Secant and Tangent Lines

ID: 11141

Time Required
15 minutes

Activity Overview

In this activity, students will observe the slopes of the secant line and tangent line as point Q on the function approaches the other point P. They will also, determine the average rate of change for an interval and approximate the instantaneous rate of change using the slope of the secant line.

Topic: The Concept of the Derivative

- *Average Rate of Change of a Function*
- *Instantaneous Rate of Change of a Function*

Teacher Preparation and Notes

- *Students are introduced to interpreting the slope of the secant line as the average rate of change of a function over a given interval and that the slope of the secant line approaches the slope of the tangent line as point Q approaches point P. Students are also introduced to the concept of the slope of the tangent line representing the instantaneous rate of change of a function at a given value of x. The instantaneous rate of change of a function can be estimated by the slope of the secant line. This estimation gets better the closer point Q gets to point P. (Note: This is only true if f1(x) is differentiable at point P.*
- *If teachers use this activity with more than one class, make sure that students DO NOT save the TI-Nspire document after moving point Q.*
- *The answers to the open response questions can be seen if the student TI-Nspire document is opened using the TI-Nspire Teacher Edition software.*
- ***To download the student TI-Nspire document (.tns file) and student extension worksheet, go to education.ti.com/exchange and enter "11141" in the quick search box.***

Associated Materials

- *SecantTangentLines.tns*
- *SecantTangentLines_Student.doc*

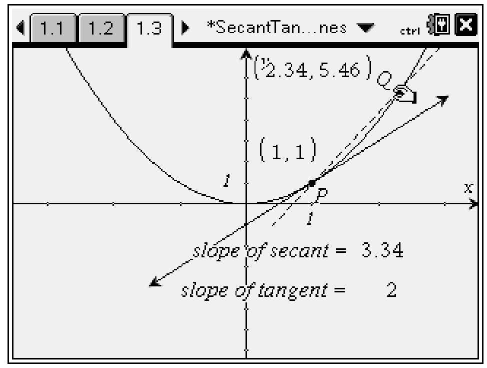
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *The Tangent Line Problem (TI-Nspire technology) — 8315*
- *The Mean Value Theorem (TI-Nspire technology) — 9896*

Part 1 – Graph of Secant and Tangent Lines

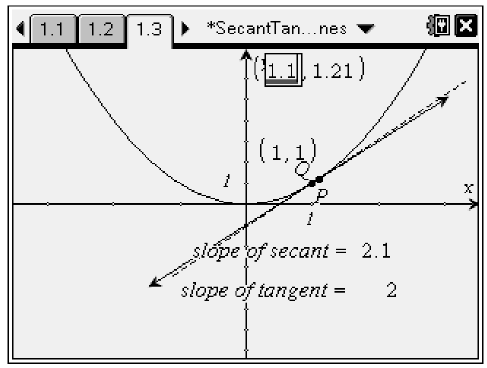
On page 1.3, the graph of $y = x^2$ is shown. Students are to grab point Q and move it toward point P. They are then asked to make an observation about the lines and the values of the slopes.



Students should see that the slope of the secant line approaches the slope of the tangent line as point Q approaches point P.

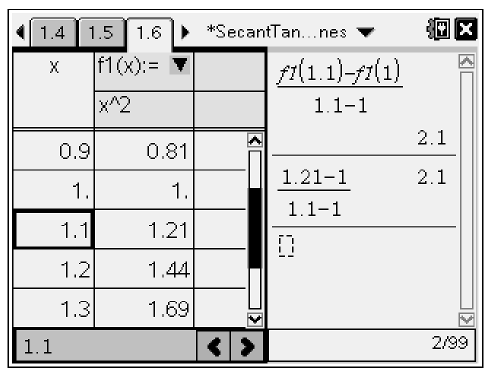
Part 2 – Finding average rate of change

Students are asked to determine the value of the average rate of change of **f1**(x) on the interval [1, 1.1] two different ways.



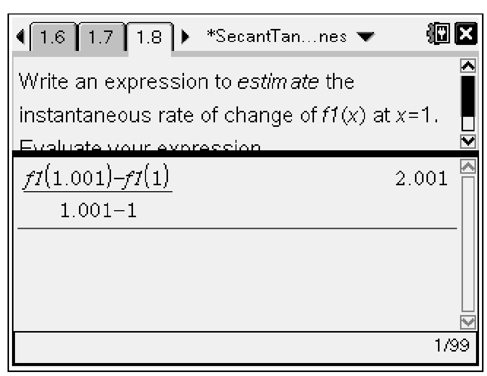
First, they need to go back to page 1.3 and change the x-coordinate of point Q to 1.1. The slope of the secant line is the average rate of change.

Then, students are to move to page 1.6 and need to use the table of values of **f1**(x) and the *Calculator* application. They should see that they get the same answer.



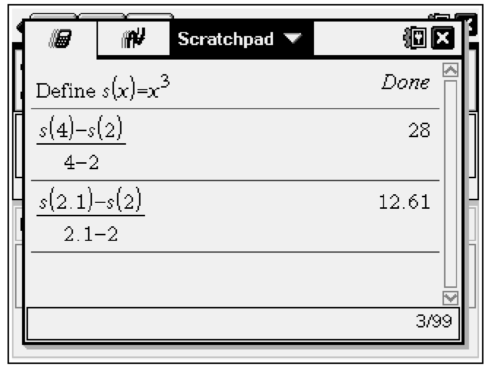
Part 3 – Finding instantaneous rate of change

On page 1.8, students are to use the calculator portion of the page to write an expression to estimate the instantaneous rate of change of **f1**(x) at $x = 1$. They need to choose a point close to $x = 1$, such as $x = 1.001$ to get a good estimate.



Extension – Rectilinear Motion Application

Students can use *Scratchpad* or insert a new problem (ctrl + ▲ menu and select **Insert Problem**) on the .tns file and work on *Calculator* pages to complete the extension. Before beginning, they will need to define the functions to make the calculations easier.



You may need to introduce the terms *average velocity* and *instantaneous velocity* for the first two questions and introduce the terms *average acceleration* and *instantaneous acceleration* for the last two questions.

Student Worksheet Solutions

1. $\dfrac{(s(4)-s(2))\text{ m}}{(4-2)\text{ s}}=\dfrac{(64-8)\text{ m}}{2\text{ s}}=28$ meters per second

2. $\dfrac{(s(2.1)-s(2))\text{ m}}{(2.1-2)\text{ s}}=\dfrac{(9.261-8)\text{ m}}{0.1\text{ s}}=12.61$ meters per second

3. $\dfrac{(v(4)-v(2))\ \frac{\text{m}}{\text{s}}}{(4-2)\text{ s}}=\dfrac{(48-12)\ \frac{\text{m}}{\text{s}}}{2\text{ s}}=18$ meters per second^2

4. $\dfrac{(v(2.1)-v(2))\ \frac{\text{m}}{\text{s}}}{(2.1-2)\text{ s}}=\dfrac{(13.23-12)\ \frac{\text{m}}{\text{s}}}{0.1\text{ s}}=12.3$ meters per second^2