Names ___________________________

Midsegment Exploration

1. Open the document *midsegment.tns* from the *MyDocuments* folder on the Home Screen. Follow the directions in the file and answer the questions on this sheet.

   Rename the file with your initials at the front of the file name. For example, *rbcmidsegment.tns*

2. Move to page 1.2 of the document. Segment $\overline{MP}$ was constructed by connecting the midpoints of $\overline{AB}$ and $\overline{AC}$. Grab a vertex of the triangle and drag it to another location. $\overline{MP}$ is called a midsegment.

   A *midsegment* of a triangle is a segment connecting the midpoints of two sides.

   What do you observe? $\overline{MP}$ moves as the vertices move. The length of the segment changes as the lengths of the sides of the triangle change.

3. Move to page 1.3 of the document. Find the slope of each side of the triangle and each midsegment. Follow the directions from the teacher on how to find the slope. Grab a vertex of the triangle and observe the slopes of each side of the triangle and each midsegment.

   What conclusion can you make? The midsegment is parallel to the third side of the triangle because the slopes are equal. (the side not containing either midpoint.)

4. Move to page 1.4 of the document. Find the length of each side of the triangle and each midsegment. Follow the directions from the teacher on how to measure the length of the segments. Grab a vertex of the triangle and observe the length of each side of the triangle and each midsegment.

   What conclusion can you make? The length of the midsegment is one-half of the length of the third side.

5. Move to page 1.5 of the document. Find the area of ΔABC and ΔAMP. Grab a vertex of the triangle and observe the relationship between the areas.

   What conclusion can you make about the area of the two triangles? The area of triangle AMP is one-fourth of the area of triangle ABC.

6. Move to page 1.6 of the document. Find the area of ΔABC and ΔCNP. Grab a vertex of the triangle and observe the relationship between the areas. The area of triangle CNP is one-fourth of the area of triangle ABC.

7. Move to page 1.7. Find the area of ΔABC and ΔBMN. Grab a vertex of the triangle and observe the relationship between the areas. The area of triangle BMN is one-fourth of the area of triangle ABC.

8. Using the information from questions 5-7, what conclusion can you make about the area of the smaller triangle and the larger triangle? The area of the smaller triangle formed by the midsegment and one of the vertices of the triangle is one-fourth the area of the larger triangle.

9. If *b* represents the base and *h* represents the height in triangle ABC, show an algebraic justification of your conclusion in question 8. ½ b = length of the base of the "midsegment" triangle
   ½ h = length of the height of the "midsegment" triangle
   Area of midsegment triangle = ½ (½ b) (½ h)
   = (1/8) bh
   ¼ of the area of the original triangle is ¼ (½ bh) = (1/8) bh which is the same as the area of the midsegment triangle.

10. Move to page 1.8 and find the area of each of the four small triangles and the area of the large triangle. Place the area of each triangle on the screen at a convenient location. Grab each vertex and move the triangle.

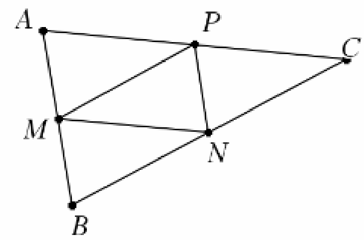
What observations can you make? All four triangles have the same area.

11. There are four triangles. Are the triangles congruent? Yes

12. If any triangles are congruent, mark all congruent segments and write the congruence relations indicated.

Students should mark segments AP, PC and MN congruent.
Students should mark segments AM, MB and PN congruent.
Students should mark segments BN, NC and MP congruent.

$\Delta AMP \cong \Delta MBN \cong \Delta PNC \cong \Delta NPM$



13. Explain your reasoning in question 12. Each triangle contains a side which is ½ the length of a side of the original triangle.

14. Roger is building a campground at Jordan Lake that is the shape of a triangle, ΔXYZ, where $\overline{YZ}$ is the base of the triangle. He wants to subdivide the camping ground into 3 smaller pieces, a quadrilateral for tent camping and two triangles for RV camping. He wants the area of the tent camping to be equal to the total area of the RV camping.

Roger first finds the midpoint M of side $\overline{XZ}$ and the midpoint N of side $\overline{XY}$. He placed a point P on side $\overline{YZ}$ but **not** at the midpoint. He forms the quadrilateral by connecting N and P and connecting M and P.

Move to page 1.9. Use the tools in the N-spire to investigate Roger's plan. Where should he locate point P?
Roger can locate point P anywhere along the side YZ.

Does the area of the tent camping equal the area of the RV camping? Justify your answer.

The RV camping area and the tent camping is the same. If point P was located at the midpoint then the two smaller triangles each are ¼ of the area of the total area. The total is ½ of the total area of the campground. If point P is located at any point along the base of the larger triangle, the height of the two triangles remains unchanged. The sum of the bases remains unchanged as well and therefore the sum of the areas of the two triangles remains the same as the area of the two triangles formed when P was the midpoint of the side.

15. Write an algebraic justification of your conclusion in problem 14.

Let b = length of MN (midsegment)
Let h = height of triangle XMN= height of triangle MNP (equal because MN is the midsegment of triangle XYZ
Area of parallelogram XNPM = area of triangle XMN + area of triangle MNP
= ½ bh + ½ bh
= bh

Length of YZ = 2b since b was the length of the midsegment which is parallel to YZ
Let x = length of YP
2b – x = length of PZ

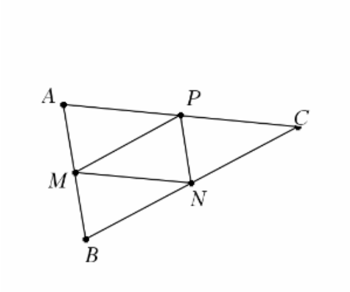
Area of triangle NYP = ½ xh and the area of triangle MPZ = ½ (2b –x)h
Area of RV camping = ½ xh + ½ (2b –x)h
= ½ xh + bh – ½ xh
= bh

Therefore, since both areas are equal to bh then RV and tent camping is the same and it does not depend on point P's location.

There are other algebraic justifications.

Follow up questions to Midsegment Investigation.

1. If M, P, and N are the midpoints of the three sides of ΔABC, how does the area of ΔMNP compare to the area of ΔABC? ¼ of the area



2. If the midpoints of the segments $\overline{MN}$, $\overline{PM}$, and $\overline{PN}$ are connected to form another triangle, how is the area of this second "midsegment triangle" related to the original triangle? It will be 1/16 of the area of the original.

3. If the process is repeated, how is the area of the third triangle related to the original triangle? It will be 1/64 of the area of the original

4. Let G represent the area of the original triangle. Write an expression to relate the area of the *nth* "midsegment triangle" to the area of the original triangle.

$$G_n = \left(\frac{1}{4}\right)^n G$$

5. How does the perimeter of ΔMNP compare to the perimeter of ΔABC? The perimeter is ½ of the perimeter of the original triangle.

6. If the midpoints of the segments, $\overline{MN}$, $\overline{PM}$, and $\overline{PN}$ are connected to form another triangle, how is the perimeter of this second "midsegment triangle" related to the original triangle? It will be ¼ of the perimeter of the original.

7. If the process is repeated, how is the perimeter of the third triangle related to the original triangle? It will be (1/8) of the perimeter of the original.

8. Let Q represent the perimeter of the original triangle. Write an expression to relate the perimeter of the *nth* "midsegment triangle" to the perimeter of the original triangle.

$$Q_n = \left(\frac{1}{2}\right)^n Q$$