

“LET’S INTEGRATE ALL THINGS EASY”

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(Please feel free to email me questions and /or comments.)

Key Topic: Integrals

Abstract:

This activity is designed to help students deal with the method of integration by parts. In this method, students usually have trouble figuring out what they should set u equal to. The title of this activity will, in most situations, answer that question.

Prerequisite Skills:

- Brief introduction to the method of integrating by parts
- Ability to evaluate derivatives
- Ability to integrate simple integrals
- Ability to evaluate integrals using a TI-89 (a review of this technique is included in this activity)

Degree of Difficulty: Easy to moderate

Needed Materials: TI-89

NCTM Principles and Standards:

- Content Standards – Algebra
 - Represent and analyze mathematical situations and structures using algebraic symbols
 - Use mathematical models to represent and understand quantitative relationships
 - Draw a reasonable conclusion about situation being modeled
- Process Standards
 - Representation
 - Connections
 - Problem Solving

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The method of integration by parts is governed by the formula

$$\int u \, dv = uv - \int v \, du .$$

When using this method, you set u equal to one part of the integral, and then set dv equal to the rest of the integral.

But are you having trouble figuring out what to set u equal to? Well, in *most* situations, the title of this activity supplies you with the answer.

How is this possible? Well, the title of this activity supplies you with an easy to remember mnemonic which tells you, in *most* situations, what you should set u equal to. Look at the first letter of each word in the title: **LIATE**. Mathematically these stand for:

L	logarithms
I	inverse functions
A	algebraic expressions
T	trigonometric functions
E	exponential functions

Set u equal to the type of expression which appears first in this list, and set dv equal to the rest of the integral.

EXAMPLE 1. Evaluate $\int x \ln x \, dx$.

We have an Algebraic expression x , and a Logarithmic expression $\ln x$. Since **L** comes before **A** in our mnemonic, we should set u equal to the Logarithmic expression $\ln x$. So the solution is:

$$\begin{aligned} &\text{With } u = \ln x \text{ and } dv = x \, dx \\ &\text{We see that } du = \frac{dx}{x} \text{ and } v = \frac{1}{2} x^2 \end{aligned}$$

So from the parts formula $\int u \, dv = uv - \int v \, du$

$$\begin{aligned} \text{We get } \int x \ln x \, dx &= (\ln x) \left(\frac{1}{2} x^2 \right) - \int \left(\frac{1}{2} x^2 \right) \frac{dx}{x} \\ &= \frac{1}{2} x^2 \ln x - \frac{1}{2} \int x \, dx \\ &= \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C \end{aligned}$$

EXAMPLE 2. Evaluate $\int xe^x dx$.

x is Algebraic and e^x is Exponential. **A** comes before **E** in the mnemonic **LIATE**.

So set $u = x$ and $dv = e^x dx$.

This gives $du = dx$ and $v = e^x$.

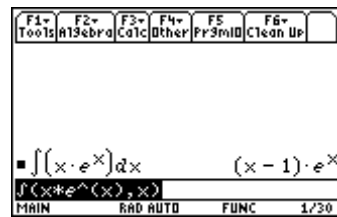
So $\int xe^x dx = xe^x - \int e^x dx = xe^x - e^x + C$

EXAMPLE 3. Evaluate the integral in example 2 using a TI-89.

The keystrokes required to evaluate this integral are:

$\boxed{F3} \boxed{2} \boxed{X} \boxed{X} \boxed{\blacklozenge} \boxed{[e^x]} \boxed{X} \boxed{)} \boxed{,} \boxed{X} \boxed{)} \boxed{ENTER}$.

Note: The TI-89 omits the constant of integration.



EXERCISES:

1. Use integration by parts to evaluate each of the following integrals.

a. $\int x \cos x dx$

b. $\int \ln x dx$

c. $\int x^2 \ln x dx$

d. $\int e^x \sin x dx$

2. Check your answers to #1 using the TI-89.