# *NUMB3RS* Activity: Two-Dimensional Trilateration
# Episode: “Convergence”

**Topic:** Locating GPS receivers **Grade Level:** 9 - 12

**Objective:** Students will use the TI-Navigator™ system along with 2-dimensional trilateration to determine the location of a GPS receiver.

**Time:** 25 - 30 minutes

**Materials:** TI-83/84 Plus graphing calculators, TI-Navigator system, and the activity settings file *Trilateration.act*.
To download this file, go to **http://education.ti.com/exchange** and search for “6583.”

## Introduction

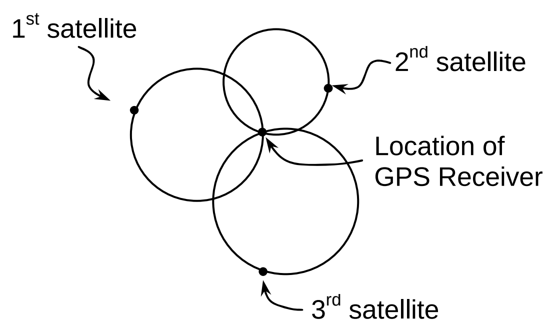
In “Convergence,” a string of thefts of high-priced collectables is being investigated. One of the mysteries surrounding the case is that the thieves seemed to know the exact locations of the victims. Charlie deduces that the criminals have somehow tapped into the Global Positioning System (GPS) tracking devices that are in the victims’ cell phones. At one point during the show, Charlie explains how the process of trilateration is used by GPS receivers to pinpoint the exact location of someone on Earth.

## Part I: How 3-Dimensional Trilateration Works

A GPS receiver calculates its distance to a satellite, knowing the amount of time it takes to receive a transmission from the satellite as well as the speed of the radio transmission (186,000 miles per second). Because the position of the satellite is known, it can be determined that the GPS receiver is located somewhere on a sphere whose radius is equal to the distance between the GPS receiver and the satellite. These calculations are repeated with 3 more satellites. The intersection of the 4 spheres is a unique point, which corresponds to the position of the GPS receiver.

## Part II: Using 2-Dimensional Trilateration to Locate a Point

In this activity, students will learn how to do 2-dimensional trilateration. This is similar to the situation described above, with the exception that students will be locating a single point on a coordinate plane. Students will need to find the intersection point of three circles (rather than 4 spheres) as indicated by the diagram below.



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**1. a.** Students should work individually during this part of the activity.

**b.** Launch TI-Navigator on the computer and press **Begin Class** to start the session.

**c.** Have students log into `NavNet` on their calculators. Log into `NavNet` as a teacher as well.

**2. a.** Load the ***Trilateration.act*** activity settings file into Activity Center. Activity Center is set up so that students can submit 1 point. They will be able to **Mark** the point but it will not be displayed in Activity Center until the **Send** command is selected.

**b.** Press **Start Activity** to begin the activity. Instruct three students to enter Activity Center and mark a point anywhere in the *xy* plane. These three points represent the three satellites.

**c.** As the teacher, you will also enter Activity Center and pick a fourth point anywhere in the *xy* plane. This point represents the GPS receiver. Do not plot the point until the conclusion of the activity. The challenge will be for students to determine the location of the GPS receiver.

**3. a.** Click the 'List-Graph' tab to display the coordinates of each of the three student points (the satellites).

**b.** Use the distance formula to calculate the distance from each satellite to the GPS receiver (the unmarked point). For example, if one satellite is located at (–3.75, 2.25) and the GPS receiver is at (0.5, –1.75), then the distance between these two points is $\sqrt{(-3.75-0.5)^2+(2.25-(-1.75))^2} \approx 5.836$.

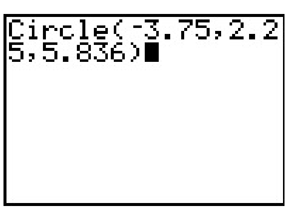
**c.** Write the three distances on the board for student reference. Make sure it is clear which distances correspond to which satellites.

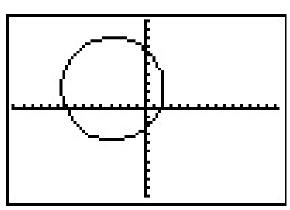
**4. a.** Have the remainder of the class exit out of `NavNet` and work from the home screen. Have these students press [2nd] [QUIT] to return to TI-Navigator Home and to exit `NavNet` select `4:EXIT APP`.

Note: Have the three students who located the satellites work with a partner for the remainder of the activity.

**b.** Have these students press [ZOOM] **6:ZStandard** and [ZOOM] **5:ZSquare** to configure their window to match the window in Activity Center.

**c.** From the home screen, have students use [2nd] [DRAW] **9:Circle** to draw three circles. For each circle, the center is the location of the satellite and the radius is the distance between the satellites and the GPS receiver. The screen below shows how to draw a circle centered at (–3.75, 2.25) with radius 5.836.





**d.** Students should notice that the three circles intersect at one specific point, which is the location of the GPS receiver.



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**e.** Have students use the arrow keys to approximate the location of this point of intersection. Their coordinates can be rounded to the nearest 0.25 since this matches the size of the step in Activity Center.

**5. a.** After the students have found the point of intersection, have them log back into `NavNet` and press [1] `Activity Center.`

**b.** The activity that was originally started in Step 2 will still be running. Students can now mark and send the point where they think the GPS receiver is located.

**c.** After all the student points are submitted, show the location of the GPS receiver by sending the point that was previously marked on the teacher's calculator. This marked point will appear in green. Students' points should be close to or match this point exactly. Discuss with students any possible reasons for differences in locations.

**d.** Press [■ **Stop Activity**].

**6.** This activity can be repeated several times, with different students marking the location of the satellites. Be sure to clear the activity data before repeating the activity.



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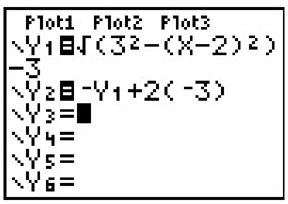
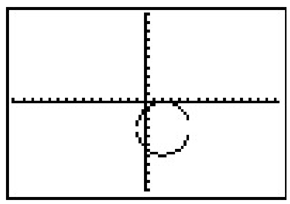
***The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.***

## Extensions

- You can also use the [Y=] editor to graph the three circles.
  **a.** To graph a circle of the form $(x-h)^2+(y-k)^2=r^2$, solve the equation for $y$ to get the equation $y=\pm\sqrt{r^2-(x-h)^2}+k$. To graph this circle in the [Y=] editor, use two equations. Enter the equation for the top half of the circle in $Y_1$ and enter $-Y_1+2k$ in $Y_2$. The example below shows how to graph a circle centered at (2, –3) with a radius of 3.



  **b.** After students graph the three circles, they will have the option of using [2nd] [CALC] **5:intersect** to find the point of intersection of the three circles.

- Additional *NUMB3RS* activities for "Convergence" can be downloaded for free from the Web sites listed below.

  **http://www.cbs.com/primetime/numb3rs/ti/activities/Act1_1plus1equals2_Convergence_final.pdf**

  **http://www.cbs.com/primetime/numb3rs/ti/activities/Act2_TheDatingGame_Convergence_final.pdf**

  **http://www.cbs.com/primetime/numb3rs/ti/activities/Act3_AirHockey_Convergence_final.pdf**

  **http://www.cbs.com/primetime/numb3rs/ti/activities/Act4_Whereisthebullet_Convergence_final.pdf**

- If you would like to learn more about TI-Navigator, visit **http://education.ti.com/navigator**.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