‘Value’able Theorems

ID: 11202

Time Required
10-15 minutes

Activity Overview

This activity is a teacher-led demonstration of the Intermediate Value Theorem and the Extreme Value Theorem from a graphical perspective. The function y1(x) is a continuous function on a finite closed interval [a, b]*. Teachers can use and move a horizontal line to show the value of k and graphically demonstrate that there is a value c on the interval [a, b] such that y1(c) = k. For the Extreme Value Theorem, two graphs are provided to contrast and show an example.*

Topic: Calculus Theorems

- *Intermediate Value Theorem*
- *Extreme Value Theorem*

Teacher Preparation and Notes

- *This activity is designed for use as a teacher demonstration. However, it may be used as a student exploration.*
- *Students are introduced to the Intermediate Value Theorem. The teacher can use a horizontal line to change the value of* **k** *and demonstrate that for every value of* **k** *in between* **y1**(**a**) *and* **y1**(**b**) *there is a value* **c** *on* [**a,b**] *such that* **y1(c) = k**.
- *Students are also introduced to the Extreme Value Theorem. For each figure, the teacher should ask whether or not the given function fulfills the hypothesis of the Extreme Value Theorem.*
- *Make sure that the operating system of all TI-89 Titanium calculators is version 3.10. To check the version of on the calculators, go to the Apps screen and select* ***F1:Menu > 3:About…****. To download the newest version of the TI-89 OS, go to education.ti.com..*
- ***To download the student worksheet, go to education.ti.com/exchange and enter “11202” in the keyword search box.***

Associated Materials

- *ValueableTheorems_Student.doc*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Absolute Extrema (TI-89 Titanium) — 3264*
- *Finding Extreme Values (TI-89 Titanium) — 3244*
- *Extrema Using Derivatives (for TI-89 Titanium) — 9413*
- *Functions and Their Extrema (TI-89 Titanium) — 6436*

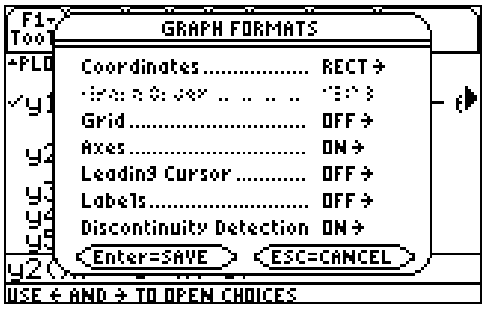
Problem 1 – Intermediate Value Theorem

Before students arrive to class, the following equation should be entered in the calculator:

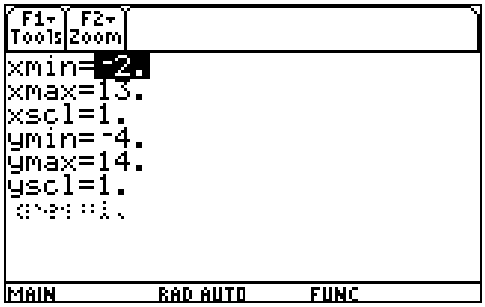
y1 = 0.15 * $(x - 7.5)^3$ + 0.6 * $(x - 6)$^2 – 2.1 * $(x - 6)$ + 5 | 2.5 ≤ x ≤ 10.5

Note: the value of *a* for this graph is 2.5 and *b* is 10.5.

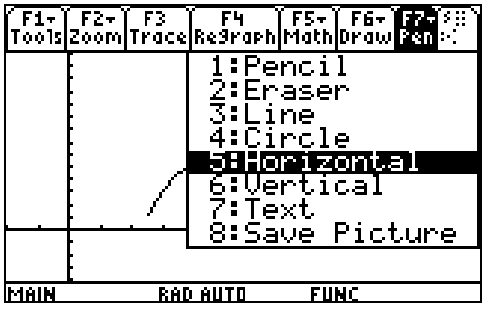
Make sure **Discontinuity Detection** is turned on. This can be found by pressing ◆ + | (the green diamond key and the vertical bar, or "such that," key) while in the *y*= screen or the graphing screen.



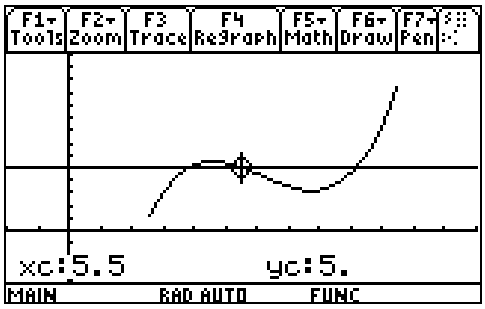
The graphing window should be set to the specifications shown to the right.



Begin the class by introducing the Intermediate Value Theorem. Demonstrate the Intermediate Value Theorem by using a horizontal line to change the value of *k.* To draw a horizontal line, press 2nd + F7, select **7:Horizontal**, and move the line up and down. A horizontal line can be dropped by pressing ENTER.



Students will see that for every value of *k* in between ***y*1**(*a*) and ***y*1**(*b*) inclusive there is a value *c* on [*a*, *b*] such that ***y*1**(*c*) = *k*. The teacher should ask the students why the function must be continuous on [*a*, *b*].



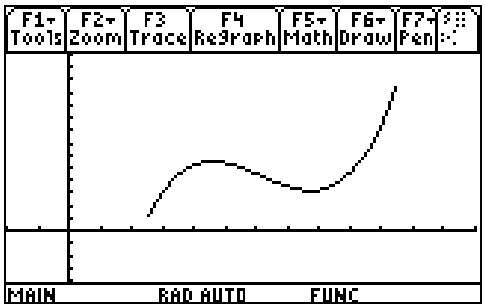
Student Solutions

1. Sample answer: If the function was not continuous, like a step function, the function would not have to take on all values between ***y*1**(*a*) and ***y*1**(*b*).
2. Sample answer: $3.16457 \le k \le 5.45766$. This does not contradict the Intermediate Value Theorem because the theorem states that there is at least one value, *c*. There may be more than one value of *c* on the given interval.

Problem 2 – Extreme Value Theorem

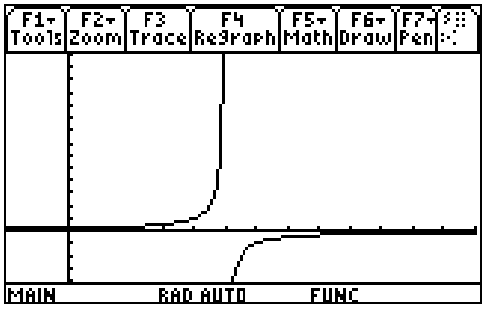
Introduce the Extreme Value Theorem.

Display or sketch an example that satisfies the conditions of the extreme value theorem (for example, show the graph of the function in ***y*1**).



Students should sketch at least two non-examples of the theorem and have them explain what condition of the EVT is not satisfied.

Demonstrate that the hypothesis is not met if the function is defined of the interval $2.5 < x \leq 10.5$ (not a closed interval).



Graph ***y*2=** $\frac{-1}{x-5}$ and ask students which of the hypotheses of the Extreme Value Theorem is not met.

Student Solutions

3. Students should describe a function that is continuous on a given interval and
4. ***y*2** is not continuous on any interval containing $x = 5$ because the function is not continuous at $x = 5$.