About the Lesson

In this activity students will explore the relationship among coordinates of points and locations on the coordinate plane, the relationships of lines with their equations, slopes and *y*-intercepts, and lastly, the slopes of parallel and perpendicular lines. As a result, students will:

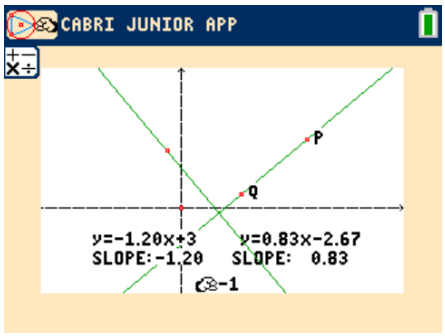
- Graph an equation of the form $y = mx + b$ and determine properties such as slope and *y*-intercept.
- Graph an equation of the form $y = mx + b$ and describe how m and b change as the graph of the line is rotated and translated.
- Understand that the coordinates of a line are the solution set to the equation of the line.
- Show that lines with equal slope are parallel.
- Graph lines whose slopes are negative reciprocals.
- Deduce that the slopes of perpendicular lines are negative reciprocals.

Vocabulary

- parallel
- perpendicular
- reciprocal
- slope

Teacher Preparation and Notes

- This activity can review coordinates and then go on to introduce or review the relationships of points, lines, slopes, and equations. The graph could also be divided into different zones by graphing $f(x) = x$ and $f(x) = -x$ and having students generalize the slope for each of these zones.
- After graphing lines and looking at relationships, another point could be placed on the plane and students could look at the coordinates of the point with respect to the line. This could be used to introduce inequalities.



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus C Silver Edition. It is also appropriate for use with the TI-84 Plus family with the latest TI-84 Plus operating system (2.55MP) featuring MathPrint™ functionality. Slight variations to these directions given within may be required if using other calculator models.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Compatible Devices:

- TI-84 Plus Family

Software Application:

- Cabri™ Jr.

Associated Materials:

- PointsLinesSlopes_Student.pdf
- PointsLinesSlopes_Student.doc
- PARALLEL.8xv
- PERPENDI.8xv

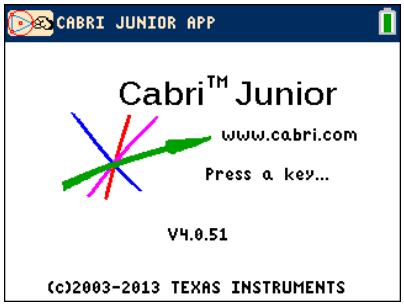
Tech Tip: Before beginning the activity, the files PARALLEL.8xv and PERPENDI.8xv need to be transferred to the students' calculators via handheld-to-handheld transfer or transferred from the computer to the calculator via TI-Connect.

Three focus questions define this activity. (Some discussion should follow the posing of these questions):

1. *What are the relationships of the coordinates of points with various locations in the Cartesian plane?*
2. *What is the relationship of a line with its slope, equation, and y-intercept?*
3. *What is the relationship between the slopes of parallel or perpendicular lines?*

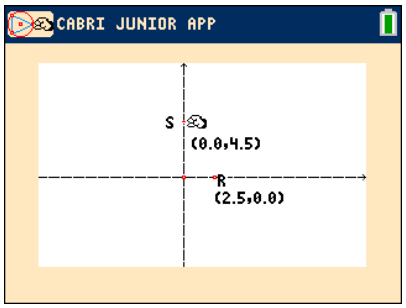
Problem 1 – Coordinates of Points

Have students open the *Cabri™ Jr* application by pressing APPS. Open a new file (Y= for F1) and make sure the axes are displayed (press GRAPH for F5 and select **Hide/Show > Axes**). Note, to undo press **F1 > Undo**.



Students should place a point, *R*, on the *x*-axis and a point, *S*, on the *y*-axis. Use **F2 > Point > Point On**. Press **F5 > Alpha-Num** and ENTER to label the points. When finished with a tool press CLEAR.

Have students display the coordinates of the points. **Use F5 > Coord & Eq** and move the cursor to a point until the point is flashing. Press ENTER to select that point, then have students move the cursor to where they want the coordinate to remain and press ENTER again.



To grab a point that is flashing, press ALPHA. To let go of a point press CLEAR. This works as Escape.

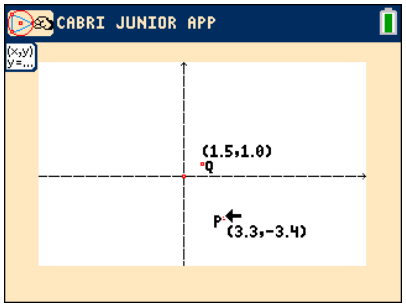
1. Explain what is common to all points on the *x*-axis.

Answer: For any *x*-value, the *y*-value is zero.

2. Explain what is common to all points on the *y*-axis.

Answer: For all *y*-values, the *x*-value is zero.

Have students delete points *R* and *S* by moving the cursor to that location and pressing DEL. Place two points, *P* and *Q*, in the top right quadrant. Drag the points around into different quadrants.



Complete the sentences by writing *positive* or *negative*.

3. A point is in Quadrant 1 (top right) when its *x*-coordinate is *positive* and its *y*-coordinate is *positive* .

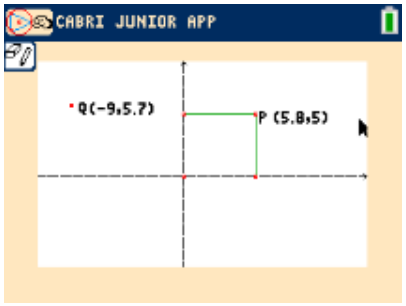
4. A point is in Quadrant 2 (top left) when its *x*-coordinate is *negative* and its *y*-coordinate is *positive* .

5. A point is in Quadrant 3 (bottom left) when its *x*-coordinate is *negative* and its *y*-coordinate is *negative* .

6. A point is in Quadrant 4 (bottom right) when its *x*-coordinate is *positive* and its *y*-coordinate is *negative* .

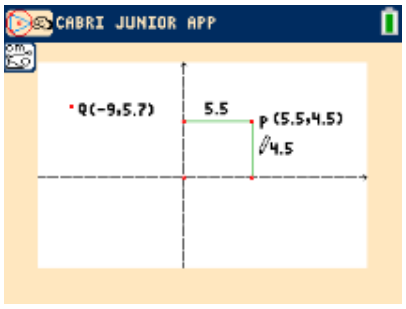
Teacher Tip: Students should now be able to make sense of exactly what the coordinates should be in different quadrants as well as on each axis. Give students the coordinates of a point and ask which quadrant it is in and visa versa. This simple activity is good for students to develop their technology skills and establish sign patterns for the four quadrants.

Have students use the **Perpendicular** tool (ZOOM **> Perp.**) to construct perpendicular lines through point *P* to each axis. After the perpendicular lines are in place, direct students to construct segments from point *P* to each axis (WINDOW **> Segment**) and then hide the "excess" perpendicular lines using the **Hide/Show** tool.



To select the intersection point of the perpendicular line and the axis using the **Segment** tool, students should select both lines so that they flash. They students should press ENTER.

Now have students measure the length of each segment using the **D. & Length** tool from **Appearance > Measure** menu. Drag point *P* and have students conjecture about the distances and the coordinates.

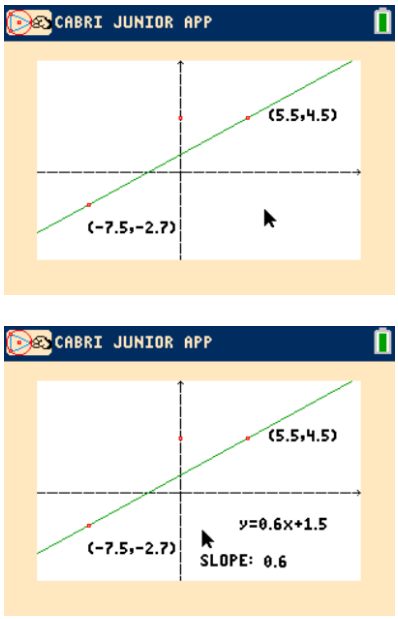


7. What is this relationship between the coordinates of point P and the distances to each axis?

   Answer: The distance from *P* to each *x*-axis is the absolute value of the *y*-coordinate. The distance from *P* to each *y*-axis is the absolute value of the *x*-coordinate.

Problem 2 – Lines, Equations, and Slopes

Delete or hide the segments and measurements. Ask students to visualize a line between points *P* and *Q* and conjecture as to whether the slope is positive or negative, small or large, etc. Then use F2 to draw a line connecting *P* and *Q*. They will use the **Line** tool from the **Creation** menu. Next, using tools from the F5 Appearance menu to display the equation and slope of the line. As they find the slope (GRAPH **> Measure > Slope**) and equation (GRAPH **> Coord. & Eq.**) of the line they can label the slope measurement as shown to the right by choosing the **Alph-Num** tool and typing **SLOPE:** in the appropriate place.



If this is a student's first introduction to slope, encourage students to discover the relationship of the displayed points and the slope. They should discover the slope of a line connecting two points is the change in the *y*-values of the points divided by the change in the *x*-values of the points.

Have students place a point on the line and then drag the point along the line. They should note the coordinates of several points along the line.

8. Place a point on the line. Drag the point along the line and record several coordinates of points. How do the coordinates relate to the equation of the line?

Answer: Coordinates will vary. Each ordered pair is a solution to the equation of the line.

Have students look for relationships between the slope and equation as they change the line by grabbing and dragging point *P* as well as grabbing and dragging the line itself.

Press + as you hover over a value to display more digits. Students should attend to precision and understand that discrepancies may be due to the number of digits shown throughout this activity.

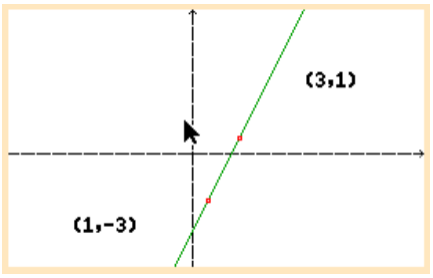
9. When dragging the line by a point, what is the relationship between the points and the slope?

Answer: The slope is the change in the *y*-values divided by the change in the *x*-values

10. When dragging the line by a point, what is the relationship of the slope and the equation?

Answer: The slope changes. The coefficient in front of the variable *x* changes.

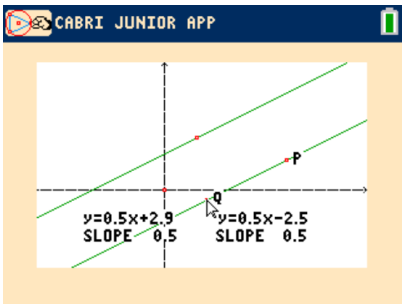
11. To the right is a graph with two points labeled. Consider the line through these points. Then, consider the graph of the equation $y = \frac{1}{2}x + 2$. Show you work and explain how the two lines compare. Especially consider the slope and *y*-intercepts.



Answer: The slope of the graph is (1 – (–3))/(3 – 1) = 4/2 = 2. This is a larger slope than the equation which has a slope of ½. The graph is much steeper than the equation. The *y*-intercept of the equation is positive 2, whereas the *y*-intercept of the graph is a negative number. From the equation of the graph, $y + 3 = 2(x - 1)$ or $y = 2x - 5$, it can be seen the *y*-intercept is –5.

Problem 3 – Slopes of Parallel and Perpendicular Lines

Open the Cabri™ Jr. file **PARALLEL**. Drag the lines by points P and Q and examine the slopes.

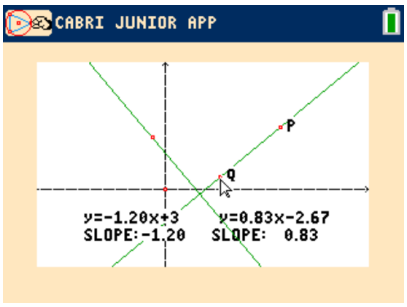


12. What can you say about the slopes of two parallel lines?

Answer: The slopes of the two parallel lines are equal.

Students are given two parallel lines, their equations, and their slopes. (This construction is done for them. Alternatively, students can construct the parallel lines, show the slopes, and explore.)

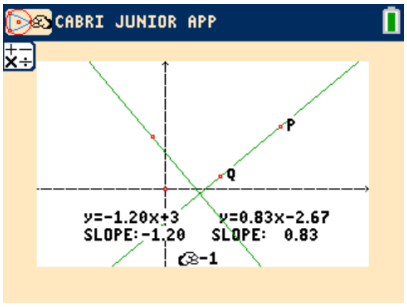
Open the Cabri™ Jr. file **PERPENDI**. Again, have students drag the lines to investigate the relationship between the slopes.



13. What can you say about the slopes of two perpendicular lines?

Answer: Students will observe that when one slope is positive, the other slope is negative. They may need to continue exploring to discover that the slopes of the perpendicular lines are the opposite reciprocals of each other.

Another way to look at the relationship of the slopes is to find the product of the two slopes. Have students select GRAPH **> Calculate**. Next, they should position the cursor over the first slope, and press ENTER. Students should then press × to specify the operation and move the cursor to the second slope and press ENTER again. This will yield the product of the two slopes—have students drag it to a convenient location on the screen. Now, change the lines by grabbing and dragging point *P*.



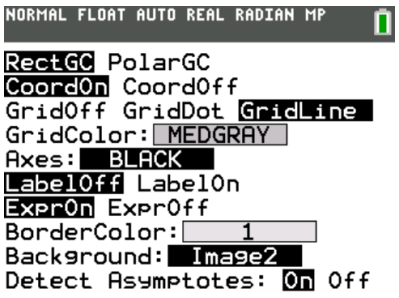
14. What do you observe about the product of the slopes?

Answer: The product is always -1. The slope of perpendicular lines is the opposite reciprocal of each other.

Extensions

As stated in the teacher preparation notes, there are many extensions to this activity depending on the level of the student. This activity can easily be manipulated to lead students into a deeper study of slopes, slopes of parallel lines and slope of perpendicular lines.

Either with a student TI-84 Plus C or the TI-SmartView TI-84 Plus C, turn on Background Image2 from the Format Graphs screen, 2nd ZOOM and view that graph with a decimal window. Press ZOOM ZDecimal.



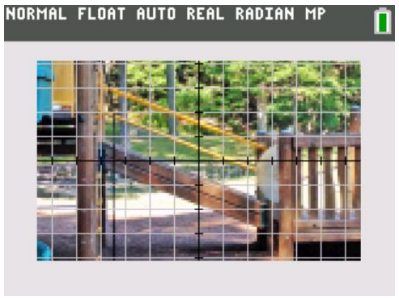
Ask students for coordinates along a line.

For example, (-3, 0) and (2, -2). They can use the point slope formula to write an equation of the line.

$$y - 0 = \frac{-2-0}{2-(-3)}(x-(-3))$$

$$y = \frac{-2}{5}(x+3)$$

$$y = \frac{-2}{5}x - \frac{6}{5}$$



Then, have them graph a line parallel to that equation.