NUMB3RS Activity: SIR Models
Episode: "Vector"

Topic: Related rates and using formulas **Grade Level:** 9 - 12
Objective: Students will use related rates to compute the effects of an epidemic and analyze results.
Time: 30 minutes
Materials: TI-83 Plus/TI-84 Plus graphing calculator and the spreadsheet and lists described in the Introduction below.

Introduction

When an unknown contaminant causes an increasing number of deaths in Los Angeles, the FBI calls in Charlie to find the source of the outbreak. In determining the source, Charlie uses an SIR model to make predictions about the epidemic and to make judgments on its strength. This activity will focus on developing the SIR Model and applying it to a population. It will also make use of the calculator to store lists and create scatter plots for analysis.

To download the spreadsheet that was used to create the tables in the student activity, as well as the lists that the students will analyze in the activity, go to **http://education.ti.com/exchange** and search for "7720."

Discuss with Students

Epidemic outbreaks are very difficult to study because of the multiple variables and parameters associated with them. However, a breakthrough came in 1927 when Kermack and McKendrick divided the affected population into three states: **S**usceptible, **I**nfected, and **R**ecovered, where people in the population move from susceptible to infected to recovered. This became known as an **SIR** model. Some SIR models are very complex, but the SIR model in this activity will be simplistic. This model will assume the population is in a closed area (no one enters or leaves), the population is fixed (no births or natural deaths), and the disease is transmitted only by direct individual contact. Emphasize to students that the values predicted by the SIR model are a product of the mathematics, not actual reality (that is, the model may predict an impossible value like 3.1 people will be infected on day 2).

In explaining the SIR formulas used in this activity (listed below), remind them not everyone who is infected will spread the disease to each person they come into contact with. They will only spread it to a factor (a) of those they come into contact with. That is why the rate of change for S is $-aSI$. The rate of change for those infected (I) will be increased by those who have just become infected (aSI), and decreased by those who have recovered from the disease (bI)

$$\begin{aligned} S' &= -aSI \\ I' &= aSI - bI \\ R' &= bI \end{aligned}$$

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Student Page Answers:
1. *Because 5 people who were infected recovered* **2.** *S' = –33.75, I' = 27.6 and R' =6.15.*
3.

Day	*S*	*I*	*R*	*S*'	*I*'	*R*'
0	1400.00	100.00	0.00	-28.00	23.00	5.00
1	1372.00	123.00	5.00	-33.75	27.60	6.15
2	1338.25	150.60	11.15	-40.31	32.78	7.53
3	1297.94	183.38	18.68	-47.60	38.43	9.17
4	1250.34	221.81	27.85	-55.47	44.38	11.09
5	1194.87	266.19	38.94	-63.61	50.30	13.31
6	1131.26	316.49	52.25	-71.61	55.78	15.82
7	1059.65	372.28	68.07	-78.90	60.28	18.61
8	980.75	432.56	86.69	-84.85	63.22	21.63
9	895.91	495.78	108.32	-88.83	64.05	24.79
10	807.07	559.82	133.10	-90.36	62.37	27.99

4. *The rate changes of all three groups are growing because the number of infected people is growing larger resulting in more people spreading the virus*. **5.** *Day 18, 847 people were infected*. **6.** *Day 16*

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Name: ________________________________ Date: ________________

NUMB3RS Activity: SIR Models

When an unknown contaminant causes an increasing number of deaths in Los Angeles, the FBI calls in Charlie to find the source of the outbreak. In determining the source, Charlie uses a model called SIR to make predictions about the epidemic and to make judgments on its strength.

Epidemics have devastating effects on society. The 1918 influenza epidemic killed between 20 and 40 million people worldwide, and the Ebola virus routinely has outbreaks in Africa for unknown reasons. Trying to predict what would happen in outbreaks can be a difficult task because of the way these pathogens spread. To better understand epidemics, SIR models were developed. In fact, the World Health Organization collects and analyzes data constantly in its efforts to detect disease spread.

An **SIR** model assumes that each person in a population is in one of three groups:

- **S**usceptible (can be infected)
- **I**nfected (has the disease)
- **R**ecovered (has built up immunity to the disease or has died from it)

As people are infected, they move from being Susceptible to Infected, and as people recover they move from Infected to Recovered.

The SIR model in this activity assumes

- The population is in a closed area (no one enters or leaves)
- The population is fixed (no births or natural deaths)
- The disease is transmitted only by direct, individual contact.

In thinking over the model, it becomes apparent that the various rates of each of these groups must continually change as people enter and leave them. In other words, the rate of people becoming infected (I') must increase as more people begin to carry the disease and infect others. Also, the rate of people not affected by the disease (S') as well as the people recovered from the disease (R') would also change over time.

For that reason, the rates of each of the groups can be mathematically modeled by

$$\begin{aligned} S' &= -aSI \\ I' &= aSI - bI \\ R' &= bI \end{aligned}$$

where a is the transmission coefficient and b is the recovery coefficient.
For example, suppose in a school of 1,500 students, 100 students are infected with the flu. Assume the recovery coefficient is 1/20 (it takes 20 days to recover) and the transmission coefficient is 0.0002 (this is the probability of a person transmitting the disease in one day).

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Because 100 students are infected, for day 0, S = 1400, I = 100 and R = 0.

Using the mathematical terms described earlier, S' = –28, I' = 23 and R' = 5. This means that 28 students became infected while 5 students recovered.

Day	S	I	R	S'	I'	R'
0	1,400	100	0	–28	23	5

1. If 28 students are no longer susceptible, why doesn't the rate change for infected (I') change to 28?

To compute the values for S, I and R for day 1, add their corresponding rate changes (S', I', and R') from day 0 to them.

2. Using the new values for S, I and R recalculate S', I', and R'. It is important to remember this is a mathematical model. To remain as close to the model as possible, numerical accuracy is essential. Even though it does not make sense to have 11.15 people recovered, it is necessary.

Day	S	I	R	S'	I'	R'
0	1,400	100	0	–28	23	5
1	1,372	123	5			

3. Complete the chart for the first 10 days of the outbreak.

Day	S	I	R	S'	I'	R'
0	1,400	100	0	–28	23	5
1	1,372	123	5	-33.75	27.6	6.15
2	1338.25	150.6	11.15	-40.31	32.78	7.53
3						
4						
5						
6						
7						
8						
9						
10						

4. What is happening to the rate changes (S', I' and R') and why?

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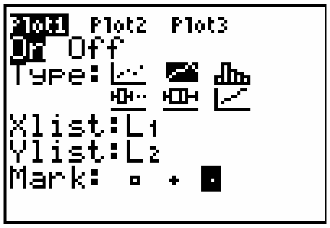
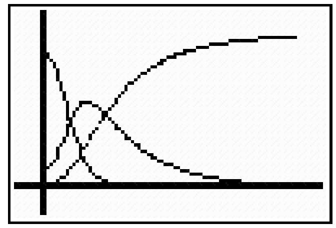
With the data list it is possible to simultaneously view the graphs of susceptible, infected, and recovered. Your teacher has data lists of *S*, *R* and *I* for the first 100 days. Using the data in the graphing calculator lists L_1, L_2, L_3, and L_4 from your teacher, produce scatter plots for *S*, *I* and *R* each vs day following the directions below. The ordered pairs for the scatter plots will be the number of individuals (Susceptible, Infected or Recovered) vs. day of the outbreak.



Press [2nd] [STAT PLOT] and select **Plot1**. Set your calculator to match the settings shown above. This will produce the susceptible graph. Do the same with the other stat plots, but set the **YList** to L_3 for infected and L_4 for recovered.



Press [ZOOM] and select **9:ZoomStat** to view the plots of *S* (Plot 1), *I* (Plot 2) and *R* (Plot 3). Press [TRACE] and use the arrow keys to navigate the three plots.

5. Using the trace button, decide what day has the largest number of infections and how many.

6. When does the number of recovered people exceed those not infected?

The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research

Extensions

Activity: SIS Models

Introduction

SIR models cannot be used to model all epidemics. Some pathogenic infections (like the common cold) behave differently – a person with a cold does not gain immunity after the pathogen runs its course. Therefore a person will never become recovered, only susceptible to the same cold again. This type of situation requires an SIS model which can be mathematically modeled by

$$\begin{aligned} S' &= -aSI + bI \\ I' &= aSI - bI \end{aligned}$$

Use the population of your school to model this. Assume a transmission coefficient of 0.0002 and a recovery coefficient of 1/10. How many students are infected by the end of the week if 15 of them were infected at the start? Vary the numbers and recalculate.

Additional Resources

Learn more about using computer simulations of infectious diseases as well as view videos of the simulations at the Web site below:
http://www.jhsph.edu/publichealthnews/press_releases/2005/cummings_midas.html#video

The Web site **www.nctm.org/mathhereandnow/pandemic** contains an applet that simulates the spread of disease and allows students to change some key factors (number of contacts, days infectious, probability of contraction).

For the Student

If epidemiology is an area of interest, form a team and compete in the Young Epidemiology Scholars Competition (first prize is $50,000).
http://www.collegeboard.com/yes/index.html

Related Topic

To download a *NUMB3RS* activity that uses TI Navigator™ to simulate the spread of a pathogen in real time, go to **http://education.ti.com/exchange** and search for "7709."

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