2. Find the x-intercept of this tangent line.

$\left(\frac{x_1}{2},0\right)$

We will call this point G.

3. Recall that the definition of a parabola is the locus of points in a plane which are equidistant from a given point, called the focus, and a given line, called the directrix. Let f be the distance from the focus to the vertex. Let the focus be point F with coordinate (0,f). Let Q be a point on the directrix such that $\overline{PQ}$ is perpendicular to the directrix. Point Q will have coordinates $(x,-f)$ ____________________

4. Use the Midpoint Formula to find the midpoint of $\overline{FQ}$ and show that the midpoint of $\overline{FQ}$ is point G.

$$\left(\frac{0+x_1}{2},\frac{f+(-f)}{2}\right)=\left(\frac{x_1}{2},0\right)$$

5. Since G is the midpoint of $\overline{FQ}$, $\overline{FG} \cong \overline{QG}$.

6. By the definition of a parabola, P is equidistant from F and Q, so $\overline{PF} \cong \overline{PQ}$.

7. $\overline{PG} \cong \overline{PG}$ by the __Reflexive___Property of Congruence.

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Proof (see Page 1.5)

1. Consider the parabola $y=ax^2$. Let the point P be a point on $y=ax^2$ with coordinates (x_1, y_1). Write the equation of the tangent line to $y=ax^2$ at the point $P(x_1,y_1)$.

$$y=\frac{2y_1}{x_1}(x-x_1)+y_1$$

2. Find the x-intercept of this tangent line.

$$\left(\frac{x_1}{2},0\right)$$

We will call this point G.

3. Recall that the definition of a parabola is the locus of points in a plane which are equidistant from a given point, called the focus, and a given line, called the directrix. Let f be the distance from the focus to the vertex. Let the focus be point F with coordinate (0,f). Let Q be a point on the directrix such that $\overline{PQ}$ is perpendicular to the directrix. Point Q will have coordinates $(x,-f)$ ____________________

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6. By the definition of a parabola, P is equidistant from F and Q, so $\overline{PF} \cong \overline{PQ}$.

Reflective Property of Parabolas

7. $\overline{PG} \cong \overline{PG}$ by the __Reflexive____Property of Congruence.

8. Δ FPG $\cong$ Δ QPG by the Side Side Side Postulate.

9. $\angle FPG \cong \angle$ QPG because corresponding parts of congruent triangles are congruent.

10. Let R be a point on $\overleftrightarrow{PQ}$, and T be a point on $\overleftrightarrow{PG}$ (see Page 1.5). $\angle RPT \cong \angle$ QPG because vertical angles are congruent.

11. From Step 9, $\angle QPG \cong \angle FPG$. Therefore $\angle$ RPT $\cong$ $\angle$ FPG by the Transitive Property of Congruence.

12. Therefore any light ray that travels down a vertical line parallel to the axis of symmetry will bounce off of the parabola at point P and be reflected towards point F, the focus of the parabola.