Student Handout Name _____

Dog Pen Problem

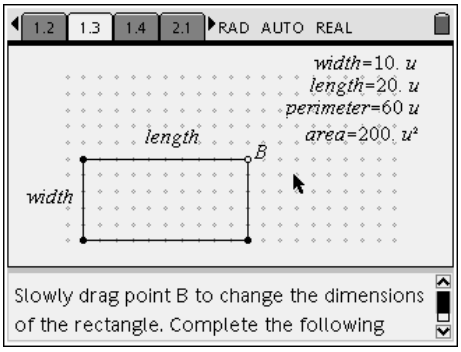
Problem 1—Launch the Problem

A farmer wants to make the largest possible rectangular pen for his dogs. He has 60 feet of fencing. What is the largest area the pen can have? What should the length and width of the pen be?

Slowly drag point B to change the dimensions of the rectangle.

Note: if you get "dependant object locked" message press and release the NavPad to make the point move, do not hold it down.

Complete the following table for at least 6 different locations of point B.



Width	Length	Perimeter	Area

Questions

1. What happens to the perimeter as you drag point B?

2. What happens to the area as the width increases?

3. What happens to the area as the width decreases?

4. From your observations, what conjecture(s) can you make about the maximum area?

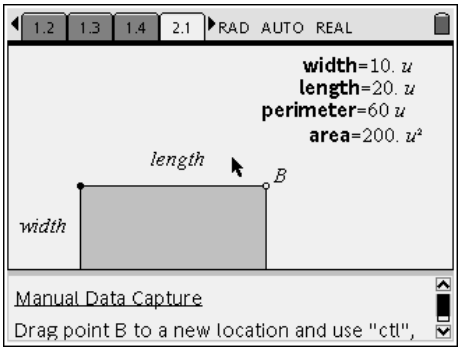
Problem 2— Find the Rectangle with Maximum Area

In this problem you continue to explore the following problem:

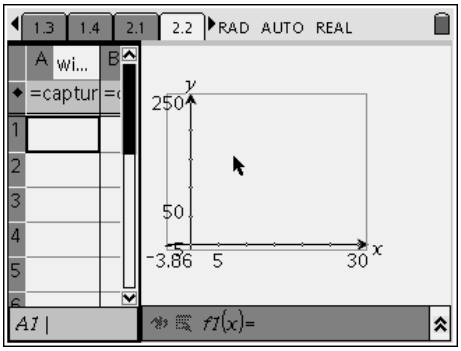
A farmer wants to make the largest possible rectangular pen for his dogs. He has 60 feet of fencing. What is the largest area the pen can have? What should the length and width of the pen be?

However, in this problem you will use the TI-Nspire CAS to manually collect data in a spreadsheet, make a scatterplot of the data, and make observations based on these representations.

In page 2.1, slowly drag point B to a new location and to press ctrl . to manually capture the rectangle's width, length, perimeter, and area data for that location of point B. Repeat this process to capture at least 10 different locations for point B.



Each new location of point B captured will generate another row of data in the spreadsheet and plot the (length, area) data in a scatterplot (page 2.2).



Questions

1. What shape does the scatterplot on page 2.2 appear to be?

2. Find the maximum area using as many different methods as you can. What should the length and width of the pen be? Explain each of the methods that you used. Be prepared to share your methods with the class.

3. What conjecture(s) can you make about the rectangle that maximizes the area of any pen with a fixed perimeter?

Problem 3—Symbolic Proof

In this problem you continue to explore the following problem:

A farmer wants to make the largest possible rectangular pen for his dogs. He has 60 feet of fencing. What is the largest area the pen can have? What should the length and width of the pen be?

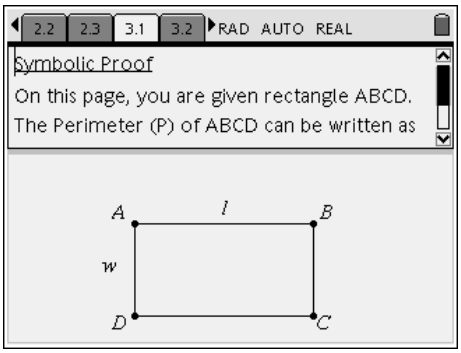
However, in this problem you will use the TI-Nspire CAS to symbolically prove this conjecture:

If a rectangle has a fixed perimeter, then the shape that maximizes the rectangle's area is a square.

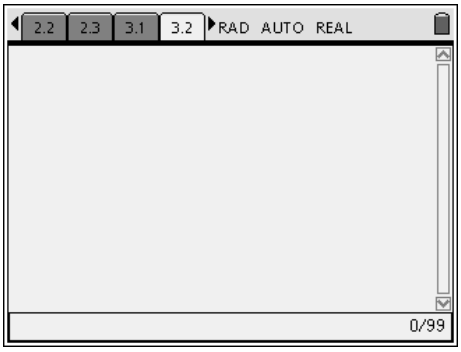
In page 3.1, You are given rectangle ABCD.

The Perimeter (P) of ABCD can be written as P=2w+2l.

The Area (A) of ABCE can be written as A=w*l.



In page 3.2, complete the following process.



a. Solve P=2w+2l for l.
b. Substitute this value of l into A=wl to find an equation for A in terms of the variable (w).
c. Find the 1st derivative of this new equation.
d. Set this new equation equal to zero and solve for w.
e. Substitute this value of w into P=2w+2l and solve for l.

Questions

1. Explain each step of this process.

2. Why did you take the 1st derivative equation, set it equal to zero, and solve for w?

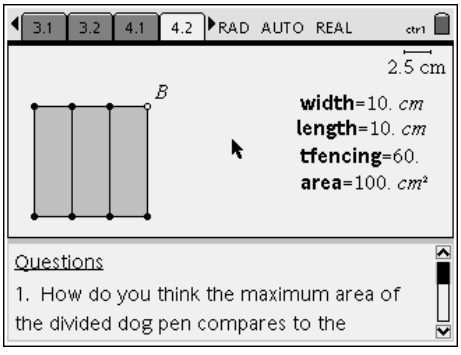
3. Why does this process prove the conjecture?

   If a rectangle has a fixed perimeter, then the shape that maximizes the rectangle's area is a square.

Problem 4—Divided Pen

In this problem you explore the following problem:

The farmer decides he wants to divide his dogs' pen up into three congruent rectangular pens with the fencing running parallel to the width (see page 4.2). He has 60 feet of fencing.



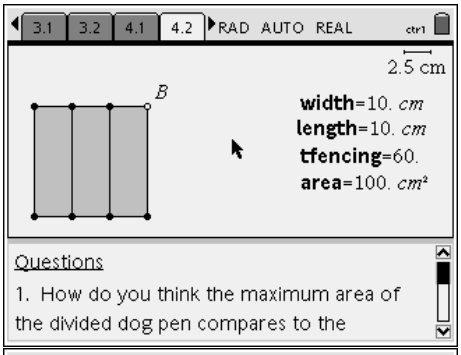
You will use the TI-Nspire CAS to automatically collect data in a spreadsheet, make a scatterplot of the data, and make observations based on these representations.

Questions

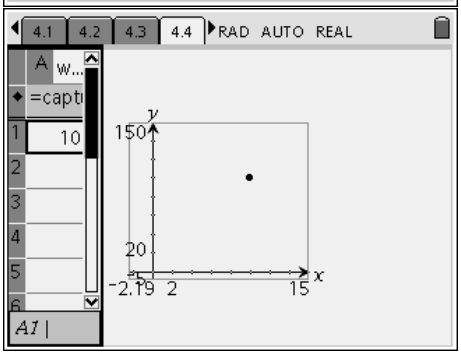
1. How do you think the maximum area of the divided dog pen compares to the maximum area in the dog pen without dividers?

2. How do you think the outer shape of the divided dog pen compares to the outer shape of the dog pen without dividers when the areas are maximized?

In page 4.2, slowly drag point B to a new location to auto capture the width, perimeter, and area of the rectangle. Repeat this process.



Each new location of point B captured will automatically generate another row of data in the spreadsheet and plot the (width, area) data in a scatterplot (page 4.4).



Questions

1. What shape does the scatterplot on page 4.4 appear to be?

2. Find the maximum area using as many different methods as you can. What should the outer length and width of the pen be? Explain each of the methods that you used. Be prepared to share your methods with the class.

3. What conjecture can you make about the rectangle that maximizes the area of the divided pens perimeter rectangle? What should the length and width of the pen be? Explain.

4. How is this conjecture similar and different to the original dog pen conjecture?

Similarities	Differences

5. Find a general rule for the width of a divided pen with a fixed perimeter. Explain.

WARNING: At the end of the activity, do not save the changes. Close the document and answer NO to saving changes. You may want to be able to open up the file again without all the data that you gathered and graphed.