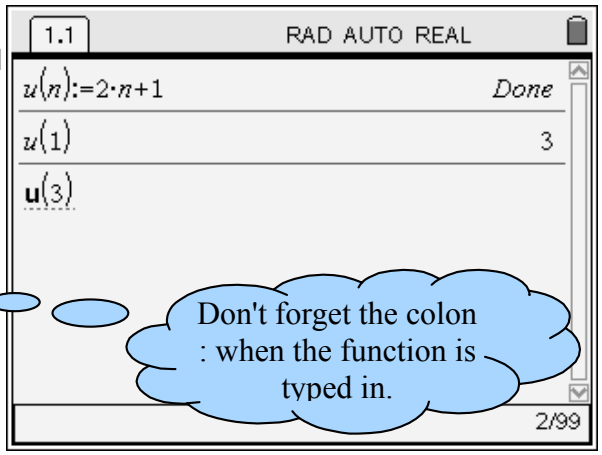
Sums of Infinite Series

Notes for Teachers

This is an activity which can be used across one or two lessons. These notes are intended as guidance for teachers. There are instructions for students which the teacher should give, together with suggested discussion points for the teacher. Also, key vocabulary is suggested.

Getting Started

Students should create a new blank document and save it as: **sums**



Starter Activity

Students should create a new calculator page and define a function u(n):=2n+1

Teacher Questions
What is u(1), u(3), u(102) .... ?

Students should make a table in their exercise books to see the sequence generated by this function:

n	1	2	3	4	5	6	n
u(n)							

Students should invent a new function and call it v(n). Then make a table to show the sequence it generates.

They should ake more functions ... w(n), x(n) etc.

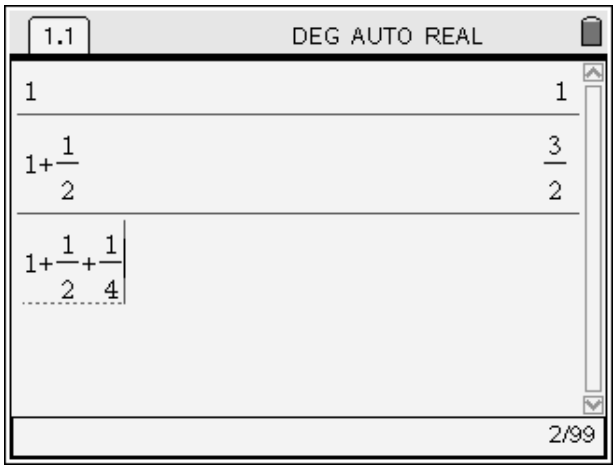
Vocabulary:

- We read “u of 3” to mean what we get when we **substitute** 3 for n in the **function**.
- n is the position of the term in the sequence.
- The letter u is the most common to use for sequences.
- The 3rd **term** is u(3) which is ...

Main Activity

The idea is to investigate adding the terms of the sequence:

$$1, 1?\frac{1}{2}, 1?\frac{1}{2}?\frac{1}{4}, 1?\frac{1}{2}?\frac{1}{4}?\frac{1}{8}, etc.$$



Vocabulary
Properly this is called a **series** because we are adding up a collection of terms of the sequence:

$$\frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$$

Operation hints:
The handheld naturally reports the results as fractions. This is what is intended!
Use the fraction template (ctrl + ÷) to enter the fractions.
To get the next entry, press ▲ twice to highlight the previous entry, then press enter.
Now you can just add the extra fraction.

Part 1

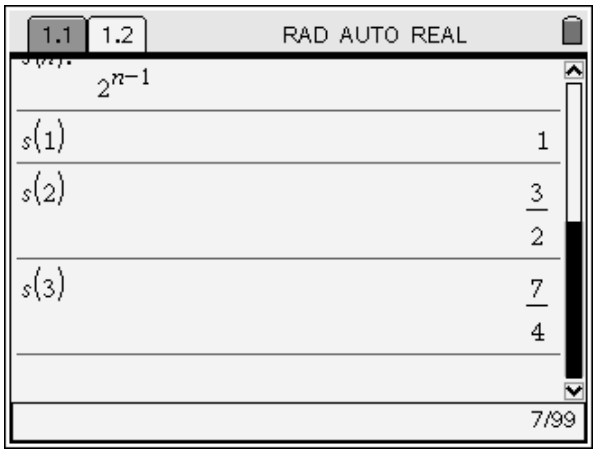
Students should make a table of their results (in exercise books):

n	1	2	3	4	5	6	7
	1	$1?\frac{1}{2}$	$1?\frac{1}{2}?\frac{1}{4}$	$1?\frac{1}{2}?\frac{1}{4}=\frac{1}{8}$	Etc.		
s(n)	1	$\frac{3}{2}$					

Teacher Prompts:
- Describe what you see in the sequence and the outcomes.
- Work out what the next term will be before your work it out.
- Check your answer with the handheld.
- Describe how to work out the numerator and denominator of the answers.

Part 2

Teacher Prompts:
- The aim is to devise a rule to work out the next number in the series.
- What would the 100th number be?
- Work out rules to work out the 100th denominator and numerator.
- Re-write your rules using n for the number of the term.



Students should define a new function

$$s(n) := \frac{(\text{your rule for the numerator})}{(\text{your rule for the denominator})}$$

Teacher Prompts:
Check that it gives the answers in your table by working out f(1), f(2) etc.
Check that your answer for the 100th term was correct by working out f(100)

The rules are both powers of 2 which need to be suitably modified so the term is correct.
It is best find the denominator first.
The numerator is 2^n-1
The denominator is $2^{(n-1)}$

Part 3

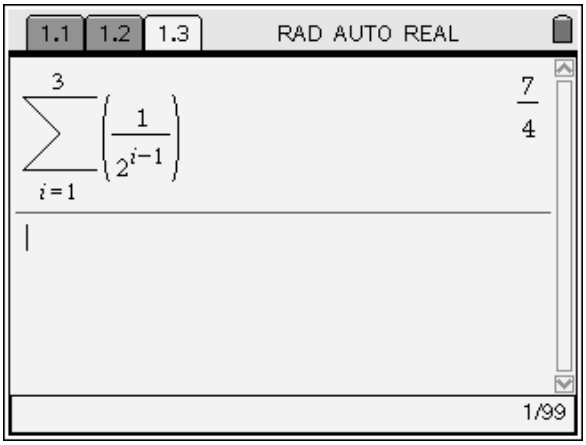
Teacher Prompts:

- What is the rule for the nth term of the original sequence $\frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \cdots$?
- We can use a special notation for adding up the terms of this sequence.
- This should give the same answers as you had in your table.

Students should use the template key (ctrl +) to enter this:

$$\sum_{i=1}^{3}\left(\frac{1}{\text{your rule for the sequence using n}}\right)$$

[Notice that it is common to use the letter *i* to distinguish this from the terms in the sequence].



Teacher Prompts:

- Check that it gives the same answer as the third term in your table.
- Check the other values in your tables.
- Check that it gives the same values as u(n).
- Check lots of different values (including large or strange ones!).

Part 4

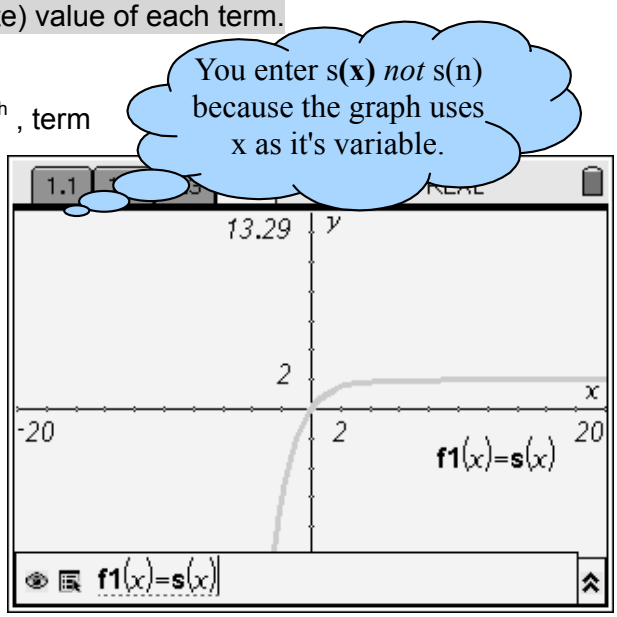
Teacher Prompts:
The fractional representation is very convenient for comparing with the sequence and the series. However the decimal representation makes comparisons of size easier.

- How much is each term worth?
- How much would the 10th term be?
- How much would the 100th term be?
- Discuss your ideas and write them down.

Students should add an extra row to their table and work out the value of each term.
Operation: use ctrl + enter to find the decimal (approximate) value of each term.

Teacher Prompts:

- What do you think the 10th, 100th, 1000th , 10000th , term will be now?
- Write down your ideas.



Students should open a new Graphs & Geometry page.
They should Graph **f1(x)=s(x)**
[that's the function created earlier].

They should use **menu/Window/Window settings** to see what happens when n gets very large.

Students can then write about their observations in their exercise books..