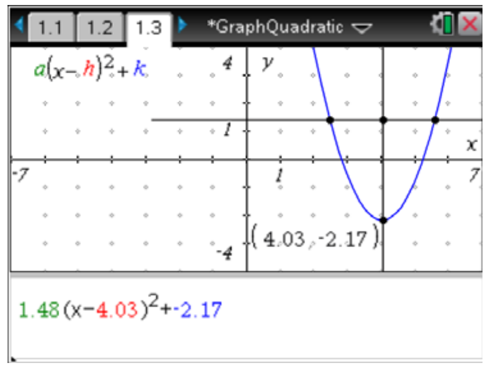
TI-Nspire Navigator Opportunity: ***Screen Capture***

See Note 2 at the end of this lesson.

As before, have students draw a line parallel to the *x*-axis that passes through the parabola twice, plotting the intersection points. Then, they will find the midpoint of the segment connecting the two intersection points. Once students select the **Midpoint** tool (from the Construction menu), clicking once on each endpoint/intersection point will place the midpoint.

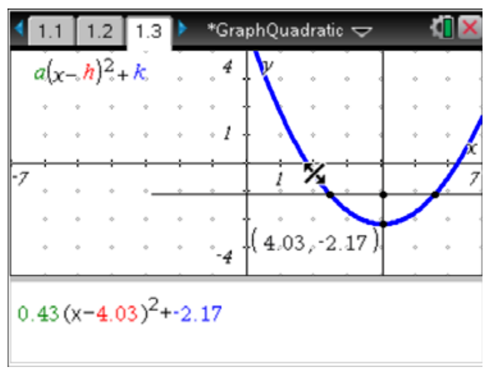


Next, display the axis of symmetry by constructing the perpendicular line (**MENU > Construction > Perpendicular**) through the midpoint. Then have them construct the intersection of the perpendicular line with the parabola.



Explain that this point is the ***vertex*** of the parabola and since the vertex is the lowest point on their graph, it is also a ***minimum***. Labeling the coordinates of the vertex and translating the parabola, ask how the vertex is related to the general equation. (coordinates are (h, k))

Now, have students adjust the "width" of the parabola by moving their cursor over the upper part of the parabola (so the cursor looks like this:).



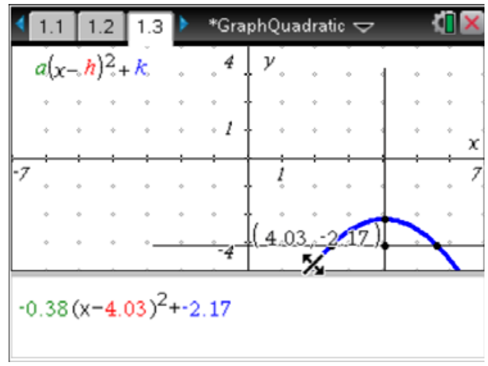
Pressing ctrl , they can begin to drag and stretch the parabola. Depending on which way they move it, the parabola will open or close. Ask students what changes in the equation as they do this (the value of a). Let students continue to change the parabola and answer questions such as, *What happens if 'a' is positive? negative? What happens when 'a' gets larger? smaller?*

Discuss that when the graph opens downward (when a is negative), the vertex is a ***maximum*** because it is the highest point on the graph.

TI-Nspire Navigator Opportunity: *Quick Poll*

See Note 3 at the end of this lesson.

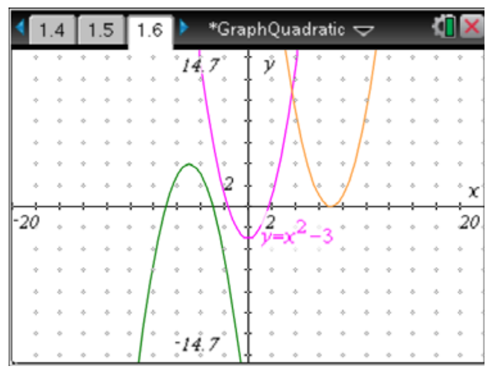
They can now display the equation of the axis of symmetry by using the **Coordinates and Equations** tool. Allow time for students to move and change their parabolas, observing how the equation changes with it. Circulate around the room and assist as needed.



Once students understand how an equation in vertex form is related to the graph of the function, they can sketch the graphs of the three functions shown on their worksheet. Stress that this is only a sketch, and will not be exact, although they should be able to place the location of the vertex exactly. While they should know the direction of the parabola (up or down) from *a*, the exact width should be an educated guess.

(They could locate several points by substituting values, but the purpose of this activity is to learn general characteristics of parabolas.)

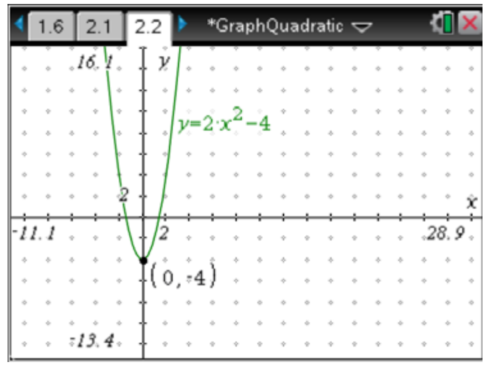
After creating their sketches, they may check their work by graphing the functions on page 1.6 and graphing the functions. Entering each rule in a text box (**MENU > Actions > Text**) and dragging the expression to the *y*-axis will display its graph.



If time permits, students can pair up to play a graphing game. Clearing the graphs on page 1.6, each student should graph any quadratic function (written in vertex form, where *a*, *h*, and *k* are integers) and then hide the equation (**MENU > Actions > Hide/Show**). With only the parabola showing, each student should then trade handhelds with their partner. Now, they should make their best guess for the equation of the function that is graphed. They should graph their guess and see how close they are.

Students that make a match in one try get 3 points; two tries, 2 points; three tries, 1 point. To confirm that their graphs match up exactly, students can use the **Hide/Show** tool again to reveal the hidden equation.

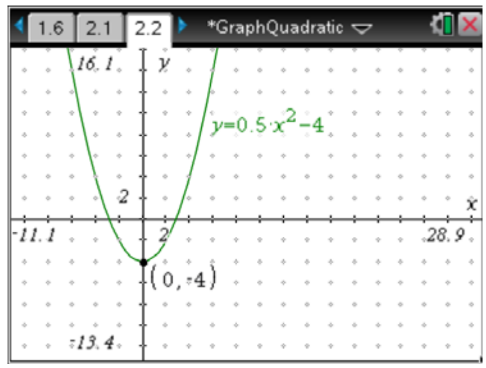
Problem 2 – Standard form



On page 2.2, students should open the text box, substitute values for *a* and *c*, and drag the expression to the *x*-axis, as before.

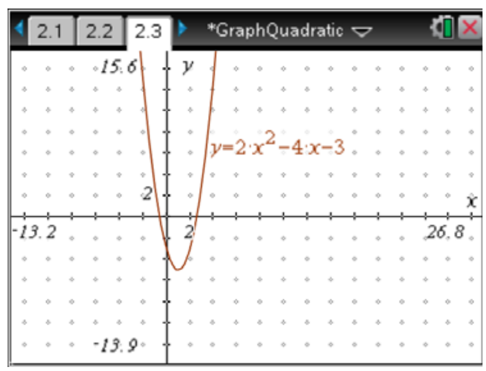
Next, have them construct the vertex point by finding the intersection of the parabola and *y*-axis, also displaying its coordinates.

Then have them return to the equation and change the value of the constant term a few times, encouraging them to try both positive and negative values. Ask, *How is the y-intercept related to the equation?*



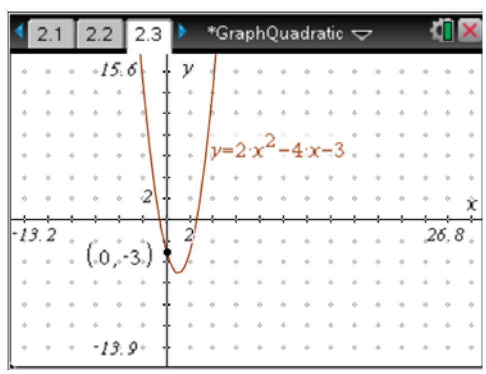
Now ask how they think changing the coefficient of x^2 might affect the parabola. Have them do so, prompting them to observe that the vertex remains unchanged while the parabola "opens and closes."

Discuss how the coefficient of x^2 is related to the shape of the parabola.



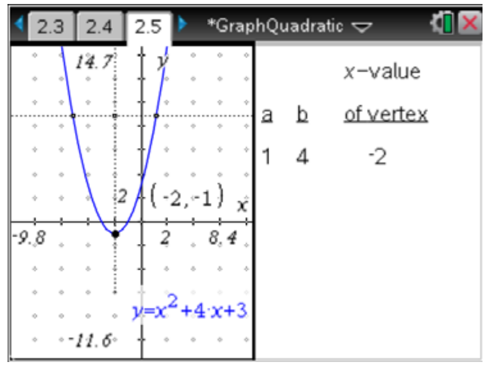
On page 2.3, students will see the ***standard form*** of a quadratic equation, $y = ax^2 + bx + c$. Ask, *Is your equation on page 2.2 in standard form?* (Yes, $b = 0$).

Students are to substitute values for *a*, *b*, and *c* and observe the changes in the parabola.



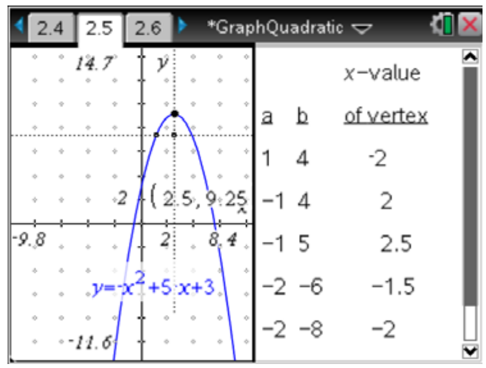
Have students plot and label the coordinates of the *y*-intercept of a new parabola. Then have them change the values of *a*, *b*, and/or *c*. Ask them to formulate a conjecture about the equation of a quadratic function in standard form and the *y*-intercept of the graph of the function. (*c* is the *y*-intercept)

On page 2.5, students will explore the coordinates of the vertex, and therefore the axis of symmetry. On the left, a quadratic function is graphed and the vertex and equation are displayed. On the right, the values of *a*, *b*, and the *x*-value of the vertex are shown.



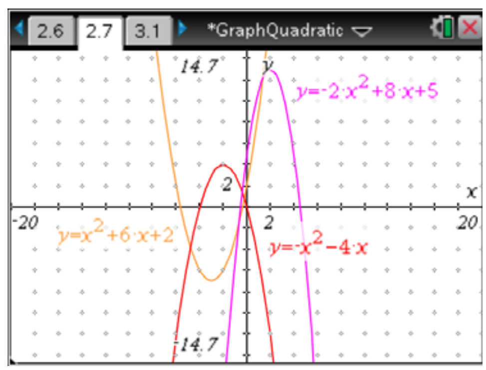
Students will change the values of *a* and *b* by double-clicking the equation and modifying the equation. They should observe how the coordinates of the vertex change and track these values on the right side of the page. Make sure they test positive and negative values, and also values of *a* other than 1 and –1.

Ask students how the *x*-value of the vertex can be found from the values of *a* and *b* in the equation (take the opposite of *b* divided by twice *a*, and have them write a general equation for the axis of symmetry $\left(x=-\dfrac{b}{2a}\right)$.



Then ask how the *y*-value of the vertex can be determined (substituting the *x*-value into the equation).

After mastering the exploration of standard form, students should sketch the graphs of the three functions shown on their worksheet and then check their work by graphing the functions on page 2.7.



If time permits, students can pair up to play the guessing game with quadratic functions written in standard form on page 2.7.

Extension

Have students expand the vertex form of a general quadratic function and group the terms so that they can compare it to the standard form. They should use their results to explain why $x = \frac{b}{-2a}$ and why c is the y-intercept.

$$y = a(x-h)^2 + k$$
$$y = a(x^2 - 2xh + h^2) + k$$
$$y = \boxed{a}x^2 \boxed{-2ah}x + \boxed{ah^2 + k}$$

$$y = \boxed{a}x^2 + \boxed{b}x + \boxed{c}$$

$$b = -2ah$$
$$h = -\frac{b}{2a}$$
x-coordinate of vertex is $-\frac{b}{2a}$

When $x = 0$, $y = a(0-h)^2 + k$
$$y = a(-h)^2 + k$$
$$y = ah^2 + k$$

TI-Nspire Navigator Opportunities

Note 1

Problem 1, *Live Presenter*

Use *Live Presenter* so that as students move the parabola, they can demonstrate the affect the value of *h has on the graph*. Ask: *When is h positive? negative? When does the absolute value of h get larger? smaller?*

Note 2

Problem 1, *Screen Capture*

Use *Screen Capture* for the students to help see the effect the value of h has on the graph. Ask students what they see in common among all the displayed graphs.

Note 3

Problem 1, *Quick Poll*

Send multiple *Quick Polls* asking students What happens if ‘a’ is positive? negative? What happens when ‘a’ gets larger? smaller?