Circumference and Area of a Circle

Overview

Students explore how to derive pi (π) as a ratio. Students also study the circumference and area of a circle using formulas.

Math Concepts

- numbers and operations
- two-dimensional geometry

Materials

- TI-30XS MultiView™

Activity

Demonstrate how finding the “perimeter” of a circle is different than finding the perimeter of a polygon.

Given a square and a ruler, can you find how far it is around the square? That’s called the "perimeter." Now, given a circle and a ruler, can you tell me how far it is around the circle? Try it. Why is this not a good approach?

The students should realize it’s very difficult to get an exact answer using a ruler, because a ruler is straight and a circle is round. It is a good idea to mention that in their later math studies, they will see it is possible to measure curves with straight lines, but at this level, it is not possible. Now, ask them how it would be possible to measure the distance around a circle.

If we had a flexible tape measure, we could easily measure around a circle. (Show this using a 3D object such as a lid.) *However, it’s not often that we have access to tape measures in math class, plus, it’s difficult to get a precise measurement, especially on a two-dimensional circle drawn on a piece of paper. That means that we really need a better method to find the distance around a circle. Let’s figure that method out.*

Put students in groups, and give each group a tape measure and two circular objects such as lids or canisters. Have students measure the distance around each object and then the diameter. Instruct students to try four things with their results: add, subtract, multiply, and divide them.

Now, share your results. When you added, subtracted, multiplied, and divided, did you see anything interesting? Is there any number that shows up repeatedly? Does everyone see 3.(something)…?

Anytime we measure around a circle, that’s called the "circumference." The distance across a circle is called the "diameter." Any time we take the circumference and divide it by the diameter, we get the number 3.14159…. That’s called pi, which is written π.

Pi is a tremendously important number in mathematics. Since we found above that $\frac{C}{d} = \pi$, we also can prove through algebra that $C = \pi d$.

What students will need help understanding is that while measuring the distance around a circle (the circumference) is difficult, as mentioned before, now it’s not necessary since there is a formula.

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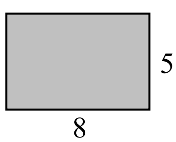
Let's see how this works. Earlier, we measured around this lid and got a circumference, or distance around, of almost $9\frac{1}{2}$ inches. If we hadn't had a tape measure, we would have needed to use the formula we just found. Since the distance across was 3 inches, we can find that $C = \pi d$, so $C = \pi(3)$, so $C = 3\pi$. Using our calculator, we see that $3\pi \approx 9.42$ inches, which makes sense, as we recorded a measurement of about 9.5 inches.

Now introduce the formula for area of a circle, and show students why, again, π is a crucial part of finding the area. Use a polygon such as a rectangle to show how you can measure and count squares to find the area but how that won't work on a circle, since the perimeter is curved.

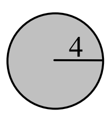
(Diagrams are included at the end of this activity for use on the projector.)

We could take this figure and find the area a few different ways. We know its dimensions, so we could easily divide the rectangle into equal-sized squares, and count them. We could also use the formula $A = lw$.

Either way, we'll find that $A = 40$ units^2.



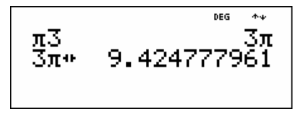
However, could you use the same process on this circle?



No, because there's no way to divide it into equal-sized squares. Again, we need the number π. The area of a circle can be expressed as $A = \pi r^2$, where r is the radius.

Follow these steps:

1. Press $\boxed{\pi}$ 3 **enter** to input the information.
2. Now press ◂▸ to see the approximate answer
3. The screen should show this:



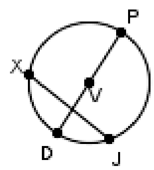
4. Press ◂▸ one more time to see 3π again.
5. The calculator can toggle between the more precise answer of 3π and the approximation.

Circumference and Area of a Circle

Name ____________________

Date ____________________

Directions: Use your TI-30XS MultiView™ calculator to find the area and circumference of each circle. Use the correct units.

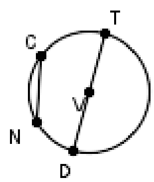
1. 

VP = 6 mm

Use exact units.

C = ______

A = ______

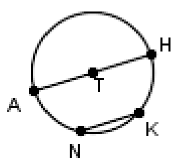
2. 

DT = 4 in.

Round to the nearest hundredth.

C = ______

A = ______

3. 

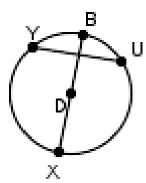
NK = 11 cm

AT = 12 cm

Round to the nearest tenth.

C = ______

A = ______

4. 

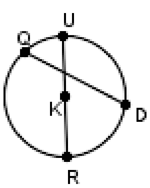
XB = 8.5 in.

UY = 6 in.

Use exact units.

C = ______

A = ______

5. 

QD = 15 mm

KU = 9 mm

Find the exact circumference and the area of *half* the circle.

C = ______

A = ______

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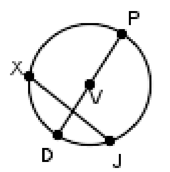
Answer Key

Directions: Find the area and circumference of each circle. Use the correct units.

1.



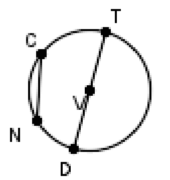
VP = 6 mm

Use exact units.

C = 12π mm

A = 36π mm^2

2.



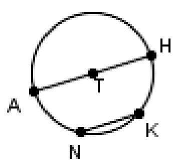
DT = 4 in.

Round to the nearest hundredth.

C = 12.57 in.

A = 12.57 in^2

3.



NK = 11 cm

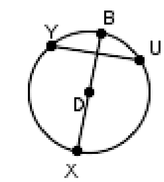
AT = 12 cm

Round to the nearest tenth.

C = 75.4 cm

A = 452.4 cm^2

4.



XB = 8.5 in.

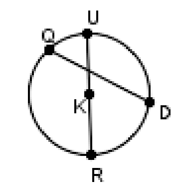
UY = 6 in.

Use exact units.

$C = \dfrac{17\pi}{2}$ in.

$A = \dfrac{289\pi}{16}$ in^2

5.



QD = 15 mm

KU = 9 mm

Find the exact circumference and the area of *half* the circle.

C = 18π mm

$A = \dfrac{81\pi}{2}$ mm^2

Circumference and Area of a Circle

Transparency 1

Circumference and Area of a Circle

Transparency 2